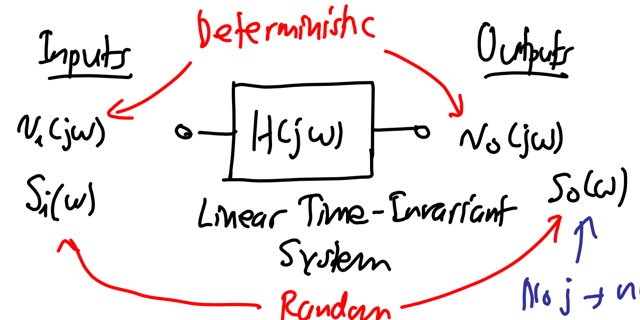
Lecture 26: Noise

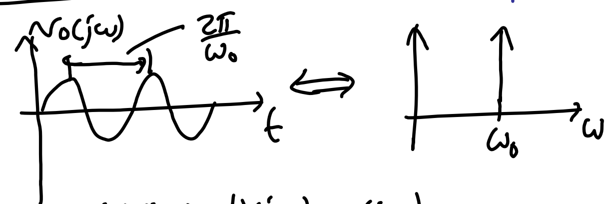
- **Announcements:**
- This is a recorded lecture, since I am presently on travel
- HW#7 online since Tuesday and due Friday, May 4, 10 a.m.
- Project slide #3 due Friday, April 27
- Project outbrief sign up sheet will be on Prof. Nguyen's office door this coming Thursday
  - Slots will be on Monday and Tuesday of Finals week
- ------------------------------------
- Reading: Senturia Chpt. 16
- Lecture Topics:
  - Minimum Detectable Signal
  - Noise
    - Circuit Noise Calculations
    - Noise Sources
    - Equivalent Input-Referred Noise
  - Gyro MDS
    - Equivalent Noise Circuit
    - Example ARW Determination
- ------------------------------------
- **Last Time:**
- Finished MEMS/transistor integration
- Now, move on to noise starting with Module 17, slides 1-7

Circuit Noise Calculations



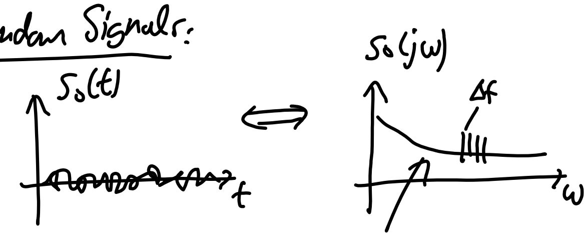
No j → noise has random phase, so j is pointless!

Deterministic Signals:



$$v_o(j\omega) = H(j\omega)\, v_i(j\omega)$$

Random Signals:



Mean-Square Spectral Density

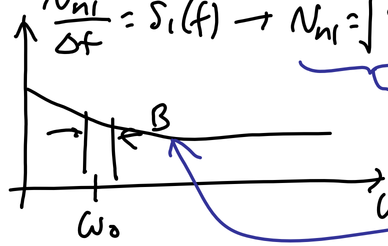
$$S_o(\omega) = \left[H(j\omega)H^*(j\omega)\right]S_i(\omega) = |H(j\omega)|^2 S_i(\omega)$$

$$\sqrt{S_o(\omega)} = |H(j\omega)|\sqrt{S_i(\omega)}$$ → How is it we can do this?

Root mean-square amplitudes

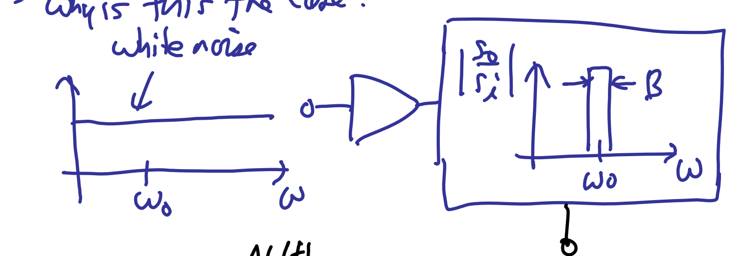
Handling Noise Deterministically

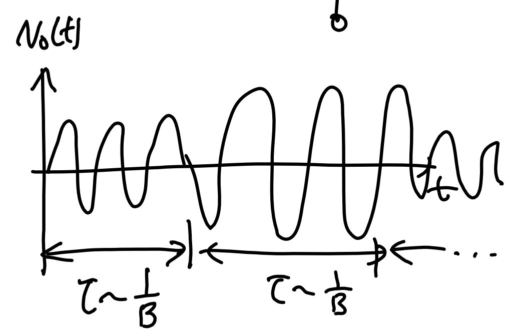
$$\frac{\overline{v_{n1}^2}}{\Delta f} = S_1(f) \rightarrow v_{n1} = \sqrt{S_1(f) B}$$



Can approximate this by a sinusoidal voltage generator (esp. when B is small, say 1Hz)

Why is this the case?

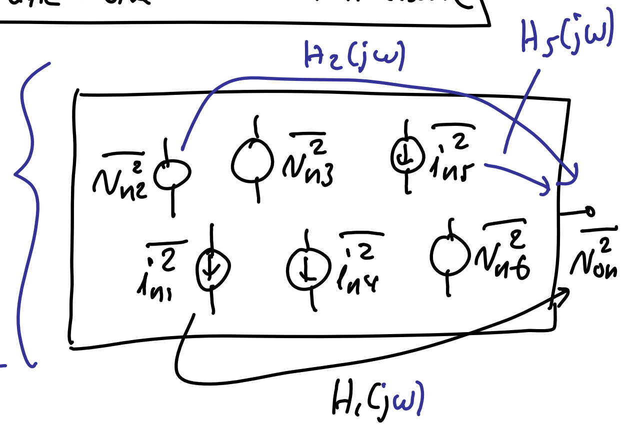




Notice the amplitude and the phase of a signal can change appreciably within a time period 1/B!

Systematic Noise Calculation Procedure

General Ckt. w/ Several Noise Sources



Assume noise sources are uncorrelated.

① For $\overline{i_{n1}^2}$, replace w/ a deterministic source of value $i_{n1} = \sqrt{\frac{\overline{i_{n1}^2}}{\Delta f} \cdot (1\text{Hz})}$

② Calculate $v_{on1}(\omega) = i_{n1}(\omega) H_1(j\omega)$ (treating it like a deterministic signal)

③ Determine $\overline{v_{on1}^2} = \overline{i_{n1}^2} \cdot |H_1(j\omega)|^2$

④ Repeat for each noise source: $\overline{v_{n2}^2}, \overline{v_{n3}^2}, \overline{i_{n4}^2}, \ldots \rightarrow$ output

⑤ Add noise powers (mean-square values)

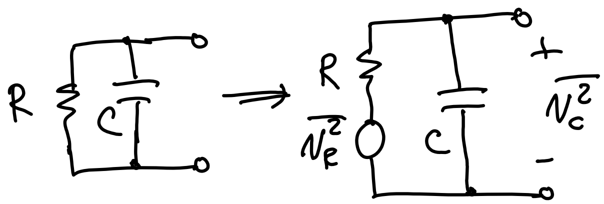
$$\overline{N_{oNTOT}^2} = \overline{N_{on1}^2} + \overline{N_{on2}^2} + \overline{N_{on3}^2} + \overline{N_{on4}^2} + \dots$$

$$\underset{\substack{\uparrow \\ \text{total rms value}}}{N_{oNTOT}} = \sqrt{\overline{N_{on1}^2} + \overline{N_{on2}^2} + \overline{N_{on3}^2} + \overline{N_{on4}^2} + \dots}$$

- Go through Module 17, slides 12-16

$\boxed{\text{Why } \frac{\overline{N_{nR}^2}}{\Delta f} = 4kTR?}$ (a heuristic argument)

Consider an RC ckt:



$$E = \frac{1}{2}kT = \frac{1}{2}C\overline{N_c^2}$$

$\therefore \boxed{\overline{N_c^2} = \frac{kT}{C}}$ ← integrated noise over all freqs. (total mean-square voltage integrated over all freqs.)

*

Question: What value of $\frac{\overline{N_R^2}}{\Delta f}$ gives us this (*
(assuming white noise)

$$\overline{N_c^2} = \int_0^\infty \left|\frac{1}{1+j\omega RC}\right|^2 \frac{\overline{N_R^2}}{\Delta f}\, df$$

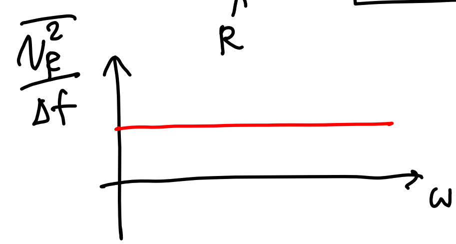
[noise is white] [$\omega_b = \frac{1}{RC}$] $\rightarrow$ $$= \frac{1}{2\pi}\frac{\overline{N_R^2}}{\Delta f}\int_0^\infty \frac{\omega_b^2}{\omega_b^2+\omega}\, d\omega$$

$$\left[\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)\right]$$

$$= \frac{1}{2\pi}\frac{\overline{N_R^2}}{\Delta f}\frac{\omega_b^2}{\omega_b}\tan^{-1}\left(\frac{\omega}{\omega_b}\right)\Bigg|_0^\infty$$

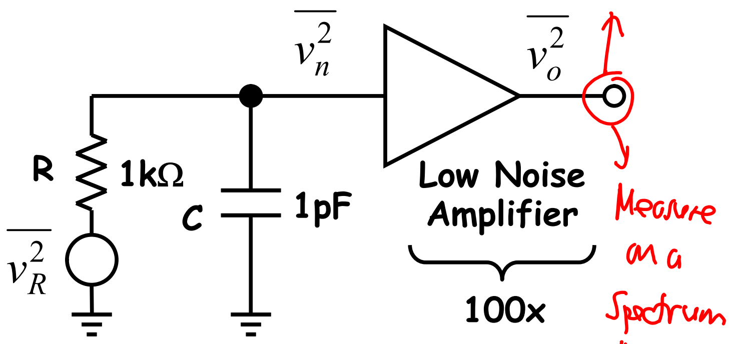
$$= \frac{1}{2\cancel{\pi}}\frac{\overline{N_R^2}}{\Delta f}\left(\frac{\cancel{\pi}}{2}\omega_b - 0\right) = \frac{1}{4}\omega_b\frac{\overline{N_R^2}}{\Delta f} = \frac{kT}{C}$$

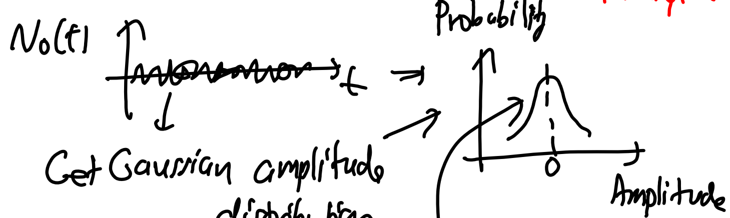
$$\frac{\overline{N_R^2}}{\Delta f} = 4kT\Big(\underbrace{\frac{\omega_b}{C}}_{R}\Big) \Rightarrow \boxed{\frac{\overline{N_R^2}}{\Delta f} = 4kTR} \checkmark$$



- Go through Module 17, slides 19-20

Example. Typical Noise Numbers

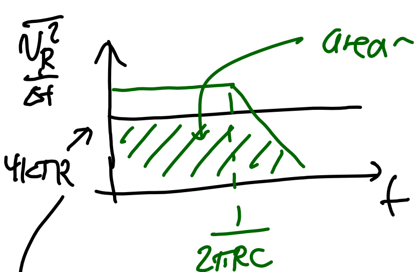




Get Gaussian amplitude distribution

68% within $\pm\sigma$

99.7% within $\pm 3\sigma$



$R = 1k\Omega \rightarrow \sqrt{(1.66\times10^{-20})(1K)}$ *

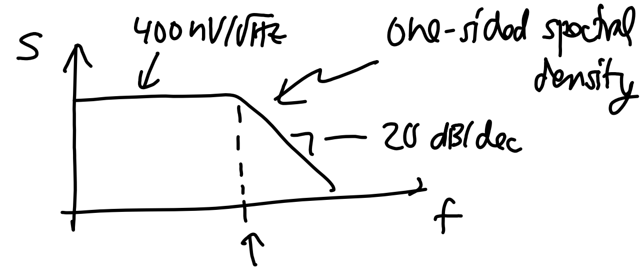
* — $1k\Omega$: $4\,nV/\sqrt{Hz}$ (for every 1K of R)

$1pF$: $\sqrt{\frac{kT}{C}} = 64\mu V_{rms}$

Case: AC Voltmeter

$\sqrt{\overline{N_o^2}} = (100)(64\mu V_{rms}) = 6.4\,mV_{rms}$

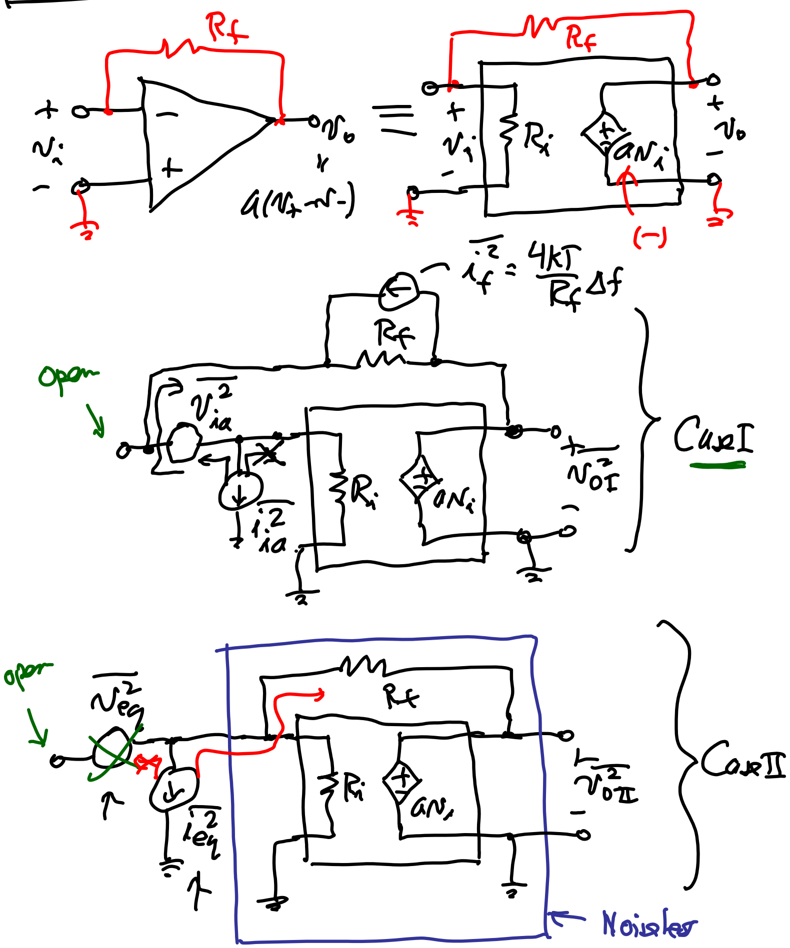
Case: Spectrum Analyzer



$\frac{1}{2\pi(1K)(1p)} = 60MHz$

- Go through Module 17, slides 23-29

Example: TransR Amplifier Input-Referred Noise



Input-Referred Current Noise:

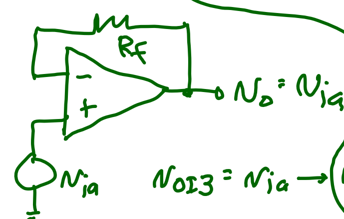
Open inputs; equate output voltage noise for Case I & Case II → solve for $\overline{i_{eq}^2}$

Case I: (use superposition)

$\overline{i_{ia}^2}$: $v_{oI1} = i_{ia} R_f \rightarrow \overline{v_{oI1}^2} = \overline{i_{ia}^2} R_f^2$

$\overline{i_f^2}$: $v_{oI2} = i_f R_f \rightarrow \overline{v_{oI2}^2} = \overline{i_f^2} R_f^2$

$\overline{v_{ia}^2}$:



$v_{oI3} = v_{ia} \rightarrow \overline{v_{oI3}^2} = \overline{v_{ia}^2}$

power @ output generated by noise sources

$\therefore \overline{v_{oI}^2} = \overline{i_{ia}^2} R_f^2 + \overline{i_f^2} R_f^2 + \overline{v_{ia}^2}$

Case II: $v_{oII} = i_{eq} R_f \rightarrow \overline{v_{oII}^2} = \overline{i_{eq}^2} R_f^2$

Now, set $\overline{v_{oI}^2} = \overline{v_{oII}^2}$:

$$\boxed{\overline{i_{eq}^2} = \overline{i_{ia}^2} + \overline{i_f^2} + \frac{\overline{v_{ia}^2}}{R_f^2}}$$