

### Position-to-Voltage Conversion

UC Berkeley

- To sense position (i.e., displacement), use a capacitive load

Again, have port-to-port I/O symmetry:

Brute force approach:

$$\frac{N_o}{V_i}(s) = \frac{1}{R_x + \frac{1}{sC_x} + sLx + \frac{1}{sC_D}}$$

$$\frac{N_o}{V_i}(s) = \frac{AC_x}{sR_xC_x + 1 + s^2LxC_x + \frac{AC_x}{sC_D}} = \frac{C_x/C_D}{1 + \frac{sR_xC_x}{1+C_x/C_D} + s^2 \frac{LxC_x}{1+C_x/C_D}}$$

$$\omega_0^2 = \frac{1}{LxC_x} \rightarrow (\omega_0)^2 = \omega_0^2 (1 + C_x/C_D)$$

$$Q = \frac{\omega_0 Lx}{R_x} \rightarrow \frac{R_x}{Lx} = \frac{\omega_0^2}{Q^2}, Q = Q\sqrt{1 + C_x/C_D}$$

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### Position-to-Voltage Conversion

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- To sense position (i.e., displacement), use a capacitive load

DC Gain Term

Low-Pass Biquad

$$\frac{N_o}{V_i}(s) = \frac{C_x/C_D}{1 + C_x/C_D} \frac{(\omega_0)^2}{s^2 + (\frac{\omega_0}{Q})s + (\omega_0)^2}$$

To maximize gain  $\rightarrow 1$ , need  $C_D \ll C_x$ . (must minimize  $C_D$ )

Note: Can we similar short-cut to the R case.

- Get DC response  $\rightarrow C$ 's dominate.
- Then:

$$\frac{N_o}{V_i}(s) = (\text{DC Gain}) \cdot \frac{1}{s} \cdot \Theta(s, \omega_0, Q')$$

Voltage Representing Position

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### Velocity Sensing Circuits

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### Velocity-to-Voltage Conversion

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- To convert velocity to a voltage, use a resistive load

Since this structure has completely symmetrical I/O ports:

Work @ resonance: (to simplify the analysis)

$$\frac{N_o}{V_i} = \frac{R_D}{R_x + R_D} \text{ (@ resonance)}$$

Then, generalize to off resonance:

$$\frac{N_o}{V_i} = \frac{R_D}{R_x + R_D} \Theta(s, Q'), \text{ where } Q' = Q \left( \frac{R_x}{R_x + R_D} \right)$$

Voltage Representing Velocity

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### Problems With Purely Resistive Sensing

Now, we get: (Approximately)

$$\frac{N_0}{N_i}(s) \sim \frac{R_0}{R_x + R_0} \cdot \frac{1}{1 + \frac{s}{\omega_p}} \cdot \Theta(s, \omega_b, Q')$$

$\omega_p = \frac{1}{(R_x || R_0) C_p}$  Depend on both  $R_0$  &  $C_p$ .

Impact depends on where  $\omega_p$  is relative to  $\omega_b$ .

Includes  $C_o$ , line C, bond pad C, and next stage C

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### Problems With Purely Resistive Sensing

In general, the sensor output must be connected to the inputs of further signal conditioning circuits → input  $R_i$  of these circuits can load  $R_D$ .

These change w/ hook-up → not good.

**Problem:** need a sensing circuit that is immune to parasitics or loading.

**Soln:** use op amps.

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### The TransR Amplifier Advantage

The virtual ground provided by the ideal op amp eliminates the parasitic capacitance  $C_p$  and  $R_i$ .

The zero output resistance of the (ideal) op amp can drive virtually anything

Virtual Ground ⇒ No voltage across  $C_p$   
 ⇒  $C_p$  effectively isn't there!

$$\frac{N_0}{N_i} = -\frac{R_2}{R_x} \Theta(s) N_i$$

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### Position Sensing Circuits

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### Problems With Pure-C Position Sensing

• To sense position (i.e., displacement), use a capacitive load

$$\frac{N_o}{N_i}(s) = \frac{C_x C_b}{1 + C_x C_D} \cdot \frac{1}{s} \cdot \Theta(s, \omega_b, Q) \cdot \omega_b^2$$

Integration yields displacement.  
To maximize gain, minimize  $C_b$ .  
⇒ Problem: parasitic capacitance  
 $C_b \rightarrow C_b + C_{P_i} + C_{P_b}$   
⇒ DC Gain:  $\frac{C_x / (C_b + C_{P_i} + C_{P_b})}{1 + C_x / (C_b + C_{P_i} + C_{P_b})}$   
Output will get smaller!  
Remedy: Suppress  $C_p$  via use of op amps.

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### The Op Amp Integrator Advantage

• The virtual ground provided by the ideal op amp eliminates the parasitic capacitance  $C_p$

$R_2 \gg \frac{1}{sC_2}$  (for biasing)  
 $R_0 = 0\Omega$   
Can drive next stages  $R_i$  w/o interference to transfer function!  
negl. since  $R_2 = \text{large}$  well defined → good!  
 $N_o = -i_o (R_2 \parallel \frac{1}{sC_2})$   
 $x = \frac{N_i}{R_x} \Theta(s) \frac{1}{sC_2} \Rightarrow \frac{N_o}{N_i}(s) = -\frac{1}{R_x C_2} \frac{\Theta(s)}{s}$

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### Differential Position Sensing

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### Differential Position Sensing

• Example: ADXL-50

Proof Mass  
Sense Finger  
Applied Acceleration  
Fixed Electrodes  
Suspension Beam in Tension  
Capacitive divider  
 $C_1 \gg C_2$   
 $V_o = -V_p + (2V_p) \frac{C_1}{C_1 + C_2}$   
 $= \frac{-V_p C_1 - V_p C_2 + 2V_p C_1}{C_1 + C_2} = V_p \frac{C_1 - C_2}{C_1 + C_2} = V_o$   
Issue: Parasitic Capacitance  
 $V_o = \frac{C_1 - C_2}{C_1 + C_2 + C_p} V_p$   
Arbitrary,  $C_p$  reduces gain!  
Solution: use op amp!

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### Buffer-Bootstrapped Position Sensing

Includes capacitance from interconnects, bond pads, and  $C_{gs}$  of the op amp

Unity Gain Buffer

$C_{gd}$  = gate-to-drain capacitance of the input MOS transistor

• Bootstrap the ground lines around the interconnect and bond pads  
 ↳ No voltage across  $C_p$   
 ↳ It's effectively not there!

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### Effect of Finite Op Amp Gain

Total ADXL-50 Sense  $C \sim 100\text{fF}$

Unity Gain Buffer

Get  $Z_i = \frac{V_i}{I_i}$ :  $I_i = (V_i - V_o) s C_p = V_i \left(1 - \frac{A_o}{1+A_o}\right) s C_p = V_i \frac{1}{1+A_o} s C_p$

$\therefore \frac{V_i}{I_i} = Z_i = \frac{1}{s \left[ \frac{C_p}{1+A_o} \right]}$   $\rightarrow C_{\text{eff}} = \frac{C_p}{1+A_o}$

Ex:  $A_o = 100, C_p = 2\text{pF}$   
 $\Rightarrow C_{\text{eff}} = \frac{2\text{pF}}{101} = 20\text{fF}$   
 ↳ Not negligible compared w/ ADXL-50  $C_{\text{tot}} \sim 100\text{fF}$ !

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### Integrator-Based Diff. Position Sensing

$R_2 \gg \frac{1}{s C_2}$  (for biasing)

$R_o = 0\Omega$

Can drive next stage's  $R_i$  w/o interference to transfer function!

$i_o = i_1 + i_2 = N_p(s C_1) - N_p(s C_2) = V_p s (C_1 - C_2)$

$\therefore V_o = -i_o \left( \frac{1}{s C_F} \right) = -N_p \left( \frac{C_1 - C_2}{C_F} \right)$

$\frac{V_o}{V_p} = - \frac{C_1 - C_2}{C_F} \Rightarrow$  A seemingly perfect differential sensor/amplifier output! ... but only when the op amp is ideal...

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