Lecture 27: Gyro Minimum Detectable Signal (MDS)

• Announcements:
  • This is our last lecture
  • HW#7 online since Tuesday and due Friday, May 4, 10 a.m.
  • Project slide #3 due Friday, April 27
  • Project outbrief sign up sheet will be on Prof. Nguyen’s office door later today
    ❥ Slots will be on Monday and Tuesday of Finals week
  • Old Final Exams passed out
  • Final Exam Info Sheet will be online
  • Review Session at a time and location TBD

• Reading: Senturia Chpt. 16

• Lecture Topics:
  ❥ Minimum Detectable Signal
  ❥ Noise
    — Circuit Noise Calculations
    — Noise Sources
    — Equivalent Input-Referred Noise
  ❥ Gyro MDS
    — Equivalent Noise Circuit
    — Example ARW Determination
  ❥ Course Wrap Up (Final Exam Info)

• Last Time:
  • Going through input referred noise
  • Now, continue with this
Input-Refereed Current Noise:

Open inputs; equate output voltage noise to

Case I + Case II ≈ some in $i_{eq}$

Case I: (we symmetrize)

\[
\frac{\overline{i}_{o1}}{2} = N_{O1} = i_{eq} \frac{R_f}{4} \Rightarrow \overline{N_{O1}} = \frac{\overline{i}_{eq}^2 R_f}{2}
\]

\[
\frac{\overline{i}_{o2}}{2} = N_{O2} = i_{eq} \frac{R_f}{4} \Rightarrow \overline{N_{O2}} = \frac{\overline{i}_{eq}^2 R_f}{2}
\]

\[
\overline{N_{o3}} = N_{o3} = \overline{i}_{eq}
\]

\[
\frac{\overline{N_{o3}}}{2} = \overline{N_{o3}^2} = \frac{\overline{i}_{eq}^2}{2} + \frac{\overline{i}_{eq}^2}{R_f^2} + \frac{N_{i_o}^2}{2}
\]

Case II:

\[
\overline{N_{o1}} = i_{eq} \frac{R_f}{4} \Rightarrow \overline{N_{o1}^2} = \overline{i}_{eq}^2 \frac{R_f^2}{4}
\]

Now, set $\overline{N_{o1}^2} = \overline{N_{o2}^2}$:

\[
\overline{i}_{eq} = \overline{i}_{i_a} + \overline{i}_f + \frac{N_{i_o}}{R_f^2}
\]

Now, get the input-referred voltage-noise:

Short input; equate output voltage noise .

\[
\overline{N_{o1}^2} = a N_{i_o} \Rightarrow \overline{N_{o1}^2} = a^2 \overline{N_{i_o}^2}
\]

\[
\overline{N_{o2}^2} = a N_{eq} \Rightarrow \overline{N_{o2}^2} = a^2 \overline{N_{eq}^2}
\]

\[
\overline{N_{i_o}^2} = \overline{N_{i_o}^2} \Rightarrow \overline{N_{eq}^2} = \overline{N_{i_o}^2}
\]
\[ \ddot{F}_c = m \ddot{a}_c = m \cdot (2 \ddot{x}_d \times \Omega) \]

\[
\begin{align*}
\hat{v}_{io} &= \frac{1}{2} i_f^2 R_f \\
\hat{v}_{eq} &= \frac{1}{2} i_{eq}^2 R_f \\
\end{align*}
\]

\[
\ddot{x}_s = \dot{F}_c \left( \frac{\omega_d Q_s}{k_s} \cdot G_s(j \omega_d) \right) = \omega_d Q_s k_s \Omega_s(j \omega_d) \Theta_s(j \omega_d) \cdot \Omega
\]

\[
F_c = m a_c = 2 \omega_d x_d \Omega m
\]

\[
\dot{i}_f = \frac{1}{2} i_{eq} \Omega m
\]

Noise Sources

Gyro Sense Element

Output Circuit

Signal Conditioning Circuit (Transresistance Amplifier)

\[
\Theta(s) = \frac{s(\omega_0/\Omega)}{s^2 + s(\omega_0/\Omega) + \omega_0^2}
\]

\[
\begin{align*}
\omega_0 = 0: & & \Theta(s) = 0 & & |\Theta(s)| \\
\omega_0 = j \omega_0: & & \Theta(j \omega_0) = 1 & & V \downarrow \\
\omega_0 = \infty: & & \Theta(\omega) = 0
\end{align*}
\]

\[
\dot{x}_0 = \dot{x}_c = \frac{\omega_d}{\omega_f} Q_s x_d \Theta_s(j \omega_d) \cdot \Omega
\]

\[A^2 = \text{scale factor}\]

\[
\dot{x}_0^* = \dot{x}_{eq, \text{tr}} \quad \text{where} \quad A = \frac{2 \omega_d}{\omega_f} Q_s x_d \Theta_s(j \omega_d)
\]

input rotation

When \[ \omega_r = \omega_{\text{min}} \equiv \text{MDS} \rightarrow \dot{x}_0^* = \dot{x}_{eq, \text{tr}} \quad \text{noisy current enters the sense amplifier}
\]

\[
\omega_{\text{min}} = \frac{\dot{x}_{eq, \text{tr}}}{A} \left( \frac{2 \omega_d}{\omega_f} \right) \left( \frac{180^\circ}{\pi} \right) \left( \frac{\text{[deg/hr]}}{\text{[Hz]}} \right)
\]

Angle Random Walk = ARW = \frac{1}{6_0} \omega_{\text{min}} \left[ \text{[deg/hr]} \right]

Easier to determine directional error as a function of elapsed time
• Go through slides 45-49 in Module 17

• Related courses at UC Berkeley:
  - EE 143: Microfabrication Technology
  - EE 147/247A: Introduction to MEMS
  - ME 119: Introduction to MEMS (mainly fabrication)
  - BioEng 121: Introduction to Micro and Nano Biotechnology and BioMEMS
  - ME C219 - EE C246: MEMS Design
  - EE 290M?