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EE C247B - ME C218 Introduction to MEMS Design Fall 2018

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Module 17: Noise & MDS

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Lecture Outline

- Reading: Senturia Chpt. 16
- Lecture Topics:
 - ↳ Minimum Detectable Signal
 - ↳ Noise
 - Circuit Noise Calculations
 - Noise Sources
 - Equivalent Input-Referred Noise
 - ↳ Gyro MDS
 - Equivalent Noise Circuit
 - Example ARW Determination

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Determining Sensor Resolution

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Minimum Detectable Signal (MDS)

- **Minimum Detectable Signal (MDS):** Input signal level when the signal-to-noise ratio (SNR) is equal to unity

```
graph LR; S[Sensor] --> SC[Signal Conditioning Circuit]; S --> Out[Output]; SC --> Out;
```

- The sensor scale factor is governed by the sensor type
- The effect of noise is best determined via analysis of the equivalent circuit for the system

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Noise

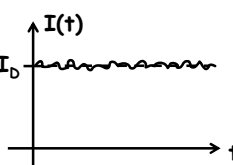
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Noise

- **Noise:** Random fluctuation of a given parameter $I(t)$
- In addition, a noise waveform has a zero average value

Avg. value
(e.g. could be
DC current)



- We can't handle noise at instantaneous times
- But we can handle some of the averaged effects of random fluctuations by giving noise a power spectral density representation
- Thus, represent noise by its mean-square value:

Let $i(t) = I(t) - I_D$

Then $\overline{i^2} = \overline{(I - I_D)^2} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T |I - I_D|^2 dt$

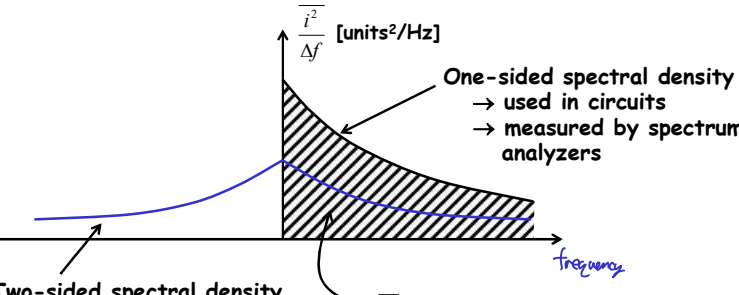
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Noise Spectral Density

- We can plot the spectral density of this mean-square value:

$\frac{\overline{i^2}}{\Delta f}$ [units²/Hz]



One-sided spectral density
 → used in circuits
 → measured by spectrum analyzers

Two-sided spectral density
 (1/2 the one-sided)
 Often used in systems courses

$\overline{i^2} =$ integrated mean-square noise spectral density over all frequencies (area under the curve)

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Noise Sources

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Thermal Noise

- Thermal Noise in Electronics:** (Johnson noise, Nyquist noise)
 - ↳ Produced as a result of the thermally excited random motion of free e-'s in a conducting medium
 - ↳ Path of e-'s randomly oriented due to collisions
- Thermal Noise in Mechanics:** (Brownian motion noise)
 - ↳ Thermal noise is associated with all dissipative processes that couple to the thermal domain
 - ↳ Any damping generates thermal noise, including gas damping, internal losses, etc.
- Properties:**
 - ↳ Thermal noise is white (i.e., constant w/ frequency)
 - ↳ Proportional to temperature
 - ↳ Not associated with current
 - ↳ Present in any real physical resistor

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Circuit Representation of Thermal Noise

- Thermal Noise can be shown to be represented by a series voltage generator $\overline{v_R^2}$ or a shunt current generator $\overline{i_R^2}$

$$\frac{\overline{i_R^2}}{\Delta f} = \frac{4kT}{R}$$

$$\frac{\overline{v_R^2}}{\Delta f} = 4kTR$$

Note: These are one-sided mean-square spectral densities! To make them 2-sided, must divide by 2.

where $4kT = 1.66 \times 10^{-20} \text{ V} \cdot \text{C}$ and where these are spectral densities.

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Noise in Capacitors and Inductors?

- Resistors generate thermal noise
- Capacitors and inductors are noiseless → why?

- Now, add a resistor:
 - ↳ Decays to zero
 - ↳ But this violates the laws of thermodynamics, which require that things be in constant motion at finite temperature

Need to add a forcing function, like a noise voltage $\overline{v_R^2}$ to keep the motion going → and this noise source is associated with R

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Why 4kTR?

- Why is $\overline{v_R^2} = 4kTR\Delta f$ (a heuristic argument)
- The **Equipartition Theorem of Statistical Thermodynamics** says that there is a mean energy $(1/2)kT$ associated w/ each degree of freedom in a given system
- An electronic circuit possesses two degrees of freedom:
 - ↳ Current, i, and voltage, v
 - ↳ Thus, we can write:

$$\frac{1}{2} Li^2 = \frac{1}{2} k_B T \quad , \quad \frac{1}{2} Cv^2 = \frac{1}{2} k_B T$$
- Similar expressions can be written for mechanical systems
 - ↳ For example: for displacement, x

$$\text{Spring constant} \quad \frac{1}{2} kx^2 = \frac{1}{2} k_B T$$

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Shot Noise

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- Associated with direct current flow in diodes and bipolar junction transistors
- Arises from the random nature by which e-'s and h+'s surmount the potential barrier at a pn junction
- The DC current in a forward-biased diode is composed of h+'s from the p-region and e-'s from the n-region that have sufficient energy to overcome the potential barrier at the junction → noise process should be proportional to DC current

pn-junction

$$\frac{\overline{i_n^2}}{\Delta f} = 2qI_D$$

Charge on an e⁻ (=1.6x10⁻¹⁹C)
DC Current

- Attributes:**
 - Related to DC current over a barrier
 - Independent of temperature
 - White (i.e., const. w/ frequency)
 - Noise power ~ I_D & bandwidth

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Flicker (1/f) Noise

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- In general, associated w/ random trapping & release of carriers from "slow" states
- Time constant associated with this process gives rise to a noise signal w/ energy concentrated at low frequencies
- Often, get a mean-square noise spectral density that looks like this:

$$\frac{\overline{i_n^2}}{\Delta f} = 2qI_D + K \left(\frac{I_D^a}{f^b} \right)$$

I_D = DC current
K = const. for a particular device
a = 0.5 → 2
b ~ 1

1/f Noise Corner Frequency

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Back to Determining Sensor Resolution

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MEMS-Based Tuning Fork Gyroscope

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[Zaman, Ayazi, et al, MEMS'06]

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Drive Axis Equivalent Circuit

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180°

180°

Drive Voltage Signal

Drive Oscillation Sustaining Amplifier

Drive Mode

Sense Mode

- Generates drive displacement velocity \dot{x}_d to which the Coriolis force is proportional

To Sense Amplifier (for synchronization)

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Drive-to-Sense Transfer Function

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Drive/Sense Response Spectra:

Amplitude

Drive Response

Sense Response

$f_o (@ T_i)$

Drive Mode

Sense Mode

Rotation-Induced Coriolis Force:

$$\vec{a}_c = 2\vec{\omega}_d \times \vec{\Omega} = 2\omega_d \dot{x}_d \times \vec{\Omega}$$

\Rightarrow Acts in the sense mode direction

$$a_s = 2\omega_d \dot{x}_d \Omega \sin 90^\circ$$

$$a_s = 2\omega_d \dot{x}_d \Omega$$

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Gyro Readout Equivalent Circuit (for a single tine)

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Noise Sources

$\vec{F}_c = m\vec{a}_c = m \cdot (2\dot{x}_d \times \vec{\Omega})$

parallel-plate sense electrodes

noise of the opamp (equivalent input noise spectrum)

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Minimum Detectable Signal (MDS)

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- Minimum Detectable Signal (MDS): Input signal level when the signal-to-noise ratio (SNR) is equal to unity

Sensed Signal

Sensor

Signal Conditioning Circuit

Output

- The sensor scale factor is governed by the sensor type
- The effect of noise is best determined via analysis of the equivalent circuit for the system

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Move Noise Sources to a Common Point

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- Move noise sources so that all sum at the input to the amplifier circuit (i.e., at the output of the sense element)
- Then, can compare the output of the sensed signal directly to the noise at this node to get the MDS

Sensed Signal

Sensor Scale Factor

Sensor Noise

Sensor

+

+

Circuit Gain

Circuit Input-Referred Noise

Signal Conditioning Circuit

Output

Includes desired output plus noise

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