Lecture 5: Benefits of Scaling III

- Announcements:
- HW#1 due this coming Thursday at 10 a.m. in the EE247B/ME218 Homework box near 140 Cory
- Hopefully, you've watched the video lectures from last week; otherwise, you'll have a hard time understanding this lecture
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- Today:
- Reading: Senturia, Chapter 1
- Lecture Topics:
  - Benefits of Miniaturization
  - Examples
    - GHz micromechanical resonators
    - Chip-scale atomic clock
    - Thermal Circuits
    - Micro gas chromatograph
- Probably won't get to it, but next up is:
- Senturia, Chpt. 3; Jaeger, Chpt. 2, 3, 6
  - Example MEMS fabrication processes
  - Photolithography
  - Etching
  - Oxidation
  - Film Deposition
  - Ion Implantation
  - Diffusion
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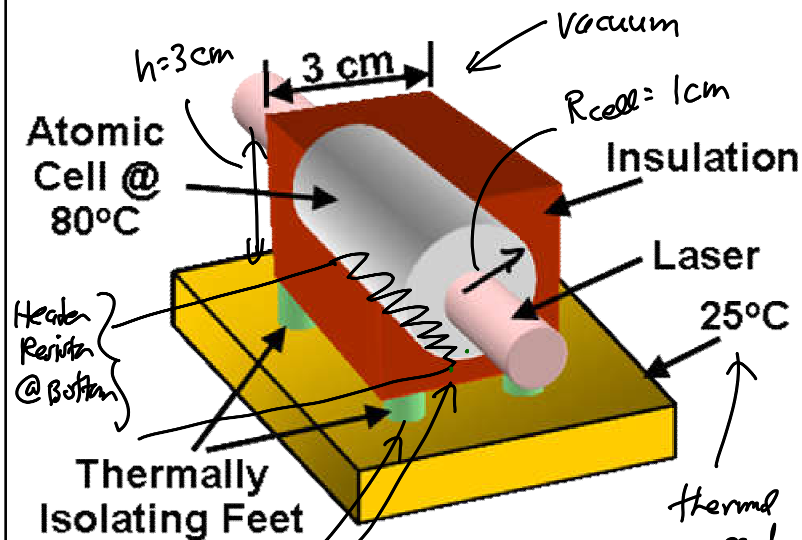
- Last Time: Thermal circuit modeling

Example: Thermal Ckt.

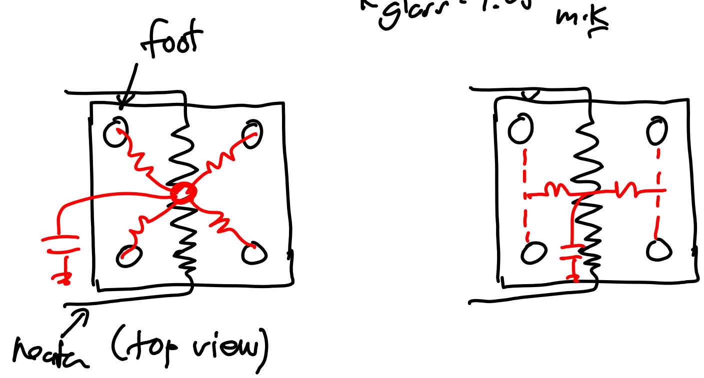
⇒ determine the power needed to get this atomic cell to 80°C (from RT) & how fast



all glass: $\rho_{glass} = 2500\ kg/m^3$

$c_{p,glass} = 0.5\ J/(g \cdot K)$

$K_{glass} = 1.05\ \frac{W}{m \cdot K}$



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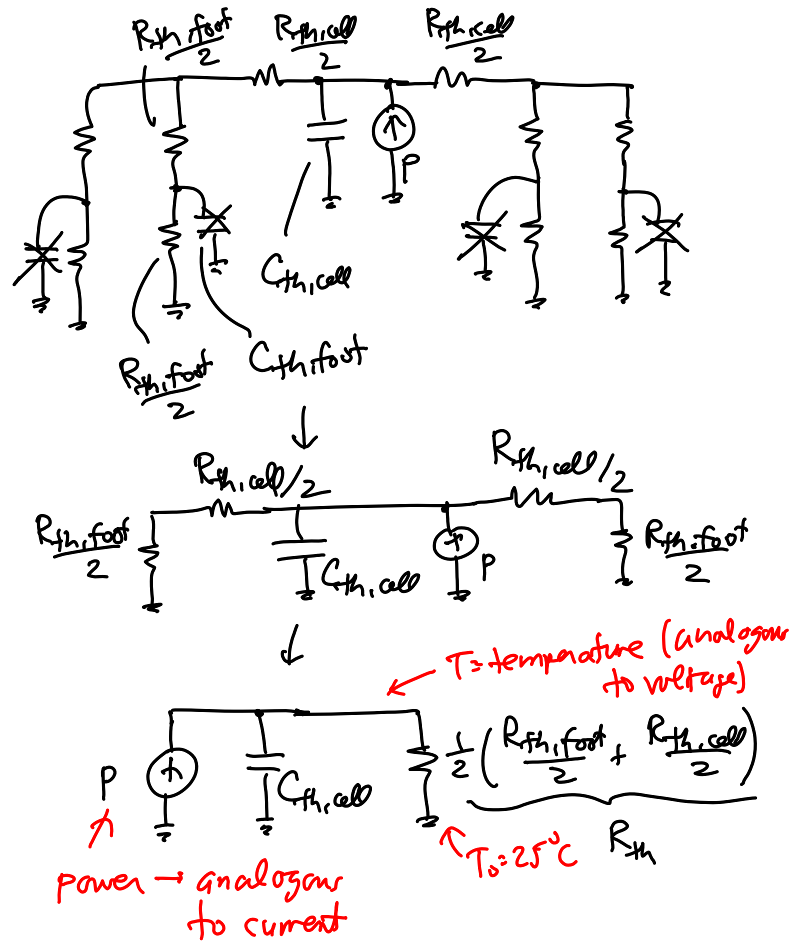
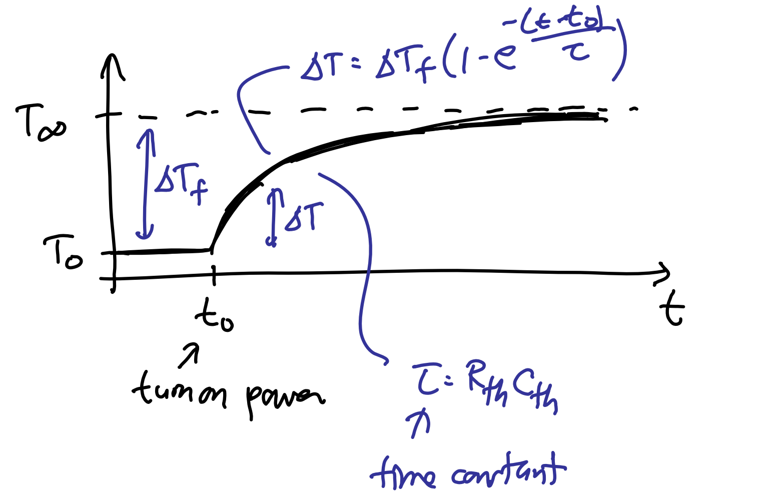
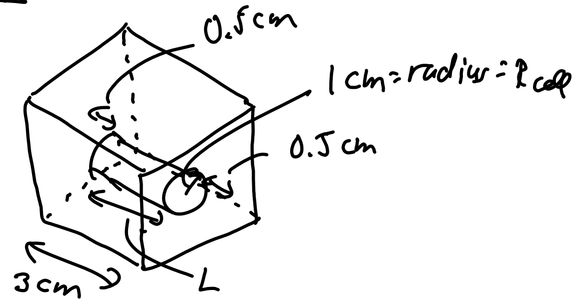


$\Delta T = T - T_0 = P R_{th}$ (in steady-state)



Find $C_{th,cell}$:

First, find the cell volume:



$V_{cell} = hWL - \pi R_{cell}^2 \cdot L$

$= (3cm)(3cm)(3cm) - \pi(1cm)^2(2cm) = 20.7\ cm^3$

$C_{th,cell} = \rho_{glass} V_{cell} C_{p,glass}$

$= \left(2500 \frac{kg}{m^3}\right)\left(1000 \frac{g}{kg}\right)\left(\frac{1}{100^3} \frac{m^3}{cm^3}\right)$

$\times (20.7\ cm^3)\left(0.5 \frac{J}{g \cdot K}\right)$

$\Rightarrow C_{th,cell} = 25.9\ J/K$

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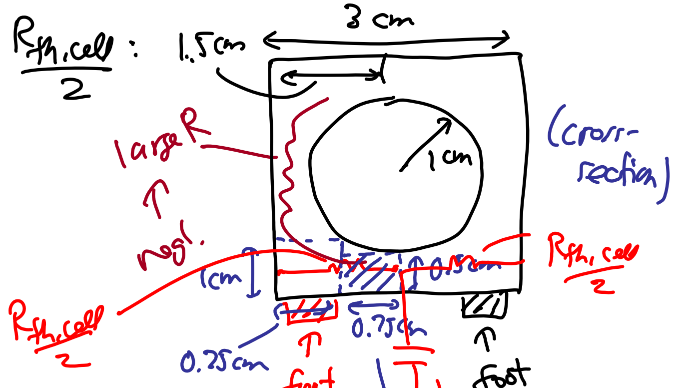
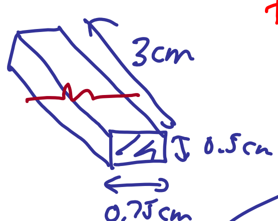
Find $\frac{R_{th,cell}}{2}$:

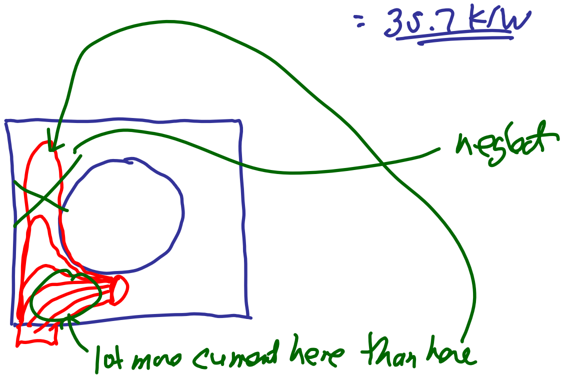




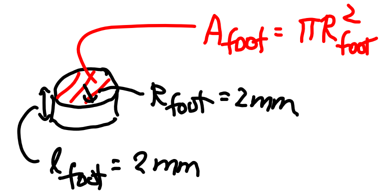
$$\frac{R_{th,cell}}{2} = \frac{\frac{3}{4}}{k(3)(\frac{1}{2})} + \frac{\frac{3}{4}}{k(3)(1)} = \frac{1}{k}\left(\frac{1}{8}+\frac{1}{4}\right) = \frac{3}{8}\frac{1}{k}$$

$\left[R_{th} = \frac{\ell}{kA}\right]$

$$\therefore \frac{R_{th,cell}}{2} = \frac{3}{8}\frac{1}{1.05} \times \left(100\,\frac{cm}{m}\right) = 35.7\ K/W$$



Find $R_{th,foot}$:



$$\therefore R_{th,foot} = \frac{\ell_{foot}}{kA_{foot}} = \frac{2\,mm}{(1.05\frac{W}{m\cdot K})\pi(2mm)^2} = 151.6\,\frac{K}{W}$$

Then:

$$R_{th} = \frac{1}{2}\left(\frac{R_{th,foot}}{2} + \frac{R_{th,cell}}{2}\right) = \frac{1}{2}\left(\frac{151.6}{2} + 35.7\right)$$

$$\Rightarrow R_{th} = 55.8\ K/W$$

⇒ Find the power req'd to maintain $T_\infty = 80°C$ in steady-state:

$$P = \frac{T_\infty - T_o}{R_{th}} = \frac{(80-25)}{55.8} = 0.99W \sim 1W$$

⇒ Find the time constant:

$$\tau = R_{th}C_{th,cell} = \underbrace{(55.8)}_{K/W}\underbrace{(25.9)}_{J/K} = 24\ min.$$

It takes ~3τ to reach steady-state
∴ must wait 72 min. before using this atomic cell

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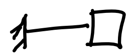
How about using MEMS?

⇒ same question as: "How about scaling this?"

⇒ much smaller cell volume → weight↓
↘ Volume↓ → C_{th}↓

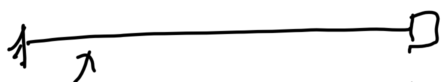
⇒ can get away from feet support.
↳ move to tethers

Macro:



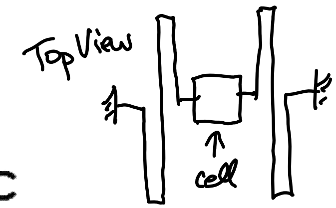
↓ shrink dimensions

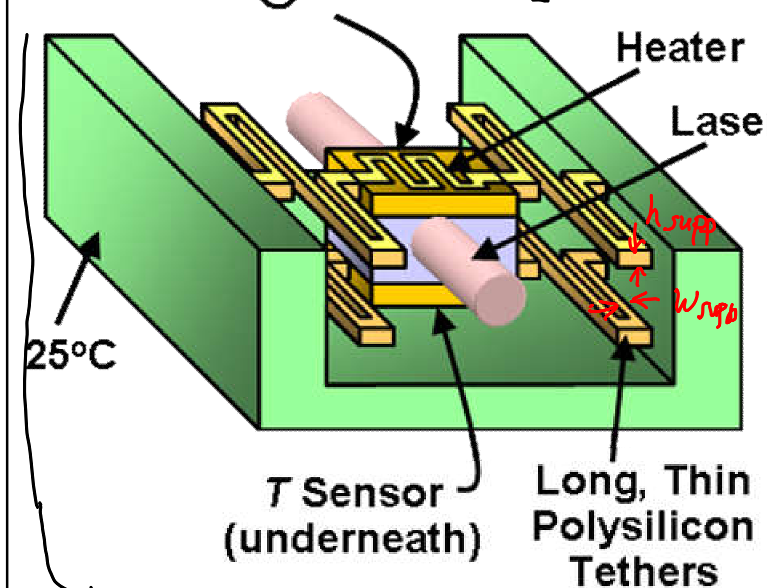
Micro:



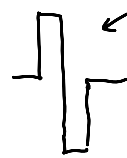
much longer, thinner support to suspend the cell (w/o droop)

MEMS Atomic Cell





Cell is hollow w/ 10μm-thick walls.



folded to save space & relieve stress
Unfolded length = 500μm-long
10μm-thick, 20μm-wide

$V_{cell} = (300\mu)(300\mu)(300\mu) - (280\mu)(280\mu)(280\mu)$
$= 5.048\times10^{-12}\ m^3$
(much smaller than macro cell!)

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$C_{th,cell} = \rho_{glass} V_{cell} C_{p,glass}$

$= (2500 \frac{kg}{m^3})(5.048 \times 10^{-12} m^3)(500 \frac{J}{kg \cdot K})$

$= 6.31 \times 10^{-6}$ J/K ← ~4 million x smaller than macro!

$R_{th,supp} = \frac{L_{supp}}{k_{polysi} W_{supp} \cdot h_{supp}} = \frac{500\mu}{(30 \frac{W}{mK})(20\mu)(10\mu)}$

→ $R_{th,supp} = 83,333$ K/W → 1493X larger than macro!

and...

$P = \frac{(80-25)}{83,333} =$ 0.66 mW ← 1493X smaller than macro!

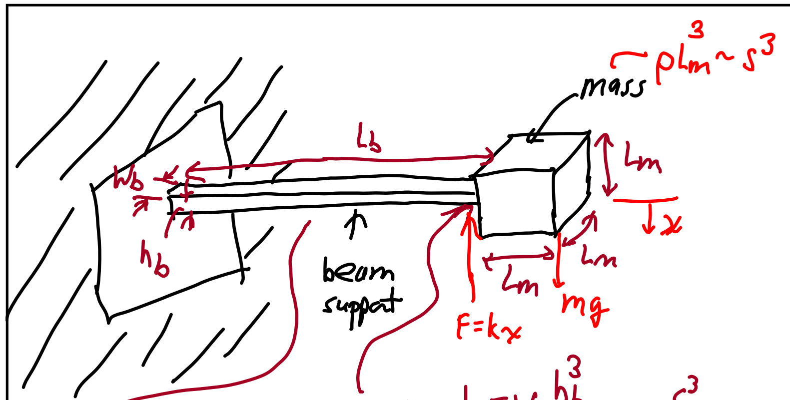
$t =$ 0.53s ← 2727X faster than macro!

All due to scaling!

Remarks. (What makes this possible?)

① Scaling reduces $C_{th} \sim l^3 \rightarrow s^3$
$s\downarrow \rightarrow C_{th}\downarrow\downarrow\downarrow$

② Scaling allows the use of long, thin tethers
$R_{th}\uparrow\uparrow\uparrow$



$k \triangleq$ stiffness @ this pt. $= \frac{1}{4} E W_b \frac{h_b^3}{L_b^3} \sim s \frac{s^3}{s^3} \sim s$

@ static equilibrium:

Force due to Gravity = Spring Force

$mg = kx$ ★ (g: acceleration due to gravity; x: displacement)

$x = \frac{m}{k} g \sim \frac{s^3}{s} \sim s^2$ → as $s\downarrow \rightarrow x\downarrow\downarrow$

(droop x)

$R_{th} = \frac{L_b}{k_{th} W_b h_b}$ → want to raise this (for a lower power consuming atomic cell) ↘ but maintain the same droop x

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$$* \leftarrow \rho L_m^3 g = \frac{1}{4} E W_b \frac{h_b^3}{L_b^3} x$$

$$\frac{L_b}{W_b h_b} = \frac{1}{4} E \frac{h_b^2}{L_b^3} x \frac{1}{\rho L_m^3 g} \sim \frac{S^2}{S^2}\frac{1}{S^3} \sim \frac{1}{S^3}$$

$\sim R_{th}$

as $S \downarrow \rightarrow \frac{L_b}{W_b h_b} \sim R_{th} \uparrow\uparrow\uparrow$

- Go through slides 30-31 and 37-48 in Module 2 to finish up Thermal Circuits and cover Micro Gas Analyzers

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