Lecture 3: Benefits of Scaling II

- **Announcements:**
- As announced last time, I am on travel right now
- This is a pre-recorded video
- The notes from last time are online, as well as the video – both in the Lecture link table
- Modules 1 & 2 are online (also, in the Lecture link table)
- Get your computer accounts by following the instructions at the end of the Course Info Sheet
- HW#1 is online and due Thursday, Feb. 1, at 10 a.m. in the EE247B/ME218 Homework Box near 140 Cory
- Kieran posted some info on where to go for office hours in the coming weeks (it's different from the course info sheet, which was tentative)
- ------------------------------------
- **Today:**
- Reading: Senturia, Chapter 1
- Lecture Topics:
  - ↳ Benefits of Miniaturization
  - ↳ Examples
    - —GHz micromechanical resonators
    - —Chip-scale atomic clock
    - —Micro gas chromatograph
- ------------------------------------
- **Last Time:**
- Going through Module 2, looking at how scaling vibrating RF MEMS provides both benefits and problems that one must circumvent
- Continue with this now

Scalings:

2x, ½x ↓

① Scale all dimensions equally by a factor S

$$f_0 \sim \frac{S}{S^2} = \frac{1}{S}$$

② If scale L only: $f_0 = \frac{1}{S^2}$ → even faster rise in freq.!

(…but problems…)

Example:

$$L = 4\mu m \rightarrow f_0 = (1.03)(8076)\frac{2\mu}{(4\mu)^2} = \boxed{1.04\ GHz}$$

↑ ignore width effects → really need ~3μm

questionable thing to do

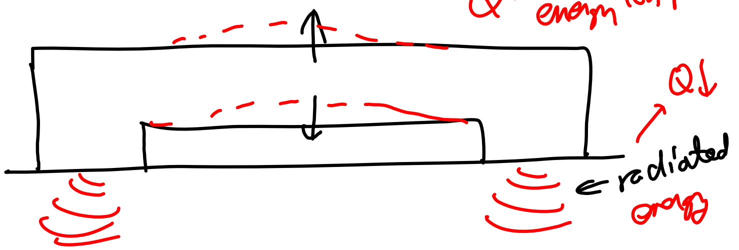
Remarks:

① Eq.(1) not accurate when $L \simeq W \simeq h$

② When $L \simeq h$ (or when it isn't more than 10×h)

↳ get anchor loss problems

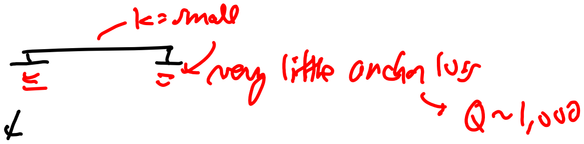
$$Q = \frac{\text{energy per cycle}}{\text{energy lost per cycle}}$$



↓ Soln:

③ Solution: use nanodimensions! ✓

↳ ex.: h=300nm, L~1μm



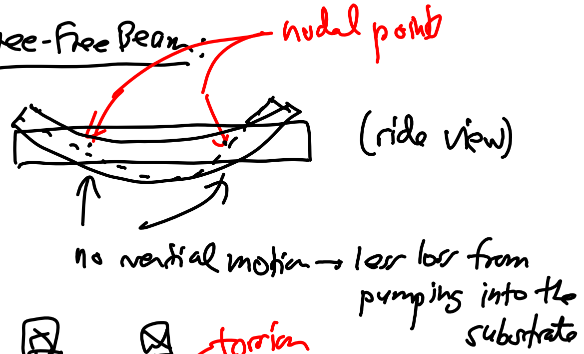
↓

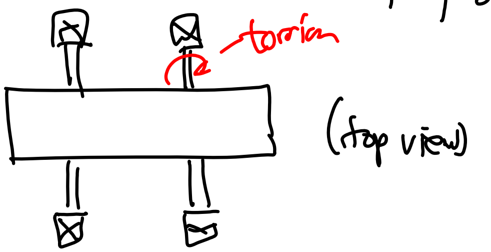
Problem: power handling ↓ when size ↓

↳ Soln: use massive numbers in arrays!

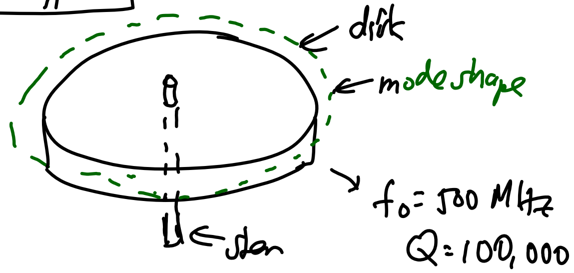
④ Better Soln: use other geometries

Free-Free Beam:



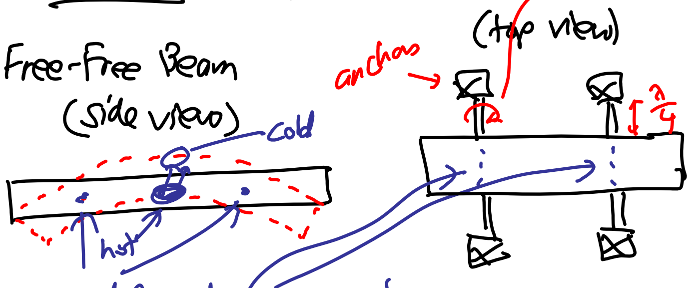


Even Better Approach



⑤ Better Soln: other geometries

Free-Free Beam
(side view)

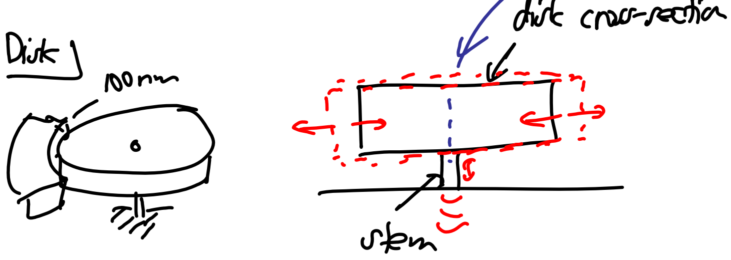


nodal locations → no motion
^ transverse

↙

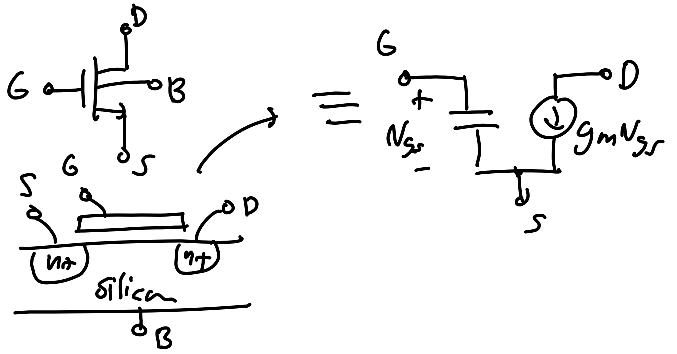
retain high Q w/ high fo ✓

Disk

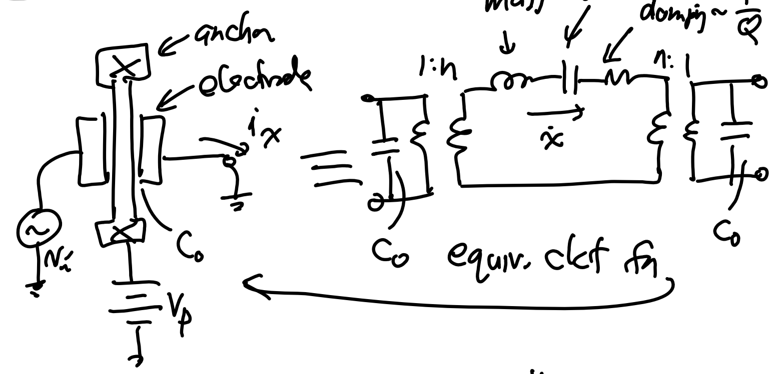


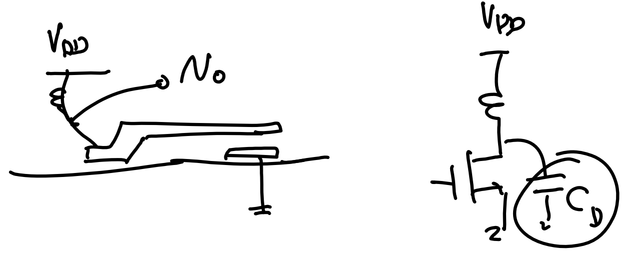
Circuit Design

Transistor Ckt.



Mechanical Ckt.

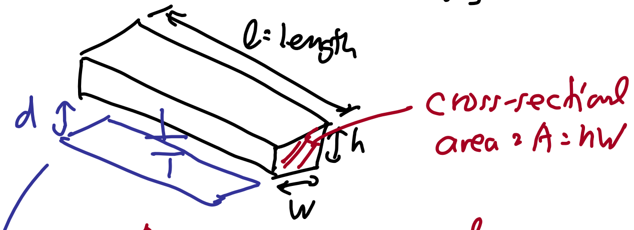




- Now go through Module 2, slides 9-30

Review Electrical Resistance First

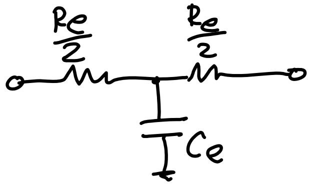
(then attack the thermal R analogy)



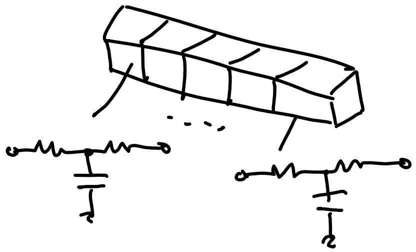
$R_e \triangleq$ electrical resistance $= \frac{l}{\sigma A}$ ← electrical conductivity

C_e = capacitance $= \frac{\epsilon_o W L}{d}$ ← permittivity

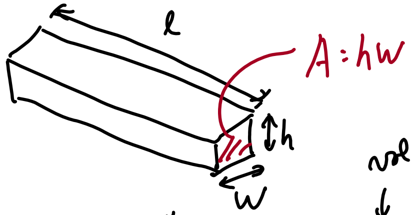
↳ Stores Energy (charge energy): $\frac{1}{2}CV^2 = E$ ← voltage across the capacitor



⇒ if want to be more exact:



Thermal Ckt.



⇒ thermal capacitance: $C_{th} = \rho V C_p$

volume → V; specific heat → C_p; density → ρ; → stores thermal energy

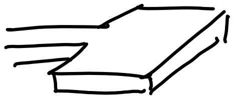
⇒ thermal resistance:

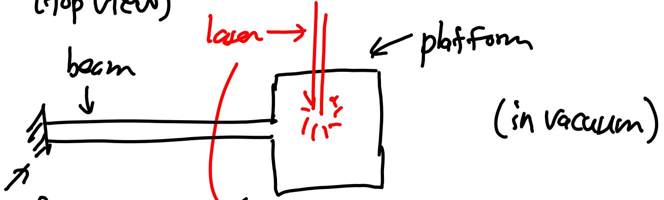
$R_{th} = \frac{l}{kA}$

l ← length; A ← cross-sectional area; k ← thermal conductivity

Examples

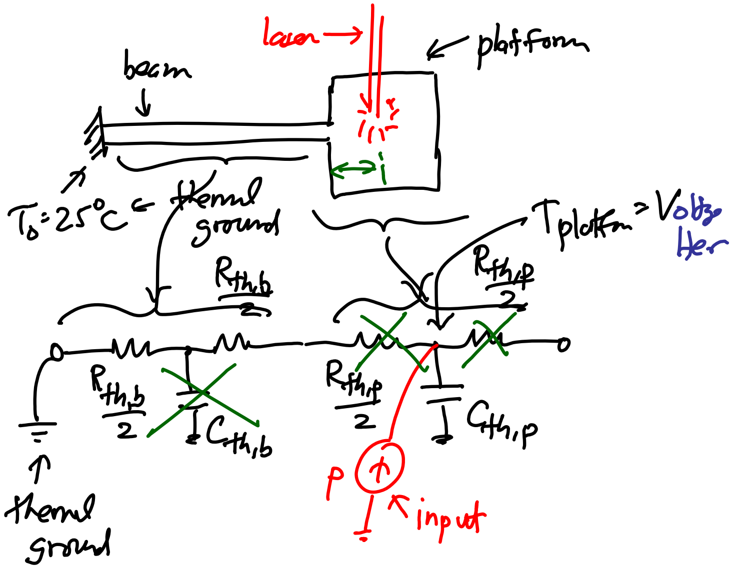
Ex. MEMS type structure (top view)





source of power: assume all energy of laser is absorbed

What is the temperature on the platform?



Analogies:

(temperature) $T \rightarrow V$ (voltage)

(power) $P \rightarrow I$ (current)

thermal electrical

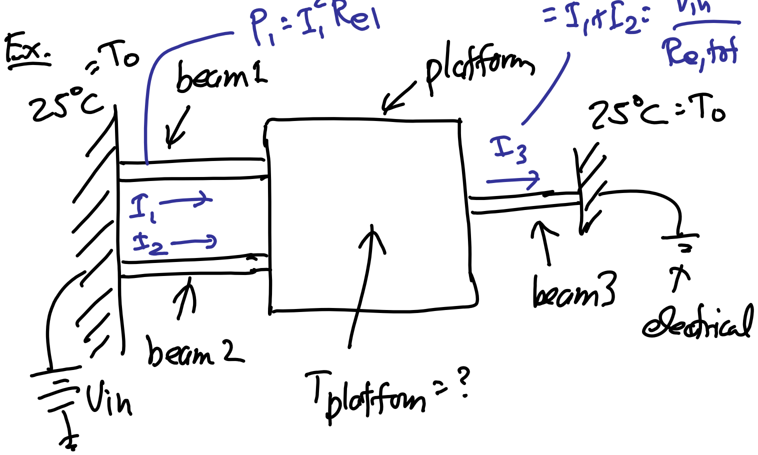
To Analyze:

① Remove small elements to simplify the ckt.

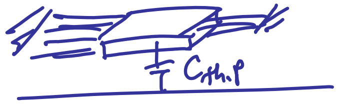
② $T_{platform} = P R_{th,b}$ (in steady-state)

(just like $V = IR$ in an electrical system)

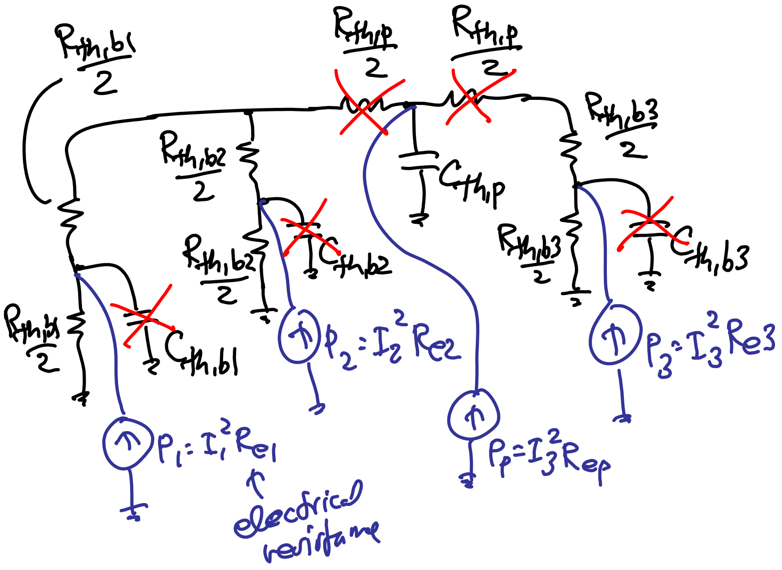
Ex.



→ coupled thermal, electrical system



⇒ Draw the thermal ckt.



To Analyze:

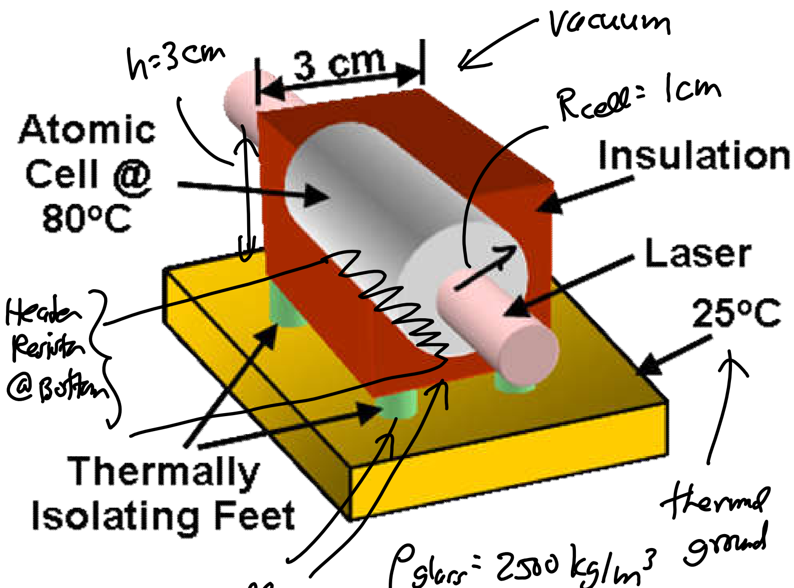
① $I_3 = I_1 + I_2 = \frac{V_{in}}{R_{e,tot}}$ (electrical analysis)

② Get P_i's. (powers)

③ Use superposition to solve the thermal ckt.
handle one power source at a time &
sum the temperatures (i.e., thermal voltages)
to get the total temperature at any
node

Example: Thermal Ckt.

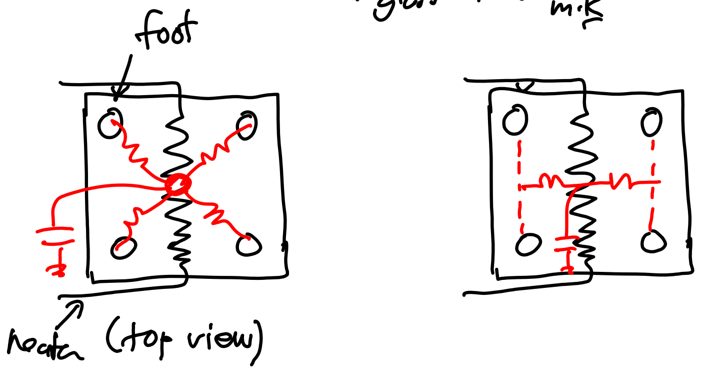
⇒ determine the power needed to get this
atomic cell to 80°C (from RT) & how fast

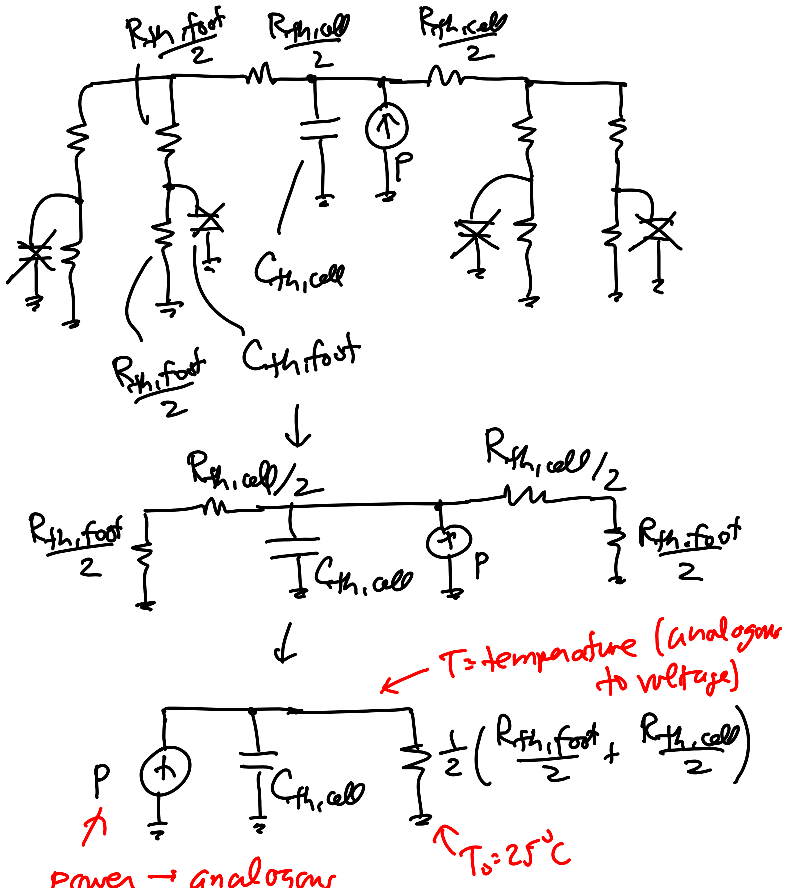


$\rho_{glass} = 2500\ kg/m^3$

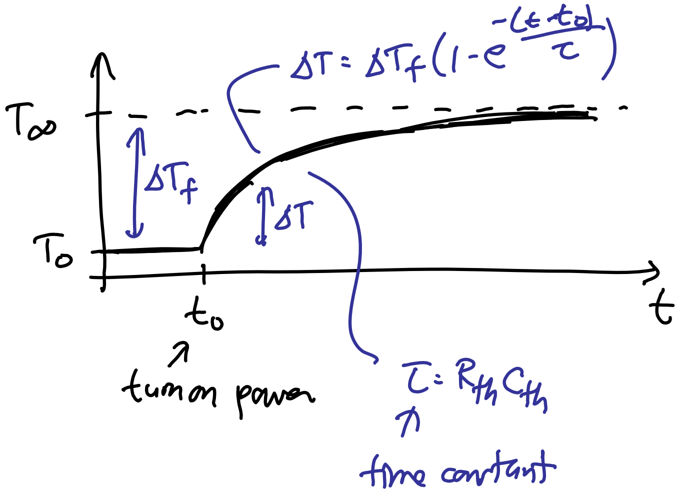
$C_{p,glass} = 0.5\ J/(g\cdot K)$

$K_{glass} = 1.05 \frac{W}{m\cdot K}$





$$\Delta T = T - T_o = PR_{th} \quad \text{(in steady-state)}$$

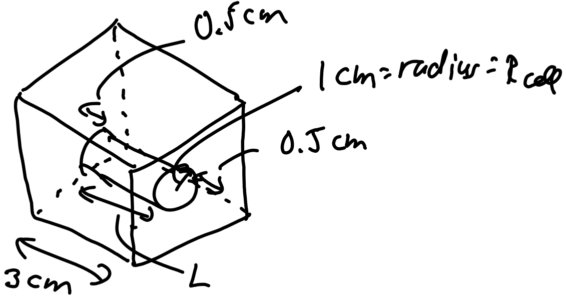


$$\Delta T = \Delta T_f\left(1 - e^{\frac{-(t-t_o)}{\tau}}\right)$$

$$\tau = R_{th}C_{th}$$

Find $C_{th,cell}$:

First, find the cell volume:

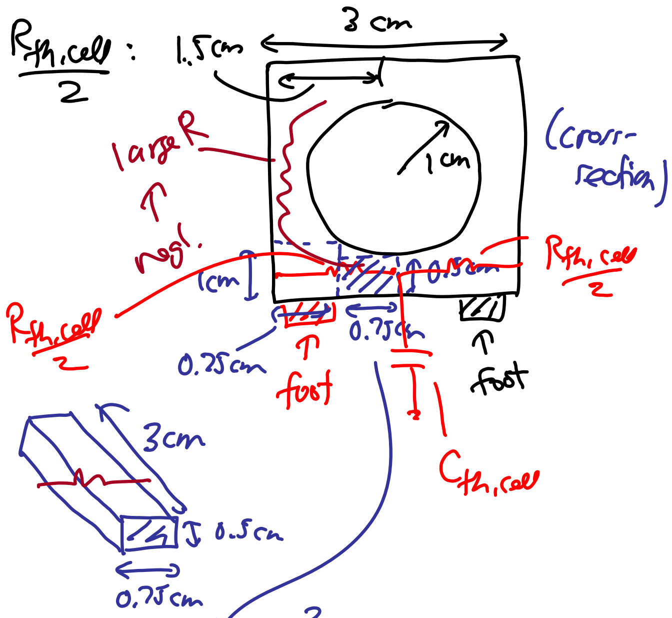


$$V_{cell} = hWL - \pi R_{cell}^2 \cdot L$$
$$= (3cm)(3cm)(3cm) - \pi(1cm)^2(2cm) = 20.7\ cm^3$$

$$C_{th,cell} = \rho_{glass} V_{cell} C_{p,glass}$$
$$= \left(2500 \frac{\cancel{kg}}{\cancel{m^3}}\right)\left(1000 \frac{\cancel{g}}{\cancel{kg}}\right)\left(\frac{1}{100^3}\frac{\cancel{m^3}}{\cancel{cm^3}}\right)$$
$$\times\ (20.7\cancel{cm^3})\left(0.5 \frac{J}{\cancel{g}\cdot K}\right)$$

$$\rightarrow C_{th,cell} = \underline{\underline{25.9\ J/K}}$$

Find $\frac{R_{th,cell}}{2}$:



$$\frac{R_{th,cell}}{2} = \frac{\frac{3}{4}}{k(3)(\frac{1}{2})} + \frac{\frac{3}{4}}{k(3)(1)} = \frac{1}{k}\left(\frac{1}{8}+\frac{1}{4}\right) = \frac{3}{8}\frac{1}{k}$$

$$\left[R_{th} = \frac{l}{kA}\right]$$

$$\therefore \frac{R_{th,cell}}{2} = \frac{3}{8}\frac{1}{1.05} \times \left(100\,\frac{cm}{m}\right)$$

$$= \underline{\underline{35.7\ K/W}}$$