Lecture 2: Benefits of Scaling I

- Announcements:
- The notes and video from last time are online – both in the Lecture link table
- Modules 1 & 2 are also online (also, in the Lecture link table)
- As announced last time, I will be traveling next week (at the IEEE MEMS Conference)
  - Next week's lectures will be by recorded video
  - The videos will be online in the Lecture link table in the far right column
  - Please watch the videos before the week after next to avoid falling behind
  - You'll need to watch them, anyway, in order to do the homework
- Get your computer accounts by following the instructions at the end of the Course Info Sheet
- You all have received invites to join the class Piazza group
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- Today:
- Reading: Senturia, Chapter 1
- Lecture Topics:
  - Benefits of Miniaturization
  - Examples
    - GHz micromechanical resonators
    - Chip-scale atomic clock
    - Micro gas chromatograph
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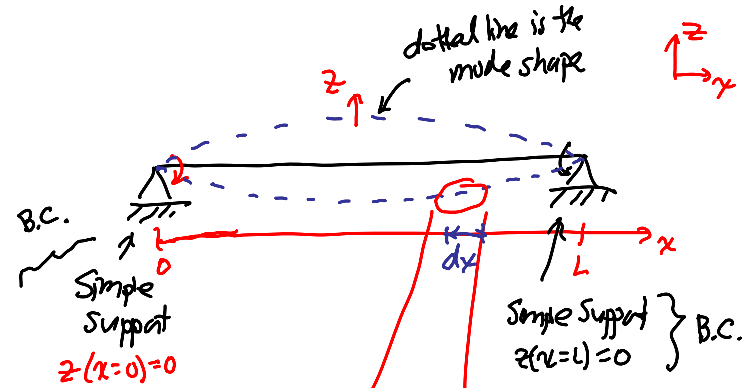
- Last Time: Going through Module 1
- Finish Module 1, then start going through Module 2

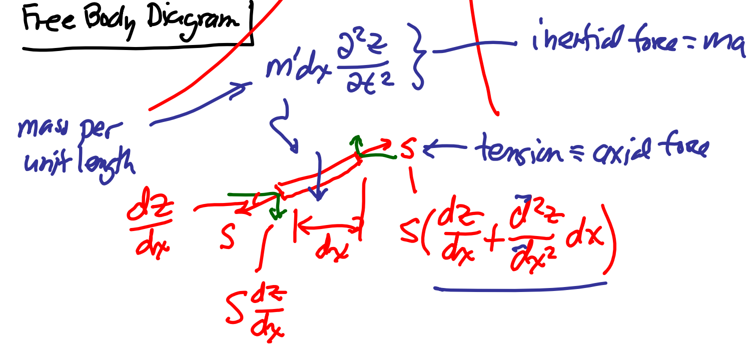
Scaling of Guitar Strings

guitar string ≡ transversely vibrating stretched wire



Get equation for resonance frequency: (fund. mode)

Free Body Diagram



⇒ condition for dynamic equilibrium:

$$S\left(\frac{dz}{dx}+\frac{\partial^2 z}{\partial x^2}dx\right)-S\frac{dz}{dx}-m'dx\frac{\partial^2 z}{\partial t^2}=0$$

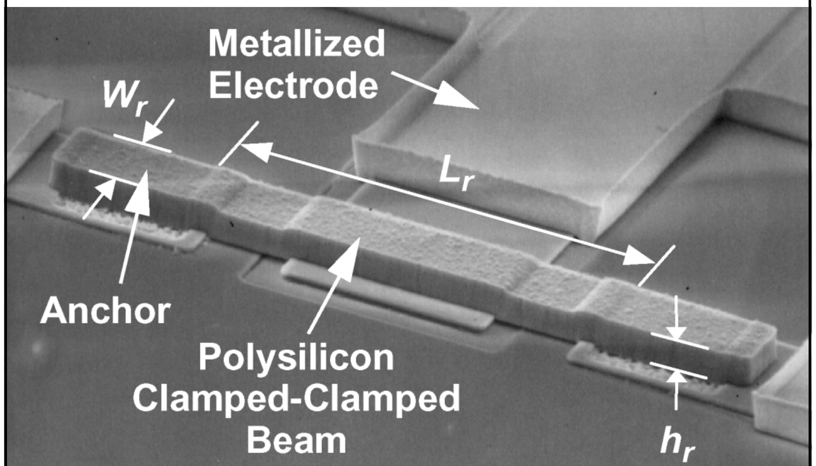
solve

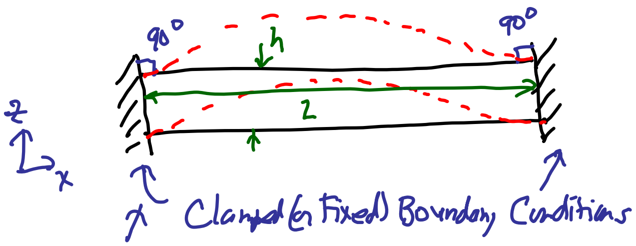
$$\boxed{f_i=\frac{i}{2L}\sqrt{\frac{S}{m'}}}$$ ← frequency

i = mode = 1, 2, 3, ...

if $L\downarrow \rightarrow f_i\uparrow$

Clamped-Clamped Beam





$z(x=0)=0$

Eq. for Resonance: $f_o = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 1.03\sqrt{\frac{E}{\rho}}\frac{h}{L^2}$ (1)

where $E \triangleq$ Young's modulus [GPa]

$\rho \triangleq$ density [kg/m³]

$h \triangleq$ thickness [m]

$L \triangleq$ length [m]

Example. $L = 40\mu m$, $h = 2\mu m$

polySi: $E = 150\,GPa$, $\rho = 2300\ kg/m^3$

$\therefore f_o = (1.03)\underbrace{\sqrt{\frac{150G}{2300}}}_{\sqrt{\frac{E}{\rho}} \triangleq \text{acoustic velocity} \to 8076\,m/s}\frac{2\mu}{(40\mu)^2} \Rightarrow \boxed{f_o = 10.4\,MHz}$

Scaling:

$L_{new} = S \cdot L_{old}$

① Scale all dimensions equally by a factor S:

$f_o \sim \frac{S}{S^2} \sim \frac{1}{S}$

② If scale L only: $f_o \sim \frac{1}{S^2}$ → even faster rise in f_o (but... problems...)

Example.

$L = 4\mu m \to f_o = (1.03)(8076)\frac{2\mu}{(4\mu)^2} \Rightarrow f_o = 1.04\,GHz$

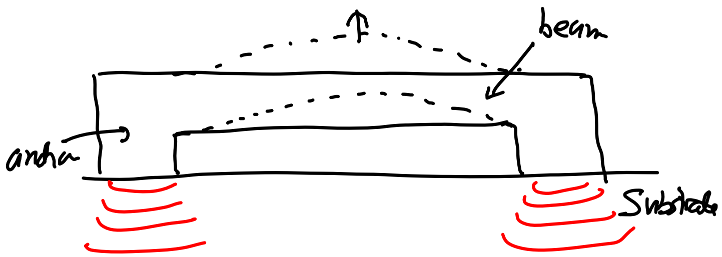
equation ignores width effects → this will be smaller

Remarks.

① Eq. (1) not accurate when $L \approx h$.

② Anchor loss when $L \approx h$! → lower Q!

Beam becomes too stiff!



energy radiates into substrate → anchor loss!

$$Q = \frac{\text{Energy per Cycle}}{\text{Energy lost per Cycle}}$$

Ex. of a "con"

(3) Solution: nanodimensions! → scale both L & h ✓

this means: $h \approx 300\,nm$, $L \sim 1\,\mu m$

Problem: power handling ↓ when size ↓

(4) Better Solution: other geometries

Free-Free Beam: