Lecture 27: Gyro Minimum Detectable Signal (MDS)

- **Announcements:**
- This is our last lecture
- HW#7 online since Tuesday and due Friday, May 4, 10 a.m.
- Project slide #3 due Friday, April 27
- Project outbrief sign up sheet will be on Prof. Nguyen's office door later today
  - Slots will be on Monday and Tuesday of Finals week
- Old Final Exams passed out
- Final Exam Info Sheet will be online
- Review Session at a time and location TBD
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- Reading: Senturia Chpt. 16
- Lecture Topics:
  - Minimum Detectable Signal
  - Noise
    - Circuit Noise Calculations
    - Noise Sources
    - Equivalent Input-Referred Noise
  - Gyro MDS
    - Equivalent Noise Circuit
    - Example ARW Determination
  - Course Wrap Up (Final Exam Info)
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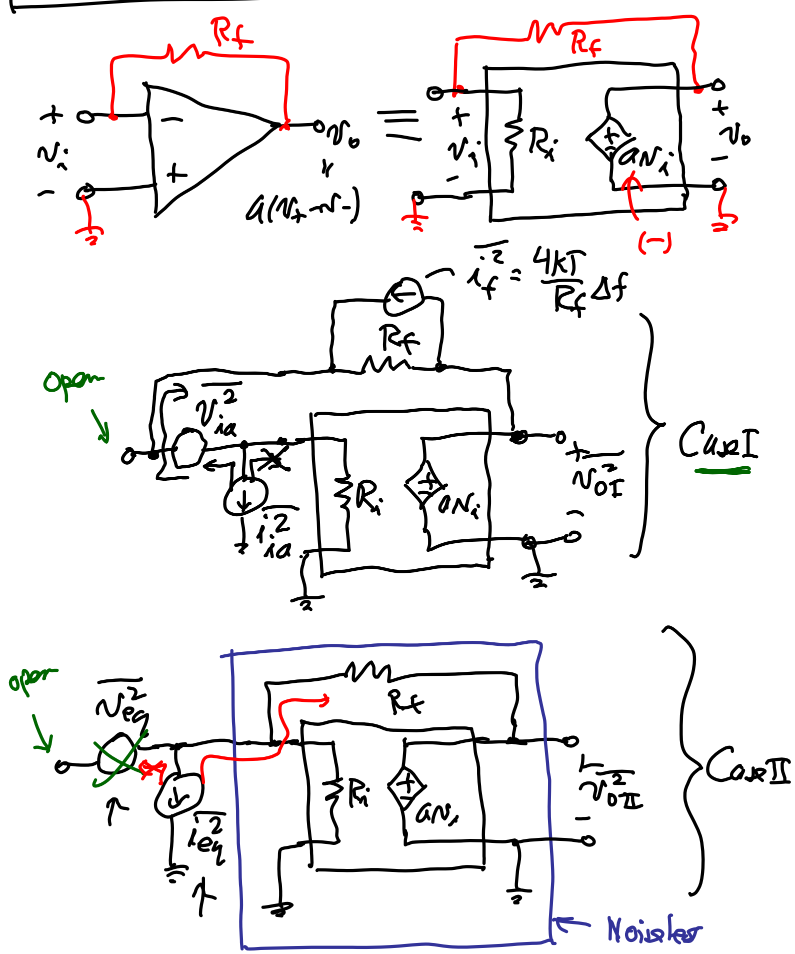
- **Last Time:**
- Going through input referred noise
- Now, continue with this

Example: Trans R Amplifier Input-Referred Noise



Input-Referred Current Noise:

Open inputs; equate output voltage noise for Case I & Case II → solve for $\overline{i_{eq}^2}$

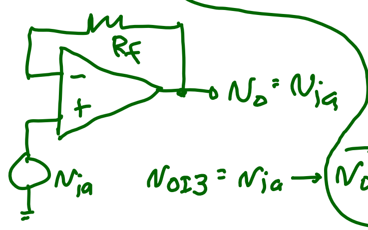
Case I: (use superposition)

$\overline{i_{ia}^2}$: $v_{oI1} = i_{ia} R_f \rightarrow \overline{v_{oI1}^2} = \overline{i_{ia}^2} R_f^2$

$\overline{i_f^2}$: $v_{oI2} = i_f R_f \rightarrow \overline{v_{oI2}^2} = \overline{i_f^2} R_f^2$

→ power @ output generated by noise sources

$\overline{v_{ia}^2}$:



$v_{oI3} = v_{ia} \rightarrow \overline{v_{oI3}^2} = \overline{v_{ia}^2}$

$\therefore \overline{v_{oI}^2} = \overline{i_{ia}^2} R_f^2 + \overline{i_f^2} R_f^2 + \overline{v_{ia}^2}$

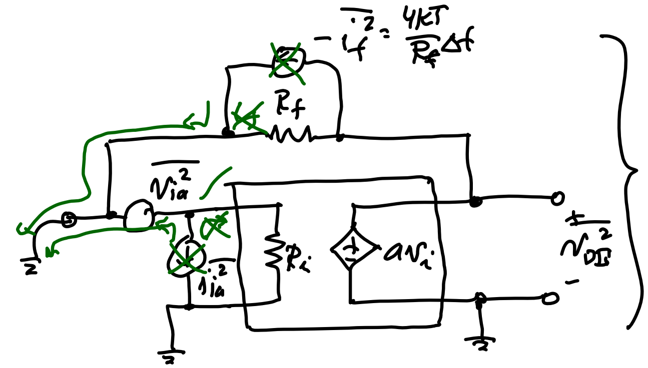
Case II: $v_{oII} = i_{eq} R_f \rightarrow \overline{v_{oII}^2} = \overline{i_{eq}^2} R_f^2$

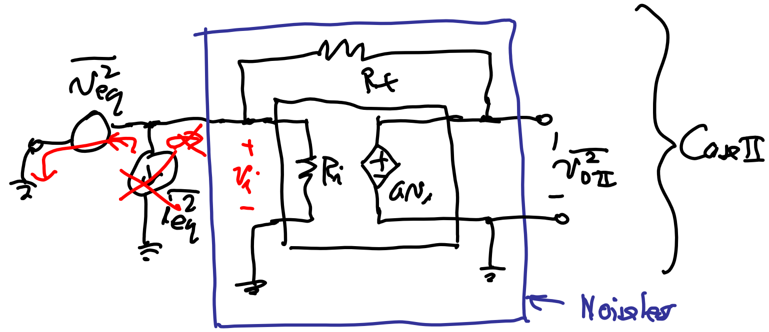
Now, set $\overline{v_{oI}^2} = \overline{v_{oII}^2}$:

$$\overline{i_{eq}^2} = \overline{i_{ia}^2} + \overline{i_f^2} + \frac{\overline{v_{ia}^2}}{R_f^2}$$

Now, get the input-referred voltage noise:

Short inputs; equate output voltage noise.



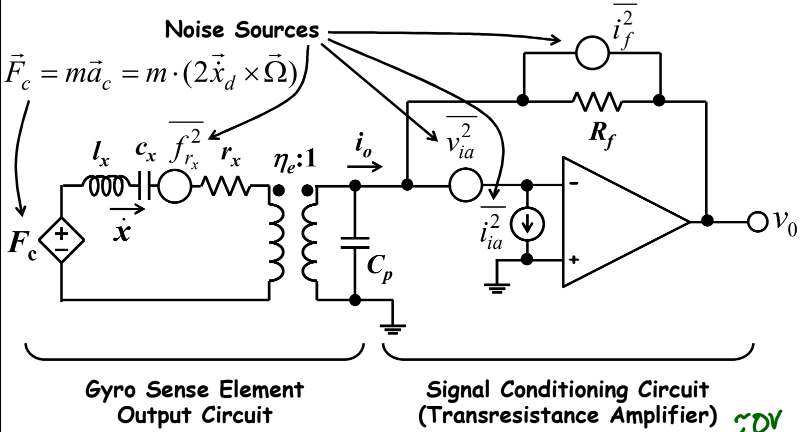


Case I: $v_{oI} = a v_{ia} \rightarrow \overline{v_{oI}^2} = a^2 \overline{v_{ia}^2}$

Case II: $v_{oII} = a v_{eq} \rightarrow \overline{v_{oII}^2} = a^2 \overline{v_{eq}^2}$

$\overline{v_{oI}^2} = \overline{v_{oII}^2} \rightarrow \boxed{\overline{v_{eq}^2} = \overline{v_{ia}^2}}$

Gyro Sensing Ckt → Want MDS



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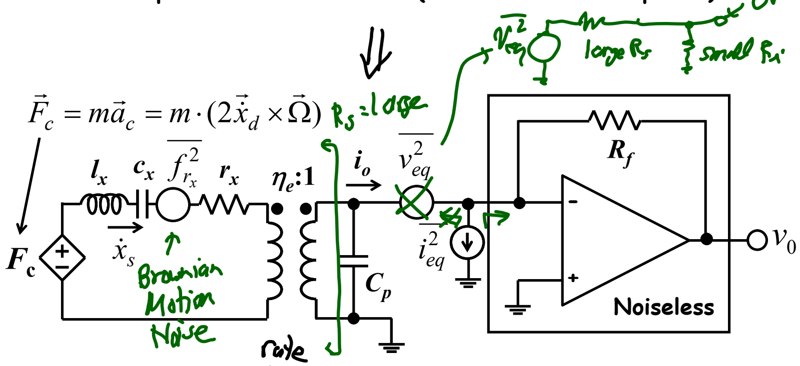


⇒ Find the rotation-to-$\hat{i}_o$ transfer function:

$$\dot{x}_s = F_c\left(\frac{\omega_s Q_s}{k_s}\,\Theta_s(j\omega_d)\right) = \frac{\omega_s Q_s}{k_s}\cdot 2\omega_d x_d \Omega m\,\Theta_s(j\omega_d)$$

(sense; drive; $\frac{m}{k_s} = \frac{1}{\omega_s}$)

$[F_c = ma_c = 2\omega_d x_d \Omega m]$

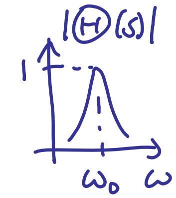
$$\dot{x}_s = 2\frac{\omega_d}{\omega_s} Q_s x_d \Theta_s(j\omega_d)\cdot\Omega$$

$$\Theta(s) = \frac{s(\omega_o/Q)}{s^2 + s(\omega_o/Q) + \omega_o^2}$$

$s=0$: $\Theta(0) = 0$

$s = j\omega_o$: $\Theta(j\omega_o) = 1$

$s=\infty$: $\Theta(\infty) = 0$



$$i_o = \eta_e \dot{x}_s = \underbrace{2\frac{\omega_d}{\omega_s} Q_s x_d \eta_e \Theta_s(j\omega_d)}_{A \,\hat{=}\, \text{scale factor}}\cdot\Omega$$

→ $i_o = A\Omega$, where $A = 2\frac{\omega_d}{\omega_s} Q_s x_d \eta_e \Theta_s(j\omega_d)$

↑ input rotation

When $\Omega = \Omega_{min} \hat{=}$ MDS → $i_o = i_{eqTOT}$ (noise current entering the sense amplifier)

$\therefore\ i_{eqTOT} = A\Omega_{min}$

$$\Omega_{min} = \frac{i_{eqTOT}}{A}\left(\frac{3600s}{hr}\right)\left(\frac{180°}{\pi}\right) \quad [(°/hr)/\sqrt{Hz}]$$

$$\text{Angle Random Walk} \hat{=} \text{ARW} = \frac{1}{60}\Omega_{min} \quad [°/\sqrt{hr}]$$

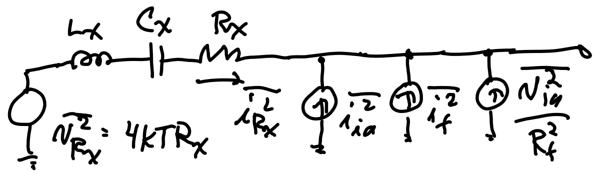
↳ Easier to determine directional error as a function of elapsed time

⇒ the total current entering the amplifier input: (i_{eqTOT})

$$i_{eqTOT} = i_{r_x} + i_{eq} \rightarrow \overline{i_{eqTOT}^2} = \overline{i_{r_x}^2} + \overline{i_f^2} + \overline{i_{ia}^2} + \frac{\overline{v_{ia}^2}}{R_f^2}$$

Brownian motion noise of the sense element
↳ determined entirely by noise in $r_x \rightarrow \overline{f_{r_x}^2}$

easiest to see when convert to all electrical ckt:



where $L_x = \frac{l_x}{\eta_e^2}$, $C_x = \eta_e^2 c_x$, $R_x = \frac{r_x}{\eta_e^2}$

$$\therefore i_{R_x} = v_{R_x}\left(\frac{1}{R_x}\right)\Theta_s(j\omega_d) \rightarrow \frac{\overline{i_{R_x}^2}}{\Delta f} = 4kTR_x\left(\frac{1}{R_x^2}\right)|\Theta_s(j\omega_d)|^2$$

$$\Rightarrow \boxed{\frac{\overline{i_{R_x}^2}}{\Delta f} = \frac{4kT}{R_x}|\Theta_s(j\omega_d)|^2}$$

← current noise from Brownian motion that goes to the transR amp input

Thus:

$$\boxed{\frac{\overline{i_{eqTOT}^2}}{\Delta f} = \frac{4kT}{R_x}|\Theta_s(j\omega_d)|^2 + \frac{4kT}{R_f} + \frac{\overline{i_{ia}^2}}{\Delta f} + \frac{\overline{v_{ia}^2}}{\Delta f}\left(\frac{1}{R_f^2}\right)}$$

$$i_{eqTOT} = \sqrt{\frac{\overline{i_{eqTOT}^2}}{\Delta f} \times BW}$$

↑ Bandwidth

- Go through slides 45-49 in Module 17
- Related courses at UC Berkeley:
  - EE 143: Microfabrication Technology
  - EE 147/247A: Introduction to MEMS
  - ME 119: Introduction to MEMS (mainly fabrication)
  - BioEng 121: Introduction to Micro and Nano Biotechnology and BioMEMS
  - ME C219 – EE C246: MEMS Design
  - EE 290M?