EE C247B – ME C218
Introduction to MEMS Design
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Lecture Module 12: Capacitive Transducers

Lecture Outline

• Reading: Senturia, Chpt. 5, Chpt. 6
• Lecture Topics:
  ☀ Energy Conserving Transducers
    ◦ Charge Control
    ◦ Voltage Control
  ☀ Parallel-Plate Capacitive Transducers
    ◦ Linearizing Capacitive Actuators
    ◦ Electrical Stiffness
  ☀ Electrostatic Comb-Drive
    ◦ 1st Order Analysis
    ◦ 2nd Order Analysis
Basic Physics of Electrostatic Actuation

- **Goal**: Determine gap spacing \( g \) as a function of input variables.
- **First**, need to determine the energy of the system.
- **Two ways to change the energy**:
  - Change the charge \( q \)
  - Change the separation \( g \)

\[
\Delta W(q, g) = V\Delta q + F_e \Delta g
\]

\[
dW = Vdq + F_e dg
\]

- **Note**: We assume that the plates are supported elastically, so they don't collapse.

Stored Energy

- Here, the stored energy is the work done in increasing the gap after charging capacitor at zero gap.

\[
W = 0 + \int_0^g F_e dg = \frac{q^2}{2 \varepsilon A}
\]

For a capacitor \( C \):
\[
q: CV \rightarrow V: \frac{q}{C}
\]

\[
W(q) = \int_0^q V dq = \int_0^q \left( \frac{q}{C} \right) dq = \frac{1}{2} \frac{q^2}{C} + \frac{1}{2} \frac{q^2}{2} \varepsilon A W(q)
\]
Charge-Control Case

* Having found stored energy, we can now find the force acting on the plates and the voltage across them:

From $dW = Vdq + Fdg$:

$F_e = \frac{\partial W(q,g)}{\partial g}$

$\Rightarrow$ Force is given by:

$F_e = \frac{1}{2} \frac{q^2}{\varepsilon A}$

$\Rightarrow$ Voltage is given by:

$V = \frac{1}{2} \frac{q^2}{\varepsilon A}$

Voltage-Control Case

* Practical situation: We control $V$

- Charge control on the typical sub-pF MEMS actuation capacitor is difficult
- Need to find $F_e$ as a partial derivative of the stored energy $W = W(V,g)$ with respect to $g$ with $V$ held constant? But can't do this with present $W(q,g)$ formula
- Solution: Apply Legendre transformation and define the co-energy $W'(V,g)$

Energy (e.g., force, voltage, ...)

$W(q_e) = \int_0^{q_e} dq_e \int_0^{q_e} \Phi(q_e) dq_e$

For a linear system, these will be equal.

$W'(e_e) = \int_0^{e_e} dq_e \int_0^{e_e} \Phi'(e_e) dq_e$

Can define co-energy as: $W'(e) = eq - W(q)$ (from this plot)
**Co-Energy Formulation**

* For our present problem (i.e., movable capacitive plates), the co-energy formulation becomes

\[ \mathcal{W}'(V, g) = q V - \mathcal{W}(q, g) \]

Differentially, this becomes:

\[ \frac{d\mathcal{W}'(V, g)}{dV} = (q dV + V dq) - d\mathcal{W}(q, g) \]

But \( d\mathcal{W}(q, g) = F_e dq + V dq \)

\[ \frac{d\mathcal{W}'(V, g)}{dV} = q dV - F_e dq \]

From which:

Charge, \( Q = \left. \frac{\partial \mathcal{W}'(V, g)}{\partial V} \right|_{g = \text{const.}} \)

Force, \( F_e = -\left. \frac{\partial \mathcal{W}'(V, g)}{\partial g} \right|_{V = \text{const.}} \)  

This gives force as a function of applied voltage.

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**Electrostatic Force (Voltage Control)**

* Find co-energy in terms of voltage (with gap held constant)

\[ \mathcal{W}' = \int_0^V g(V, V') dV' = \int_0^V \left( \frac{\varepsilon A}{g} \right) V' dV' = \frac{1}{2} \left( \frac{\varepsilon A}{g} \right) V^2 = \frac{1}{2} CV^2 \]

(as expected)

* Variation of co-energy with respect to gap yields electrostatic force:

\[ F_e = -\left. \frac{\partial \mathcal{W}'(V, g)}{\partial g} \right|_V = -\frac{1}{2} \left( \frac{\varepsilon A}{g^2} \right) V^2 = \frac{1}{2} C V^2 \]

* Strong function of gap!

* Variation of co-energy with respect to voltage yields charge:

\[ q = \left. \frac{\partial \mathcal{W}'(V, g)}{\partial V} \right|_g = \left( \frac{\varepsilon A}{g} \right) V = CV \]

as expected
Spring-Suspended Capacitive Plate

Charge Control of a Spring-Suspended Capacitive Plate

Force generated by charge \( q \) supplied by current \( I \):

\[
F_e = \frac{\partial W(q,g)}{\partial g} = \frac{q^2}{2\varepsilon A}
\]

Restoring force of spring:

\[
F_{\text{spring}} = k \Delta = F_e
\]

(@ equilibrium)

Initial gap:

\[
V = \frac{q}{C} = \frac{q}{\varepsilon A} g = \frac{q}{\varepsilon A} \left( g_0 - \frac{1}{2} \varepsilon A \frac{k}{k} \right) V
\]

\( \Rightarrow V \perp as~g \rightarrow 0 \)
**Voltage Control of a Spring-Suspended C**

Again, \( F_{\text{Spring}} = kz = F_e \)

But now:

\[
F_e = \frac{\varepsilon A V^2}{2g} \left( \frac{V_{g - g}}{g} \right)
\]

And the gap:

\[
g = g_0 - z = g_0 - \frac{F_e}{k} = g_0 - \frac{1}{2g} \frac{\varepsilon A V^2}{k} = g_0 - \frac{\varepsilon A V^2}{g} = g
\]

\( g \) shows up on both sides!

Charge (for a stable gap):

\[
Q = \frac{\varepsilon A V^2}{g} = CV = \frac{\varepsilon A V}{g} = q
\]

**Stability Analysis**

- Net attractive force on the plate:

\[
F_{\text{net}} = \frac{\varepsilon A V^2}{2g^2} \frac{k}{F_e} (g_0 - g)
\]

- An increment in gap \( dg \) leads to an increment in net attractive force \( dF_{\text{net}} \)

\[
dF_{\text{net}} = \frac{dF_{\text{net}}}{dg} \, dg = \left[ -\frac{\varepsilon A V^2}{g^3} + k \right] \, dg
\]

\( F_{\text{net}} \rightarrow dF_{\text{net}} = (-) \)

\( k > \frac{\varepsilon A V^2}{g^3} \)

For a stable state

otherwise, plate collapses
**Pull-In Voltage \( V_{PI} \)**

- \( V_{PI} \) = voltage at which the plates collapse
- The plate goes unstable when
  \[
  k = \frac{\varepsilon AV_{PI}^2}{g_{PI}^3} \quad \text{(1)} \quad \text{and} \quad F_{net} = 0 = \frac{\varepsilon AV_{PI}^2}{2g_{PI}^2} - k(g_o - g_{PI}) \quad \text{(2)}
  \]

* Substituting (1) into (2):
  \[
  0 = \frac{\varepsilon AV_{PI}^2}{2g_{PI}^2} - \frac{\varepsilon AV_{PI}^2}{g_{PI}^2}(g_o - g_{PI})
  \]

* When a gap is driven by a voltage to \( \frac{2}{3} \) its original spacing, collapse will occur!

**Voltage-Controlled Plate Stability Graph**

- Below: Plot of normalized electrostatic and spring forces vs. normalized displacement \( 1 - (g/g_o) \)

- Spring Force
- Electrical Forces

Stable Equilibrium Points

Increasing \( V \)

Normalized Displacement
Advantages of Electrostatic Actuators

• Easy to manufacture in micromachining processes, since conductors and air gaps are all that’s needed → low cost!
• Energy conserving → only parasitic energy loss through $I^2R$ losses in conductors and interconnects
• Variety of geometries available that allow tailoring of the relationships between voltage, force, and displacement
• Electrostatic forces can become very large when dimensions shrink → electrostatics scales well!
• Same capacitive structures can be used for both drive and sense of velocity or displacement
• Simplicity of transducer greatly reduces mechanical energy losses, allowing the highest Q’s for resonant structures

Problems With Electrostatic Actuators

• Nonlinear voltage-to-force transfer function
• Relatively weak compared with other transducers (e.g., piezoelectric), but things get better as dimensions scale
Linearizing the Voltage-to-Force Transfer Function

- Apply a DC bias (or polarization) voltage \( V_p \) together with the intended input (or drive) voltage \( v_i(t) \), where \( V_P \gg v_i(t) \)

\[
v(t) = V_P + v_i(t)
\]

\[
F_e(t) = \frac{\partial^2 w}{\partial x^2} \cdot \frac{\partial}{\partial x} \left( \frac{1}{2} C \left( \frac{d}{g_o} \right)^2 \right)
\]

\[
= \frac{1}{2} \frac{2C}{g_o^2} \left( V_P(t) \right)^2 + \frac{1}{2} \left( V_P + v_i(t) \right)^2 \frac{2C}{g_o^2} + \frac{1}{2} \left( V_P + v_i(t) \right)^2 \frac{2C}{g_o^2}
\]

\[
\approx \frac{1}{2} \left( V_P + 2V_P v_i(t) + [v_i(t)]^2 \right) \frac{2C}{g_o^2}
\]

\[
[V_P \gg v_i(t) \Rightarrow F_e(t) \approx \frac{1}{2} V_P^2 \frac{2C}{g_o^2} + V_P \frac{2C}{g_o^2} v_i(t)]
\]

DC Offset: \( V_P \)
AC drive signal: \( V_P + v_i(t) \)

\[
C(x) = \frac{6A}{g_o^2} \approx C_0 \left( 1 - \frac{x}{L} \right)^n \approx C_0 \left( 1 + \frac{x}{g_o} \right)
\]

\[
\Rightarrow F_e(t) \approx \frac{1}{2} C_0 \frac{2C}{g_o} V_P^2 + V_P \frac{2C}{g_o} v_i(t)
\]
Differential Capacitive Transducer

- The net force on the suspended center electrode is

\[
F_{\text{net}} = F_{\text{at}}(t) - F_{\text{st}}(t)
\]

Do the math.

\[
F_{\text{net}}(t) = \frac{\varepsilon}{2} \frac{d^2}{dt^2} \left[ \left( V_P(t) \right)^2 - \left[ V_G(t) \right]^2 \right]
\]

\[
= \frac{1}{2} \frac{\varepsilon}{d^2} \left[ \left( V_P + 2V_P N(t) + \frac{C}{2} \frac{ds}{dt} \right)^2 - \left( V_P - 2V_P N(t) + \frac{C}{2} \frac{ds}{dt} \right)^2 \right]
\]

\[
F_{\text{net}}(t) = 2V_P \frac{\varepsilon}{d^2} N(t) \quad \text{Linear if N(t) is linear in gap}
\]

Remaining Nonlinearity (Electrical Stiffness)
Parallel-Plate Capacitive Nonlinearity

- **Example**: clamped-clamped laterally driven beam with balanced electrodes
- **Nomenclature**:
  - $V_a$ or $v_A$ vs. $V_1$
  - $v_a = v_a(t) \cos \omega t$
  - $V_a$ or $v_A = V_4 + v_a$

**DC Component** (upper case variable; upper case subscript)

**AC or Signal Component** (lower case variable; lower case subscript)

**Expression for $\frac{\partial C}{\partial x}$**:

$$ C(x) \cdot \frac{\partial A}{\partial x} = C_0 \left( \frac{\partial x}{\partial x} + A_1 \frac{\partial x^2}{\partial x^2} + A_2 \frac{\partial x^3}{\partial x^3} + \cdots \right) $$

- $C_0 = \frac{C_0}{d_1}$
- $A_1 = \frac{2}{d_1}$
- $A_2 = \frac{3}{d_1^2}$
- $A_3 = \frac{4}{d_1^3}$
- $\cdots$

**Expand the Taylor series further**

$$ \frac{\partial C}{\partial x} = C_0 \left( 1 + A_1 \frac{\partial x^2}{\partial x^2} + A_2 \frac{\partial x^3}{\partial x^3} + \cdots \right) $$

**Conductive Structure**

- $k_m$

**Electrode**

- $d_1$

**Conductive Structure**

- $m$

**Force $F_{dl}$**

- $x$

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Parallel-Plate Capacitive Nonlinearity

Thus, the expression for force from the left side becomes:

\[ F_{dl} = \frac{1}{2} \frac{\partial C}{\partial x} (V_p - V_i)^2 - \frac{1}{2} \frac{\partial C}{\partial x} (V_p - V_i)^2 \]

\[ \left\{ \text{small displacements: } x \ll d_i \right\} \]

\[ F_{dl} = \frac{1}{2} \left( - \frac{C_{ol}}{d_i} \right) \left( 1 + A_i \frac{x}{d_i} \right) (V_p^2 - 2V_i^2) + A_i V_p^2 x \left( - 2A_i V_i x + A_i x v_i^2 \right) \]

@ resonance: \( \chi \approx \frac{Q F_{dl}}{j k} \approx \frac{Q}{j k} \frac{\partial C}{\partial x} V_p V_i \)

Thus:

\( V_i = (V_p \cos \omega t) \to x = V_p \sin \omega t \)

\( x = 90^\circ \) phase-shifted from \( V_i \)

• Retaining only terms at the drive frequency:

\[ F_{dl} \bigg|_{ob} = V_p \frac{C_{ol}}{d_1} |v_1| \cos \omega_o t + V_p^2 \frac{C_{ol}}{d_1^2} |x| \sin \omega_o t \]

Drive force arising from the input excitation voltage at the frequency of this voltage

Proportional to displacement

90° phase-shifted from drive, so in phase with displacement

• These two together mean that this force acts against the spring restoring force!

\( k_e = V_p^2 \frac{C_{ol}}{d_1^2} = V_p^2 \frac{\varepsilon A}{d_1^2} \)

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Electrical Stiffness, $k_e$

- The electrical stiffness $k_e$ behaves like any other stiffness.
- It affects resonance frequency:

$$\omega' = \sqrt{\frac{k}{m}} = \sqrt{\frac{k_m - k_e}{m}}$$

$$= \frac{k_m}{m} \left( 1 - \frac{k_e}{k_m} \right)^{1/2}$$

$$\omega' = \omega_p \left( 1 - \frac{V_p^2 \varepsilon A}{k_m d_1^3} \right)^{1/2}$$

Frequency is now a function of dc-bias $V_p$.

Voltage-Controllable Center Frequency

- Quadrature force $\Rightarrow$ voltage-controllable electrical stiffness:

$$k_e = \varepsilon \frac{A}{d^3} V_p^2$$

$$f_0 = \frac{1}{2 \pi} \sqrt{\frac{k_m - k_e}{m_r}}$$

Graph showing the change in frequency with DC-bias $V_p$.
* Microresonator Thermal Stability

- Thermal stability of poly-Si micromechanical resonator is 10X worse than the worst case of AT-cut quartz crystal.

- Use a temperature dependent mechanical stiffness to null frequency shifts due to Young’s modulus thermal dep.

* Problems:
  - stress relaxation
  - compromised design flexibility

[Ref: Hsu et al., IEDM'00]
Excellent Temperature Stability

Temperature Increasing

Top Electrode-to-Resonator Gap ↑
Elect. Stiffness: \( k_e \sim 1/d^3 \)  
Frequency: \( f_0 \sim (k_e - k_e')^{0.5} \)  
Counteracts reduction in frequency due to Young's modulus temp. dependence

Top Metal Electrode

[HSU et al MEMS'02]

Uncompensated \( \mu \)resonator

\[ \text{Elect.-Stiffness Compensation} \quad -0.24 \text{ppm/°C} \]

\[ \text{AT-cut Quartz Crystal at Various Cut Angles} \]

\[ \text{On par with quartz!} \]

Electrode

Resonator

Top Metal Electrode

[HSU et al MEMS'02]

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\[ \text{Temperature increasing} \]

Top Electrode-to-Resonator Gap ↑
Elect. Stiffness: \( k_e \sim 1/d^3 \)
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\[ \text{On par with quartz!} \]
**Module 12: Capacitive Transducers**

**Measured $\Delta f/f$ vs. $T$ for $k_e$-Compensated $\mu$Resonators**

- Slits help to release the stress generated by lateral thermal expansion
- Linear $TC_f$ curves
- $-0.24$ ppm/$^\circ$C

**Design/Performance:**
- $f_o=10$ MHz, $Q=4,000$
- $V_p=8$ V, $h_e=4$ $\mu$m
- $d_o=1000$ A, $h=2$ $\mu$m
- $W=8$ $\mu$m, $L=40$ $\mu$m

[Hsu et al. MEMS '02]

*Can One Cancel $k_e$ w/ Two Electrodes?*

- What if we don’t like the dependence of frequency on $V_p$?
- Can we cancel $k_e$ via a differential input electrode configuration?
- If we do a similar analysis for $F_{d2}$ at Electrode 2:

$$F_{d2}|_{\omega_o} = -V_p^2 \frac{C_{o2}}{d_2^2} |v_2| \cos \omega_o t$$

$$+ V_p^2 \frac{C_{o2}}{d_2^2} |x| \sin \omega_o t$$

**Adds to the quadrature term $\rightarrow k_e$’s add, no matter the electrode configuration!**
Problems With Parallel-Plate C Drive

• Nonlinear voltage-to-force transfer function
  - Resonance frequency becomes dependent on parameters (e.g., bias voltage $V_p$)
  - Output current will also take on nonlinear characteristics as amplitude grows (i.e., as $x$ approaches $d_o$)
  - Noise can alias due to nonlinearity
• Range of motion is small
  - For larger motion, need larger gap ... but larger gap weakens the electrostatic force
  - Large motion is often needed (e.g., by gyroscopes, vibromotors, optical MEMS)

Electrostatic Comb Drive
Electrostatic Comb Drive

- Use of comb-capacitive transducers brings many benefits
  - Linearizes voltage-generated input forces
  - (Ideally) eliminates dependence of frequency on dc-bias
  - Allows a large range of motion

Comb-Driven Folded Beam Actuator

Comb-Drive Force Equation (1st Pass)

\[ F_d = \frac{2k}{d} \frac{d^2 \phi}{dx^2} = \frac{1}{2} \frac{d^2}{dx^2} \left( v_p \cdot N_s \right)^2 - \frac{2}{d} \frac{\delta e_h}{d} \left( v_p^2 - 2 v_p v^2 + \frac{v^4}{2} \right) \approx \frac{2}{d} \frac{\delta e_h}{d} N_s^2 = F_{id} \]
Lateral Comb-Drive Electrical Stiffness

Again:

\[ C(x) = \frac{2N\varepsilon dx}{d} \rightarrow \frac{\partial C}{\partial x} = \frac{2N\varepsilon}{d} \]

* No \( (\partial C/\partial x) \) \( x \)-dependence → no electrical stiffness: \( k_e = 0 \)

* Frequency immune to changes in \( V_P \) or gap spacing!

Typical Drive & Sense Configuration

Simple Analysis:

\[ F_{d1} = \frac{1}{2} \frac{\varepsilon \partial C}{\partial x} (V_I^2 - V_P^2) \frac{V_{d1}^2}{d^2} \left( 2(2V_{d1} - V_I)(2V_{d1} - V_P)(2V_{d1} - V_{d2}) \right) \]

\[ F_{d2} = \frac{1}{2} \frac{\varepsilon \partial C}{\partial x} (V_P^2 - V_S^2) \frac{V_{d2}^2}{d^2} \left( 2(2V_{d2} - V_I)(2V_{d2} - V_P)(2V_{d2} - V_{d1}) \right) \]

\[ F_{\text{net}} = F_{d1} + F_{d2} = \frac{1}{2} \frac{\varepsilon \partial C}{\partial x} (V_I^2 - V_P^2 - 2(V_{d1}V_{d2} - V_I)(2V_{d1} - V_P) + V_{d2}^2 - V_{d1}^2) \]

\[ F_{\text{net}} = z(2N)(\varepsilon \varepsilon) V_I V_P \]
Comb-Drive Force Equation (2nd Pass)

• In our 1st pass, we accounted for:
  - Parallel-plate capacitance between stator and rotor
• ... but neglected:
  - Fringing fields
  - Capacitance to the substrate
• All of these capacitors must be included when evaluating the energy expression!

Comb-Drive Force With Ground Plane Correction

• Finger displacement changes not only the capacitance between stator and rotor, but also between these structures and the ground plane → modifies the capacitive energy

\[ F_{\phi, x} = \frac{\partial W'}{\partial x} = \frac{1}{2} \frac{dC_{sp}}{dx} V_s^2 + \frac{1}{2} \frac{dC_{sr}}{dx} V_r^2 + \frac{1}{2} \frac{dC_{rn}}{dx} (V_q - V_r)^2 \]

[Gary Fedder, Ph.D., UC Berkeley, 1994]
**Capacitance Expressions**

- Case: \( V_r = V_p = 0 \)
- \( C_{sp} \) depends on whether or not fingers are engaged

\[
C_{sp} = N \left[ C_{sp,c} x + C_{sp,u} (L - x) \right]
\]

\[
C_{ys} = NC_{ys} \frac{1}{x}
\]

* Capacitance per unit length

**Region 2**

**Region 3**

[Gary Fedder, Ph.D., UC Berkeley, 1994]

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**Comb-Drive Force With Ground Plane Correction**

- Finger displacement changes not only the capacitance between stator and rotor, but also between these structures and the ground plane → modifies the capacitive energy

\[
F_{g,x} = \frac{\partial W'}{\partial x} = \frac{1}{2} \frac{dC_{sp}}{dx} V_s^2 + \frac{1}{2} \frac{dC_{sp}}{dx} V_s^2 + \frac{1}{2} \frac{dC_{ys}}{dx} (V_q - V_p)^2
\]

\[
F_{e,x} = \frac{N}{2} (C_{ys} + C_{sp,c} - C_{sp,u}) V_s^2
\]

(for \( V_q = V_p = 0 \))

[Gary Fedder, Ph.D., UC Berkeley, 1994]
Simulate to Get Capacitors → Force

- Below: 2D finite element simulation

\[ F_{x,n} = \frac{N}{2} (C_{rs}^{e} + C_{sp,n}^{e} - C_{sp,n}^{r}) F_{e,x}^2 \]

20-40\% reduction of \( F_{e,x} \)

Vertical Force (Levitation)

\[ F_{e,z} = \frac{\partial W'}{\partial z} = \frac{1}{2} dC_{sp}^{r} V_{s}^2 + \frac{1}{2} dC_{rp}^{r} V_{r}^2 + \frac{1}{2} dC_{rs}^{r} (V_{s} - V_{r})^2 \]

* For \( V_{r} = 0V \) (as shown): \[ F_{e,z} = \frac{1}{2} N_{x} \left[ \frac{d(C_{sp,e}^{r} + C_{rs}^{r})}{dz} \right] V_{s}^2 \]
Simulated Levitation Force

* Below: simulated vertical force $F_z$ vs. $z$ at different $V_p$'s [f/ Bill Tang Ph.D., UCB, 1990]

$F_z$ is roughly proportional to $-z$ for $z$ less than $z_o$ → it's like an electrical stiffness that adds to the mechanical stiffness

$$F_z \approx \gamma_z V_p^2 \frac{(z_o - z)}{z_o} = k_e (z_o - z)$$

Equilibrium levitation, $z_o$

Vertical Levitation [mm]

Vertical Resonance Frequency

$$\frac{\omega_z}{\omega_{zo}} = \sqrt{\frac{k_z + k_e}{k_z}}$$

where $k_e = \left(\frac{\gamma_z}{z_o}\right) V_p^2$

Vertical resonance frequency

Vertical resonance frequency at $V_p = 0V$

Lateral resonance frequency

* Signs of electrical stiffnesses in MEMS:
  - Comb (x-axis) → $k_e = 0$
  - Comb (z-axis) → $k_e > 0$
  - Parallel Plate → $k_e < 0$

Applied voltage

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* Pattern ground plane polysilicon into differentially excited electrodes to minimize field lines terminating on top of comb
* Penalty: x-axis force is reduced

Force of Comb-Drive vs. Parallel-Plate

- Comb drive (x-direction)
  \( V_1 = V_2 = V_s = 1V \)
  \[ F_{e,x} = \frac{\varepsilon_o h d_o}{2} V_s^2 \]

- Differential Parallel-Plate (y-direction)
  \( V_1 = 0V, V_2 = 1V \)
  \[ F_{e,y} = \frac{1}{2} \frac{\varepsilon_o h L_d d_o}{d_o^2} V_s^2 \]
  \[ \frac{F_{e,y}}{F_{e,x}} = \frac{1}{2} \frac{\varepsilon_o h V_s^2}{d_o} \frac{L_d}{d_o} \]

Parallel-plate generates a much larger force; but at the cost of linearity