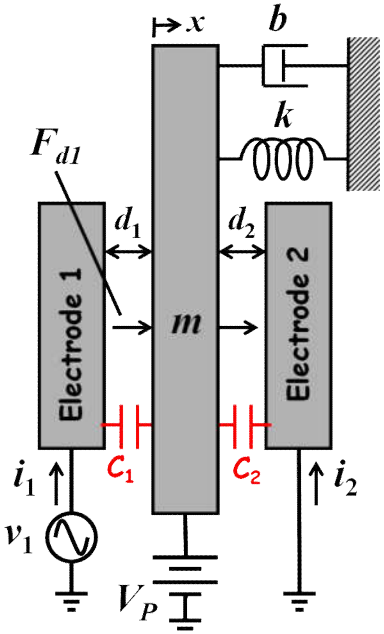
Lecture 23: Mechanical Circuit Analysis & Gyros

- Announcements:
- Module 13 on Equivalent Circuits II online
- Module 14 on Sensing Circuits online
- Module 15 on Gyros, Noise, & MDS
- HW#6 online and due Tuesday, April 17
- Project Slide Set #1 due Friday, April 13
- You do need to be in groups of three
  - Not 2, not 4
  - Part of the learning experience in any project entails how to work with others
- Next Tuesday, April 17, Eta Kappa Nu will show up at the beginning of class to guide you through course evaluations
  - Bring your computer
- ------------------------------------
- Reading: Senturia, Chpt. 6, Chpt. 14
- Lecture Topics:
  - Input Modeling
    - Force-to-Velocity Equiv. Ckt.
    - Input Equivalent Ckt.
  - Current Modeling
    - Output Current Into Ground
    - Input Current
    - Complete Electrical-Port Equiv. Ckt.
  - Impedance & Transfer Functions
- ------------------------------------
- Reading: Senturia, Chpt. 14, Chpt. 16, Chpt. 21
- Lecture Topics:
  - Gyroscopes
- Reading: Senturia, Chpt. 14
- Lecture Topics:
  - Detection Circuits
    - Velocity Sensing
    - Position Sensing
- ------------------------------------
- Last Time:
- In the midst of generating the full equivalent circuit

$\boxed{\text{Input Current Expression}}$ ⇒ then yields complete equivalent input ckt



Get $I_1(j\omega)$!

$$i_1(t) = C_1(x,t)\frac{dv_1(t)}{dt} + V_1(t)\frac{dC_1(x,t)}{dt}$$

$$(V_1(t) = v_1 - V_P) \Rightarrow i_1 = C_1\frac{dv_1}{dt} + (v_1 - V_P)\frac{\partial C_1}{\partial x}\frac{\partial x}{\partial t}$$

↑ f(t)

magnitude of $v_1 = |v_1|\cos\omega t$

$$\therefore\ I_1(j\omega) = \underbrace{j\omega C_1|v_1|}_{\text{Feedthrough Current}} + \cancel{j\omega|v_1|\frac{\partial C_1}{\partial x}X} - \underbrace{j\omega V_P\frac{\partial C_1}{\partial x}X}_{\text{Motional Current due to mass motion}} \longrightarrow *$$

@DC: $x = \frac{F_{d1}}{k} = -\frac{1}{k}V_P\frac{\partial C_1}{\partial x}v_1$

@ resonance: $x = \frac{QF_{d1}}{k} = -\frac{Q}{jk}V_P\frac{\partial C_1}{\partial x}v_1 = X$

↑ 90° phase lag

plug in to *

Thus: (@ resonance) $\rightarrow \omega_0$

$$I_1(j\omega) = j\omega_0 C_1|v_1| + j\omega_0\Big(\overbrace{V_P\frac{\partial C_1}{\partial x}}^{\eta_{e1}}\Big)^2\frac{Q}{jk}|v_1|$$

$$= \underbrace{j\omega_0 C_1|v_1|}_{} + \underbrace{\omega_0\frac{Q}{k}\eta_{e1}^2|v_1|}_{}$$

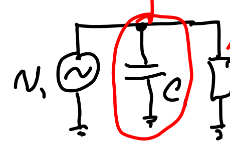
90° phase-shifted from v_1

↙ This is a capacitor in shunt w/ input!

In phase w/ v_1!

↑ This is an effective resistance @ ω_0 seen looking into the electrode!

@ resonance

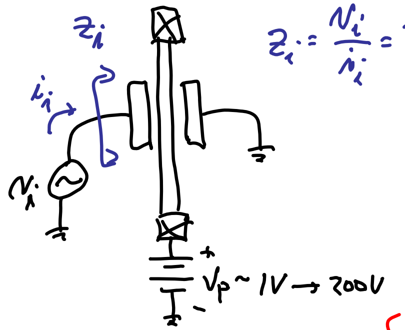


Motional Resistance:

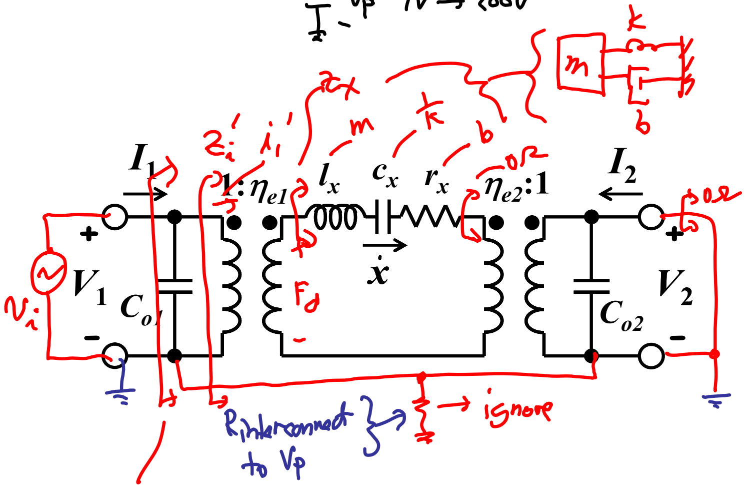
$$R_{x1} = \frac{V_1}{I_1} = \frac{k}{\omega_0 Q\eta_{e1}^2} = \frac{m\omega_0}{Q\eta_{e1}^2} = \boxed{\frac{b}{\eta_{e1}^2} = R_{x1}}$$

The equivalent ckt. better get this right!

Input Impedance Into Port 1



$Z_i = \frac{v_i}{i_i} = ?$



$$Z_i = \frac{v_i}{i_i} = C_{o1} \| Z_i'$$

$$Z_i' = \frac{v_i'}{i_i'} = \frac{e_1}{f_1}$$

$$\begin{bmatrix} e_2 \\ f_2 \end{bmatrix} = \begin{bmatrix} \eta & 0 \\ 0 & -\frac{1}{\eta} \end{bmatrix} \begin{bmatrix} e_1 \\ f_1 \end{bmatrix} \Rightarrow \begin{matrix} e_2 = \eta e_1 \rightarrow e_1 = \frac{e_2}{\eta} \\ f_2 = -\frac{1}{\eta} f_1 \rightarrow f_1 = -\eta f_2 \end{matrix}$$

$$\frac{e_1}{f_1} = Z_i' = \frac{e_2}{\eta}\left(\frac{1}{-\eta f_2}\right) = -\frac{1}{\eta^2}\frac{e_2}{f_2} \rightarrow \frac{v_i'}{i_i'} = Z_i' = \cancel{-}\frac{1}{\eta_{e1}^2}\frac{F_d}{(\cancel{-}\dot{x})}$$

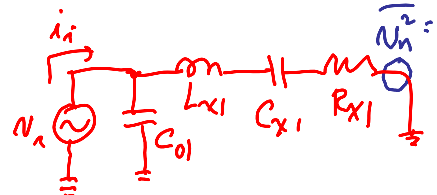
$$Z_i' = \frac{1}{\eta_{e1}^2} Z_x$$

$$Z_i' = \frac{1}{\eta_{e1}^2}\left(j\omega l_x + \frac{1}{j\omega c_x} + r_x\right)$$

$$= \underbrace{j\omega\left(\frac{l_x}{\eta_{e1}^2}\right)}_{L_{x1}} + \underbrace{\frac{1}{j\omega(\eta_{e1}^2 c_x)}}_{C_{x1}} + \underbrace{\frac{r_x}{\eta_{e1}^2}}_{R_{x1}}$$

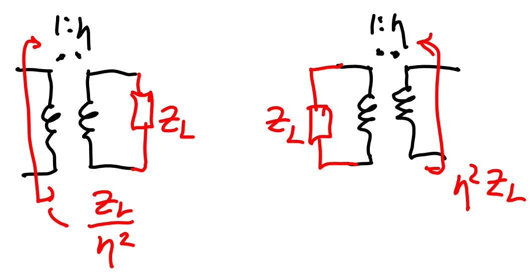
← models noise!

$\overline{v_n^2} = 4kTR_x$

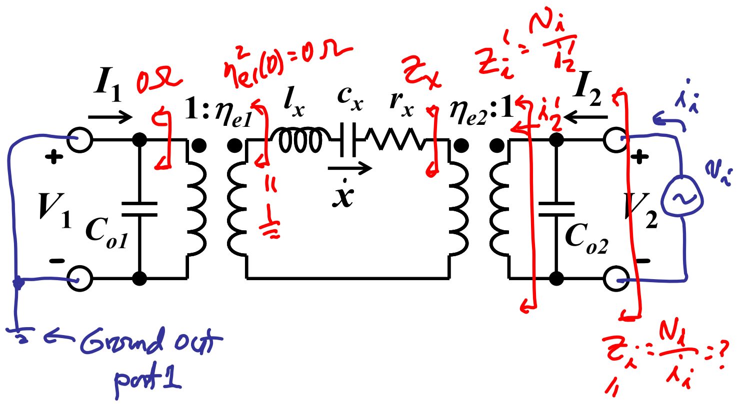


Purely Electrical Equiv. Ckt.

Xformer Inspection Analysis

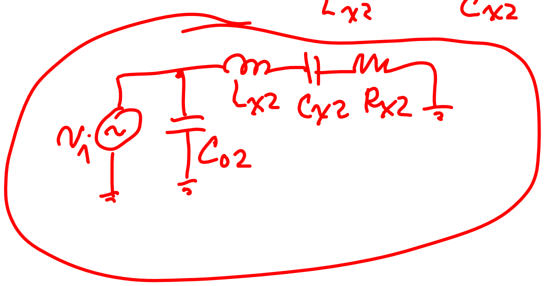


Input Impedance Into Port 2



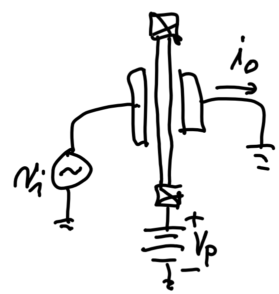
$C_{o2} \| z_i'$

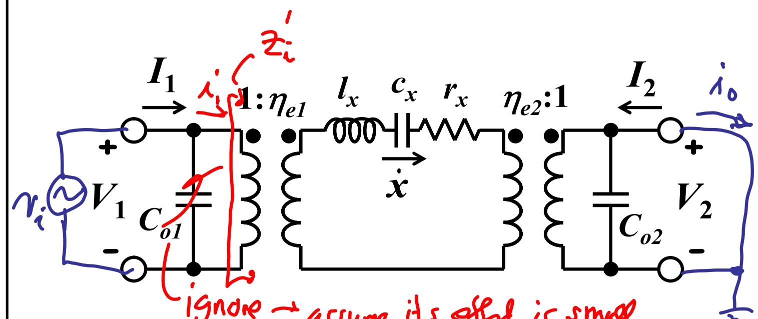
$$z_i' = \frac{v_i}{i_2'} = \frac{z_x}{\eta_{e2}^2} = \underbrace{j\omega\left(\frac{l_x}{\eta_{e2}^2}\right)}_{L_{x2}} + \underbrace{\frac{1}{j\omega(\eta_{e2}^2 c_x)}}_{C_{x2}} + \underbrace{\frac{r_x}{\eta_{e2}^2}}_{R_{x2}}$$



Port 1-to-Port 2 Transconductance

$G = \frac{i_o}{v_i} = ?$





ignore → assume its effect is small compared to motional parameters (not always the case)

$$\left.\begin{aligned} \dot{x} &= \frac{1}{\eta_{e1}} i_1 \\ i_o &= \eta_{e2}\dot{x} \end{aligned}\right\} \quad i_o = \frac{\eta_{e2}}{\eta_{e1}} i_1 = \frac{\eta_{e2}}{\eta_{e1}}\left(\frac{v_i}{z_i'}\right)$$

$$i_o = \frac{\eta_{e2}}{\eta_{e1}}\left[\eta_{e1}^2 \frac{1}{j\omega l_x + \frac{1}{j\omega c_x} + r_x}\right] v_i$$

$$\therefore \frac{i_o}{v_i}(j\omega) = \frac{\eta_{e1}\eta_{e2}}{j\omega l_x + \frac{1}{j\omega c_x} + r_x}$$

$$\boxed{\frac{i_o}{v_1}(j\omega) = \left[j\omega L_{x12} + \frac{1}{j\omega C_{x12}} + R_{x12} \right]^{-1}}$$

where $L_{x12} = \frac{l_x}{\eta_{e1}\eta_{e2}}$, $C_{x12} = \eta_{e1}\eta_{e2} c_x$

$R_{x12} = \frac{r_x}{\eta_{e1}\eta_{e2}}$

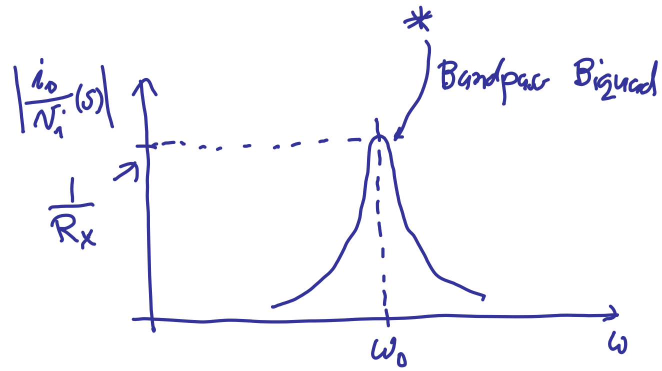
→ separate into magnitude & freq. response components

$$\frac{i_o}{v_1}(s) = \frac{1}{sL_x + \frac{1}{sC_x} + R_x} = \frac{s\left(\frac{1}{L_x}\right)}{s^2 + \frac{1}{L_x C_x} + s\left(\frac{R_x}{L_x}\right)}$$

$$\left(\frac{1}{L_x C_x} = \omega_0^2, \quad Q = \frac{\omega_0 L_x}{R_x} \rightarrow \frac{R_x}{L_x} = \frac{\omega_0}{Q} \right)$$

$$\frac{i_o}{v_1}(s) = \underbrace{\frac{1}{R_x}}_{\text{Gain Term}} \underbrace{\frac{s\left(\frac{\omega_0}{Q}\right)}{s^2 + s\left(\frac{\omega_0}{Q}\right) + \omega_0^2}}_{\text{Frequency Shaping Term}} = \underbrace{\frac{1}{R_x}}_{\text{resonance magnitude}} H(s)$$

*



To analyze mechanical ckt. problem:

1. Just solve the ckt. @ resonance → easy, since L & C cancel
2. Then multiply the result by $H(s)$!

- Now, go through slide 21 in Module 13
- Then, start gyroscopes in Module 15