

EE C247B - ME C218

Introduction to MEMS Design


Fall 2018

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Module 17: Noise & MDS

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Lecture Outline

- Reading: Senturia Chpt. 16
- Lecture Topics:
 - ↳ Minimum Detectable Signal
 - ↳ Noise
 - Circuit Noise Calculations
 - Noise Sources
 - Equivalent Input-Referred Noise
 - ↳ Gyro MDS
 - Equivalent Noise Circuit
 - Example ARW Determination

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Determining Sensor Resolution

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Minimum Detectable Signal (MDS)

- **Minimum Detectable Signal (MDS):** Input signal level when the signal-to-noise ratio (SNR) is equal to unity

Sensed Signal

Sensor Scale Factor

Sensor Noise

Sensor

Circuit Gain

Circuit Output Noise

Signal Conditioning Circuit

Output

Includes desired output plus noise

- The sensor scale factor is governed by the sensor type
- The effect of noise is best determined via analysis of the equivalent circuit for the system

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Noise

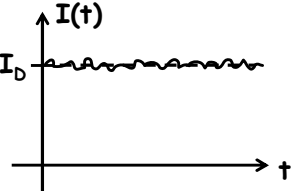
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Noise

- **Noise:** Random fluctuation of a given parameter $I(t)$
- In addition, a noise waveform has a zero average value

Avg. value (e.g. could be DC current) $\rightarrow I_D$



- We can't handle noise at instantaneous times
- But we can handle some of the averaged effects of random fluctuations by giving noise a power spectral density representation
- Thus, represent noise by its mean-square value:
Let $i(t) = I(t) - I_D$
Then $\overline{i^2} = \overline{(I - I_D)^2} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T |I - I_D|^2 dt$

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Noise Spectral Density

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- We can plot the spectral density of this mean-square value:

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Circuit Noise Calculations

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Inputs

$v_i(j\omega)$

$S_i(\omega)$

Deterministic

$H(j\omega)$

Linear Time-Invariant System

Random

Outputs

$v_o(j\omega)$

$S_o(\omega)$

No j → noise has random phase, so j is pointless!

$v_o(t)$

\leftrightarrow

$v_o(j\omega)$

$S_o(t)$

\leftrightarrow

$S_o(j\omega)$

Mean square spectral density

- Deterministic:** $v_o(j\omega) = H(j\omega)v_i(j\omega)$
- Random:** $S_o(\omega) = [H(j\omega)H^*(j\omega)]S_i(\omega) = |H(j\omega)|^2 S_i(\omega)$

$\sqrt{S_o(\omega)} = |H(j\omega)|\sqrt{S_i(\omega)}$ → How is it we can do this?

Root mean square amplitudes

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Handling Noise Deterministically

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- Can do this for noise in a tiny bandwidth (e.g., 1 Hz)

$\frac{v_{n1}^2}{\Delta f} = S_1(f) \rightarrow v_{n1} = \sqrt{S_1(f) \cdot B}$

Can approximate this by a sinusoidal voltage generator (especially for small B, say 1 Hz)

$\tau \sim \frac{1}{B}$

[This is actually the principle by which oscillators work → oscillators are just noise going through a tiny bandwidth filter] ← Why? Neither the amplitude nor the phase of a signal can change appreciably within a time period 1/B.

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Systematic Noise Calculation Procedure

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General Circuit With Several Noise Sources

- Assume noise sources are uncorrelated

- For i_{n1}^2 , replace w/ a deterministic source of value

$$i_{n1} = \sqrt{\frac{i_{n1}^2}{\Delta f}} \cdot (1 \text{ Hz})$$

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Systematic Noise Calculation Procedure


2. Calculate $v_{on1}(\omega) = i_{n1}(\omega)H(j\omega)$ (treating it like a deterministic signal)
3. Determine $\overline{v_{on1}^2} = \overline{i_{n1}^2} \cdot |H(j\omega)|^2$
4. Repeat for each noise source: $\overline{i_{n1}^2}, \overline{v_{n2}^2}, \overline{v_{n3}^2}$
5. Add noise power (mean square values)

$$\overline{v_{onTOT}^2} = \overline{v_{on1}^2} + \overline{v_{on2}^2} + \overline{v_{on3}^2} + \overline{v_{on4}^2} + \dots$$

$$v_{onTOT} = \sqrt{\overline{v_{on1}^2} + \overline{v_{on2}^2} + \overline{v_{on3}^2} + \overline{v_{on4}^2} + \dots}$$


↖
Total rms value

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Noise Sources


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Thermal Noise

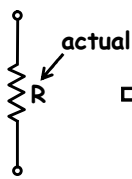
- **Thermal Noise in Electronics:** (Johnson noise, Nyquist noise)
 - ↳ Produced as a result of the thermally excited random motion of free e⁻'s in a conducting medium
 - ↳ Path of e⁻'s randomly oriented due to collisions
- **Thermal Noise in Mechanics:** (Brownian motion noise)
 - ↳ Thermal noise is associated with all dissipative processes that couple to the thermal domain
 - ↳ Any damping generates thermal noise, including gas damping, internal losses, etc.
- **Properties:**
 - ↳ Thermal noise is white (i.e., constant w/ frequency)
 - ↳ Proportional to temperature
 - ↳ Not associated with current
 - ↳ Present in any real physical resistor

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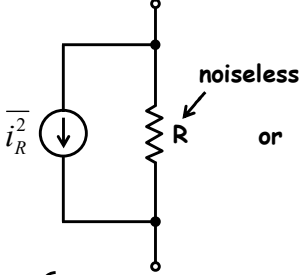
Circuit Representation of Thermal Noise

- Thermal Noise can be shown to be represented by a series voltage generator $\overline{v_R^2}$ or a shunt current generator $\overline{i_R^2}$



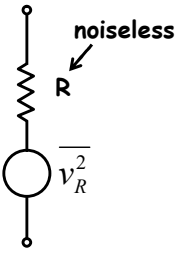
actual

⇒



noiseless

or



noiseless

$$\frac{\overline{i_R^2}}{\Delta f} = \frac{4kT}{R}$$

$$\frac{\overline{v_R^2}}{\Delta f} = 4kTR$$

}

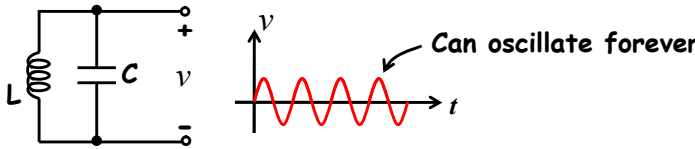
where $4kT = 1.66 \times 10^{-20} \text{ V} \cdot \text{C}$
and where these are spectral densities.

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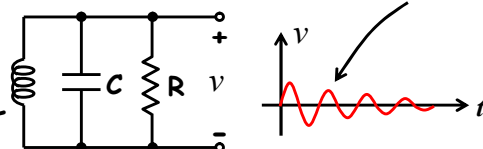
Noise in Capacitors and Inductors?

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- Resistors generate thermal noise
- Capacitors and inductors are noiseless → why?



- Now, add a resistor:



But this violates the laws of thermodynamics, which require that things be in constant motion at finite temperature

Need to add a forcing function, like a noise voltage $\overline{v_R^2}$ to keep the motion going → and this noise source is associated with R

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Why $4kTR$?

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- Why is $\overline{v_R^2} = 4kTR\Delta f$ (a heuristic argument)
- The Equipartition Theorem of Statistical Thermodynamics says that there is a mean energy $(1/2)kT$ associated w/ each degree of freedom in a given system
- An electronic circuit possesses two degrees of freedom:
 - ↪ Current, i , and voltage, v
 - ↪ Thus, we can write:

$$\frac{1}{2}Li^2 = \frac{1}{2}k_B T \quad , \quad \overbrace{\frac{1}{2}Cv^2}^{\text{Energy}} = \frac{1}{2}k_B T$$
- Similar expressions can be written for mechanical systems
 - ↪ For example: for displacement, x

$$\text{Spring constant} \quad \frac{1}{2}kx^2 = \frac{1}{2}k_B T$$

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Why 4kTR? (cont)

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- Why is $\overline{v_R^2} = 4kTR\Delta f$? (a heuristic argument)
- Consider an RC circuit:

$E = \frac{1}{2}kT = \frac{1}{2}C\overline{v_C^2}$
 $\therefore \overline{v_C^2} = \frac{kT}{C}$ ← integrated noise over all freqs.
 (total mean square voltage integrated over all freqs.)

Question: What value of $\frac{\overline{v_R^2}}{\Delta f}$ (assuming white noise) gives us this?

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Why 4kTR? (cont)


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Question: What value of $\frac{\overline{v_R^2}}{\Delta f}$ (assuming white noise) gives us $\overline{v_C^2} = \frac{kT}{C}$?

$\overline{v_C^2} = \int_0^\infty \left| \frac{1}{1+j\omega RC} \right|^2 \frac{\overline{v_R^2}}{\Delta f} d\omega$
 [noise is white] → $= \frac{1}{2\pi} \frac{\overline{v_R^2}}{\Delta f} \int_0^\infty \frac{\omega_b^2}{\omega_b^2 + \omega^2} d\omega$
 [$\omega_b = \frac{1}{RC}$]

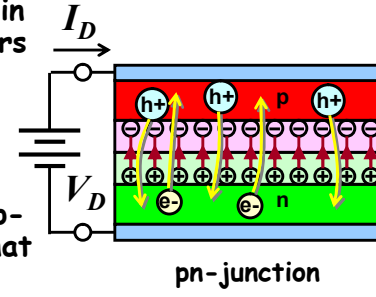
$\left[\int \frac{dx}{x^2+a^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) \right]$
 $= \frac{1}{2\pi} \frac{\overline{v_R^2}}{\Delta f} \frac{\omega_b^2}{\omega_b} \tan^{-1}\left(\frac{\omega}{\omega_b}\right) \Big|_0^\infty = \frac{1}{2\pi} \frac{\overline{v_R^2}}{\Delta f} \left(\frac{\pi}{2} \omega_b - 0 \right)$
 $= \frac{1}{4} \omega_b \frac{\overline{v_R^2}}{\Delta f} = \frac{kT}{C} \rightarrow \frac{\overline{v_R^2}}{\Delta f} = 4kT \left(\frac{\omega_b}{C} \right) \Rightarrow \frac{\overline{v_R^2}}{\Delta f} = 4kTR \quad \checkmark$

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Shot Noise

- Associated with direct current flow in diodes and bipolar junction transistors
- Arises from the random nature by which e⁻s and h⁺s surmount the potential barrier at a pn junction
- The DC current in a forward-biased diode is composed of h⁺s from the p-region and e⁻s from the n-region that have sufficient energy to overcome the potential barrier at the junction
 → noise process should be proportional to DC current
- **Attributes:**
 - ↳ Related to DC current over a barrier
 - ↳ Independent of temperature
 - ↳ White (i.e., const. w/ frequency)
 - ↳ Noise power ~ I_D & bandwidth




pn-junction

$$\frac{\overline{i_n^2}}{\Delta f} = 2qI_D$$

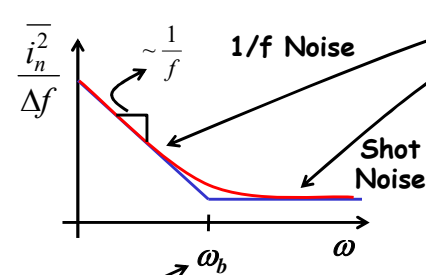
Charge on an e⁻ (=1.6×10⁻¹⁹C)
 DC Current

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Flicker (1/f) Noise

- In general, associated w/ random trapping & release of carriers from "slow" states
- Time constant associated with this process gives rise to a noise signal w/ energy concentrated at low frequencies
- Often, get a mean-square noise spectral density that looks like this:



$$\frac{\overline{i_n^2}}{\Delta f} = 2qI_D + K \left(\frac{I_D^a}{f^b} \right)$$

I_D = DC current
 K = const. for a particular device
 a = 0.5 → 2
 b ~ 1

1/f Noise Corner Frequency

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Example: Typical Noise Numbers

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• Hookup the circuit below and make some measurements

Measure w/ AC voltmeter
 Measure w/ spectrum analyzer
 ← Get Gaussian amplitude distribution

Probability
 Amplitude
 68% within $\pm \sigma$
 99.7% within $\pm 3\sigma$

area $\sim N_n^2$
 $4kTR$
 f
 $\frac{1}{2\pi RC}$

$1k\Omega: 4nV/\sqrt{Hz}$ (for every 1k of R)
 $1pF: \sqrt{\frac{kT}{C}} = 64\mu V_{rms}$

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Example: Typical Noise Numbers

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• Hookup the circuit below and make some measurements

Measure w/ AC voltmeter
 Measure w/ spectrum analyzer

AC Voltmeter
 $\sqrt{N_o^2} = (100)(64\mu V_{rms}) = 6.4mV_{rms}$

Spectrum Analyzer
 $\frac{1}{(2\pi)(1k)(1p)} = 60MHz$
 $400nV/\sqrt{Hz}$
 20dB/dec
 one-sided spectral density
 f

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Back to Determining Sensor Resolution

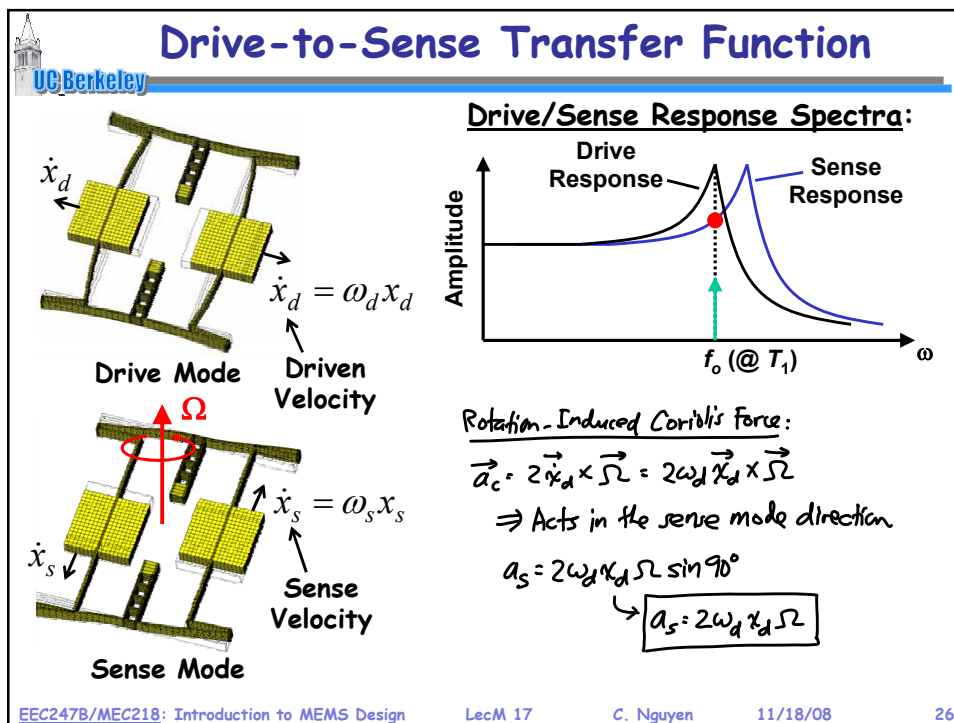
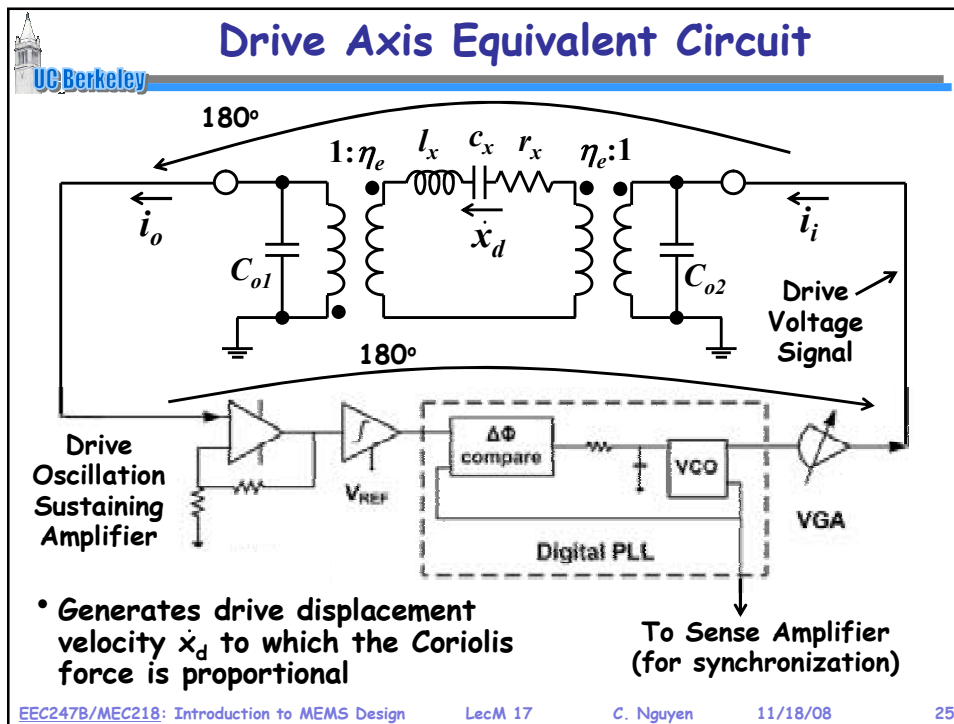
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MEMS-Based Tuning Fork Gyroscope

[Zaman, Ayazi, et al, MEMS'06]

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**Gyro Readout Equivalent Circuit
(for a single tine)**

$\vec{F}_c = m\vec{a}_c = m \cdot (2\vec{\dot{x}}_d \times \vec{\Omega})$

Noise Sources

Gyro Sense Element Output Circuit Signal Conditioning Circuit (Transresistance Amplifier)

- Easiest to analyze if all noise sources are summed at a common node

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Minimum Detectable Signal (MDS)

• **Minimum Detectable Signal (MDS):** Input signal level when the signal-to-noise ratio (SNR) is equal to unity

Sensed Signal Sensor Signal Conditioning Circuit Output

- The sensor scale factor is governed by the sensor type
- The effect of noise is best determined via analysis of the equivalent circuit for the system

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Move Noise Sources to a Common Point

- Move noise sources so that all sum at the input to the amplifier circuit (i.e., at the output of the sense element)
- Then, can compare the output of the sensed signal directly to the noise at this node to get the MDS

Sensed Signal → **Sensor** (Sensor Scale Factor, Sensor Noise) → **Signal Conditioning Circuit** (Circuit Gain, Circuit Input-Referred Noise) → **Output** (Includes desired output plus noise)

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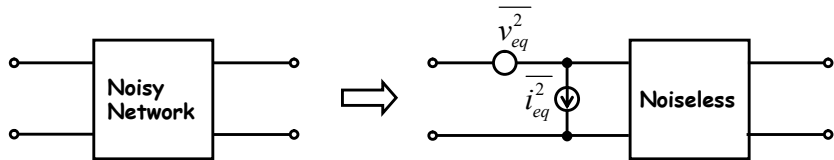
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Equivalent Input-Referred Voltage and Current Noise Sources

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Equivalent Input v , i Noise Generators

• Take a noisy 2-port network and represent it by a noiseless network with input v and i noise generators that generate the same total output noise



• **Remarks:**

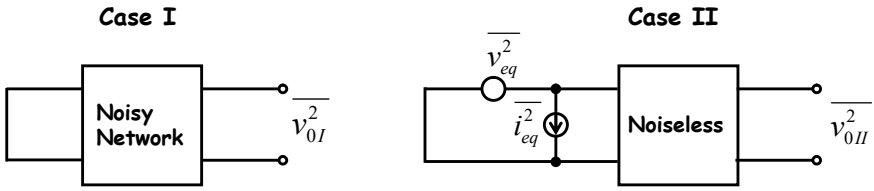
1. Works for linear time-invariant networks
2. v_{eq} and i_{eq} are generally correlated (since they are derived from the same sources)
3. In many practical circuits, one of v_{eq} and i_{eq} dominates, which removes the need to address correlation
4. If correlation is important \rightarrow easier to return to original network with internal noise sources

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Calculation of v_{eq}^2 and i_{eq}^2

a) To get v_{eq}^2 for a two-port:

Case I **Case II**



- 1) Short input, find $\overline{v_{0I}^2}$ (or $\overline{i_{0I}^2}$)
- 2) For eq. network, short input, find $\overline{v_{0II}^2}$ (or $\overline{i_{0II}^2}$)

\parallel \parallel
 $f(\overline{v_{eq}^2})$ $f(\overline{v_{eq}^2})$

- 3) Set $\overline{v_{0I}^2} = \overline{v_{0II}^2} \rightarrow$ solve for $\overline{v_{eq}^2}$ (or $\overline{i_{0I}^2} = \overline{i_{0II}^2}$)

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Calculation of $\overline{v_{eq}^2}$ and $\overline{i_{eq}^2}$ (cont)

b) To get $\overline{i_{eq}^2}$ for a 2-port:

- 1) Open input, find $\overline{v_{0I}^2}$ (or $\overline{i_{0I}^2}$)
- 2) Open input for eq. circuit, find $\overline{v_{0II}^2}$ (or $\overline{i_{0II}^2}$)
- 3) Set $\overline{v_{0I}^2} = \overline{v_{0II}^2} (\overline{i_{eq}^2}) \rightarrow$ solve for $\overline{i_{eq}^2}$ (or $\overline{i_{0I}^2} = \overline{i_{0II}^2} (\overline{i_{eq}^2})$)

- Once the equivalent input-referred noise generators are found, noise calculations become straightforward as long as the noise generators can be treated as uncorrelated

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Cases Where Correlation Is Not Important

- There are two common cases where correlation can be ignored:
 1. Source resistance R_s is **small** compared to input resistance $R_i \rightarrow$ i.e., voltage source input
 2. Source resistance R_s is **large** compared to input resistance $R_i \rightarrow$ i.e., current source input

1) $R_s =$ small (ideally = 0 for an ideal voltage source):

\therefore For $R_s =$ small, $\overline{i_{eq}^2}$ can be neglected \rightarrow only $\overline{v_{eq}^2}$ is important!
(Thus, we need not deal with correlation)

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Cases Where Correlation Is Not Important

2) $R_s = \text{large}$ (Ideally = ∞ for an ideal current source)

Voltage $\overline{v_{eq}^2}$ effectively "opened" out!

$$v_i = \frac{R_{in}}{\infty + R_{in}} v_{eq} = 0!$$

\therefore For $R_s = \text{large}$, $\overline{v_{eq}^2}$ can be neglected!
 → only $\overline{i_{eq}^2}$ is important!
 (... and again, we need not deal with correlation)

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Example: TransR Amplifier Noise

Case I

Input-referred current noise:
 Open inputs; equate output voltage noise.

Case I:
 $N_{oI1} = i_{ia} R_f$
 $N_{oI2} = i_f R_f$
 $N_{oI3} = N_{ia}$
 Have N_{ia} through R_{in} to N_{ia} This is unity gain!

$\therefore N_{oI}^2 = i_{ia}^2 R_f^2 + i_f^2 R_f^2 + N_{ia}^2$

Case II: $N_{oII}^2 = \overline{i_{eq}^2} R_f^2$

$\therefore \overline{i_{eq}^2} = i_{ia}^2 + i_f^2 + \frac{N_{ia}^2}{R_f^2}$

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Example: TransR Amplifier Noise (cont)

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Input-referred voltage noise:
 Short inputs; equate output voltage noise

Case I:
 $\overline{N_{oI}^2} = \overline{N_{ia}^2} a^2$
 (Both i_{ia}^2 & i_f^2 are shorted out.)

Case II:
 $\overline{N_{oII}^2} = \overline{N_{eq}^2} a^2$
 $\therefore \overline{N_{eq}^2} = \overline{N_{ia}^2}$

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Example: TransR Amplifier Noise (cont)

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- To summarize, for a transresistance amplifier, the equivalent input-referred current and voltage noise generators are given by:

$$\overline{i_{eq}^2} = \overline{i_{ia}^2} + \overline{i_f^2} + \frac{\overline{v_{ia}^2}}{R_f^2}$$

$$\overline{v_{eq}^2} = \overline{v_{ia}^2}$$

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Back to Gyro Noise & MDS

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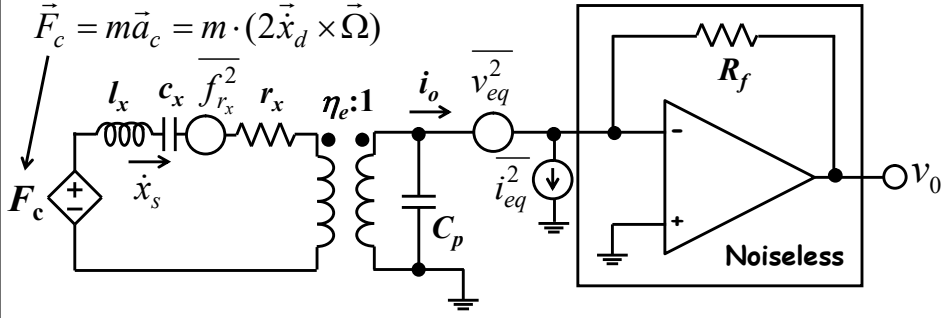
Example: Gyro MDS Calculation

$$\vec{F}_c = m\vec{a}_c = m \cdot (2\vec{\dot{x}}_d \times \vec{\Omega})$$

- The gyro sense presents a large effective source impedance
 - ↳ Currents are the important variable; voltages are "opened" out
 - ↳ Must compare i_o with the total current noise i_{eqTOT} going into the amplifier circuit

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Example: Gyro MDS Calculation (cont)

• First, find the rotation to i_o transfer function:


$$\dot{x}_s = \frac{\omega_s Q_s}{k_s} (H_s(j\omega_d)) F_s = \frac{\omega_s Q_s \cdot 2\omega_d \kappa_d \Omega m}{k_s} (H_s(j\omega_d)) \cdot \frac{1}{\omega_s^2}$$

$$[F_s = F_c = 2\omega_d \kappa_d \Omega m]$$

$$\dot{x}_s = 2 \frac{\omega_d}{\omega_s} Q_s \kappa_d (H_s(j\omega_d)) \cdot \Omega$$

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Example: Gyro MDS Calculation (cont)



$$i_o = \eta_e \dot{x}_s = 2 \frac{\omega_d}{\omega_s} Q_s \kappa_d \eta_e (H_s(j\omega_d)) \cdot \Omega \Rightarrow i_o = A \Omega$$

$A \triangleq$ scale factor

Where $A = 2 \frac{\omega_d}{\omega_s} Q_s \kappa_d \eta_e (H_s(j\omega_d))$

When $\Omega = \Omega_{min} \triangleq$ MDS, $i_o = i_{eqTOT}$ ← input-referred noise current entering the sense amplifier → in pA/√Hz


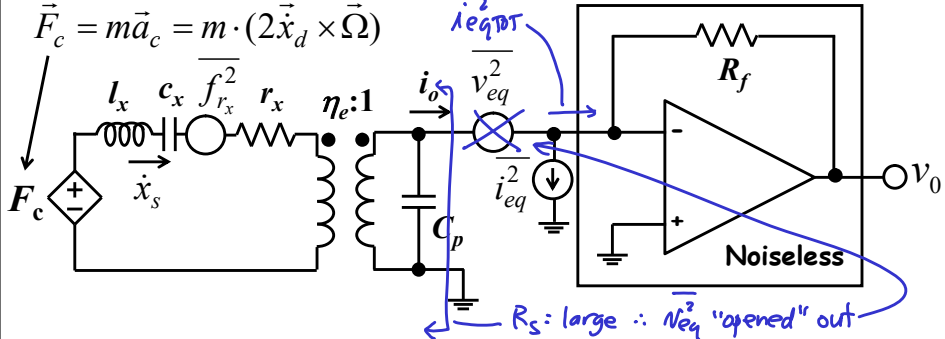
$$\therefore i_{eqTOT} = A \Omega_{min} \rightarrow \Omega_{min} = \frac{i_{eqTOT}}{A} \left(\frac{3600s}{hr} \right) \left(\frac{180^\circ}{\pi} \right) [(\%hr)/\sqrt{Hz}]$$

Angle Random Walk: ARW = $\frac{1}{60} \Omega_{min} [^\circ/\sqrt{hr}]$

↪ Easier to determine directional error as a function of elapsed time.

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Example: Gyro MDS Calculation (cont)

$\vec{F}_c = m\vec{a}_c = m \cdot (2\vec{\dot{x}}_d \times \vec{\Omega})$

• Now, find the i_{eqTOT} entering the amplifier input:


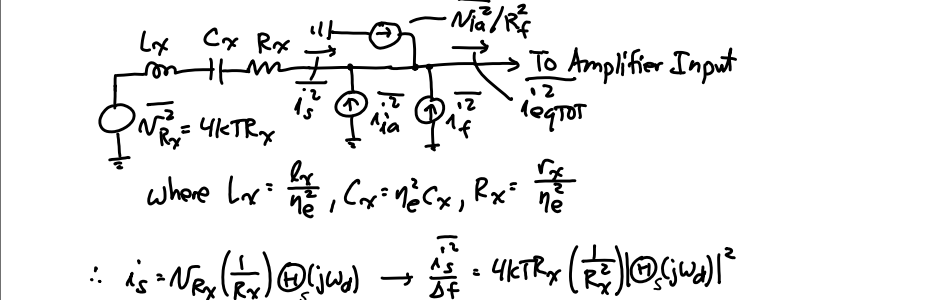
$$i_{eqTOT}^2 = i_s^2 + i_{eq}^2 \rightarrow i_{eqTOT}^2 = i_s^2 + i_f^2 + i_{ia}^2 + \frac{N_{ia}^2}{R_f^2}$$

$\frac{f_{rx}^2}{\Delta f} = 4kT r_x$
 ↗ Brownian motion noise of the sense element → determined entirely by the noise in $r_x \rightarrow f_{rx}^2$
 ↘ easiest to convert to an all electrical equiv. ckt.

R_s: large ∴ N_{eq} "opened" out

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Example: Gyro MDS Calculation (cont)

Where $L_x = \frac{R_x}{\eta_e^2}$, $C_x = \eta_e^2 C_x$, $R_x = \frac{r_x}{\eta_e^2}$

$$\therefore i_s = N_{R_x} \left(\frac{1}{R_x} \right) |H_s(j\omega_d)|^2 \rightarrow \frac{i_s^2}{\Delta f} = 4kTR_x \left(\frac{1}{R_x^2} \right) |H_s(j\omega_d)|^2$$

$$\Rightarrow \frac{i_s^2}{\Delta f} = \frac{4kT}{R_x} |H_s(j\omega_d)|^2$$

Thus:

$$\frac{i_{eqTOT}^2}{\Delta f} = \frac{4kT}{R_x} |H_s(j\omega_d)|^2 + \frac{4kT}{R_f} + \frac{i_{ia}^2}{\Delta f} + \frac{N_{ia}^2}{\Delta f} \left(\frac{1}{R_f^2} \right)$$

Learn to get there from EE240.
 ↘ or just get them from a data sheet ...

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LF356 Op Amp Data Sheet

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LF155/LF156/LF256/LF257/LF355/LF356/LF357 JFET Input Operational Amplifiers

General Description
 These are the first monolithic JFET input operational amplifiers to incorporate well matched, high voltage JFETs on the same chip with standard bipolar transistors (BI-FET™ Technology). These amplifiers feature low input bias and offset currents/low offset voltage and offset voltage drift, coupled with offset adjust which does not degrade drift or common-mode rejection. The devices are also designed for high slew rate, wide bandwidth, extremely fast settling time, low voltage and current noise and a low 1/f noise corner.

Common Features

- Logarithmic amplifiers
- Photocell amplifiers
- Sample and Hold circuits
- Low input bias current: 30pA
- Low Input Offset Current: 3pA
- High input impedance: $10^{12}\Omega$
- Low input noise current: $0.01 \text{ pA}/\sqrt{\text{Hz}}$ (Handwritten: $\sqrt{\frac{i_{ia}^2}{\Delta f}} = 0.01 \text{ pA}/\sqrt{\text{Hz}}$)
- High common-mode rejection ratio: 100 dB
- Large dc voltage gain: 106 dB

Uncommon Features

	LF155/ LF355	LF156/ LF256/ LF356	LF257/ LF357 ($A_v=5$)	Units
Extremely fast settling time to 0.01%	4	1.5	1.5	μs
Fast slew rate	5	12	50	V/ μs
Wide gain bandwidth	2.5	5	20	MHz
Low input noise voltage	20	12	12	nV/ $\sqrt{\text{Hz}}$ (Handwritten: $\sqrt{\frac{v_{ia}^2}{\Delta f}} = 12 \text{ nV}/\sqrt{\text{Hz}}$)

Features

Advantages

- Replace expensive hybrid and module FET op amps
- Rugged JFETs allow blow-out free handling compared with MOSFET input devices
- Excellent for low noise applications using either high or low source impedance—very low 1/f corner
- Offset adjust does not degrade drift or common-mode rejection as in most monolithic amplifiers
- New output stage allows use of large capacitive loads (5,000 pF) without stability problems
- Internal compensation and large differential input voltage capability

Applications

- Precision high speed integrators
- Fast D/A and A/D converters
- High impedance buffers
- Wideband, low noise, low drift amplifiers

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Example ARW Calculation

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• **Example Design:**

↳ **Sensor Element:**

$$m = (100\mu\text{m})(100\mu\text{m})(20\mu\text{m})(2300\text{kg}/\text{m}^3) = 4.6 \times 10^{-10}\text{kg}$$

$$\omega_s = 2\pi(15\text{kHz})$$

$$\omega_d = 2\pi(10\text{kHz})$$

$$k_s = \omega_s^2 m = 4.09 \text{ N/m}$$

$$x_d = 20 \mu\text{m}$$

$$Q_s = 50,000$$

$$V_p = 5\text{V}$$

$$h = 20 \mu\text{m}$$

$$d = 1 \mu\text{m}$$

↳ **Sensing Circuitry:**

$$R_f = 100\text{k}\Omega$$

$$i_{ia} = 0.01 \text{ pA}/\sqrt{\text{Hz}}$$

$$v_{ia} = 12 \text{ nV}/\sqrt{\text{Hz}}$$

The diagram shows a 3D perspective of a MEMS sensor element. It features a central yellow 'Proof Mass' supported by four 'Proof Mass' pillars. Surrounding the proof mass are 'Sense Electrodes' (yellow) and 'Tuning Electrodes' (red). A 'Drive Electrode' (red) is also shown. Blue arrows indicate the 'Sense' and 'Drive' directions. A coordinate system with Ω_z is shown.

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Example ARW Calculation (cont)

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Get rotation rate to output current scale factor:

$$A = Z \frac{\omega_d}{\omega_s} Q_s \kappa_d \eta_e |\Theta_s(j\omega_d)| = Z \left(\frac{10K}{15K} \right) (50K) (20\mu) (5) (2000\epsilon_0) (0.000024) = \underline{2.83 \times 10^{-12} C}$$

$$\Theta_s(j\omega_d) = \frac{(j\omega_d)(\omega_s/Q_s)}{-\omega_d^2 + \frac{j\omega_d\omega_s}{Q_s} + \omega_s^2} = \frac{j(10K)(15K)/(50K)}{(15K)^2 - (10K)^2 + \frac{j(10K)(15K)}{50K}} = \frac{j(3K)}{1.25 \times 10^8 + j(3K)}$$

$$\Rightarrow |\Theta_s(j\omega_d)| = \frac{3K}{\sqrt{(1.25 \times 10^8)^2 + (3K)^2}} = 0.000024 \quad 8.854 \times 10^{-8} F/m$$

$$\frac{\partial C}{\partial x} = \frac{C_0}{d} = \frac{\epsilon_0 h \omega_p}{d} = \frac{\epsilon_0 (20\mu)(100\mu)}{(1\mu)^2} = 2000\epsilon_0 \rightarrow \eta_e = V_p \frac{\partial C}{\partial x} = 5(2000\epsilon_0) \quad 8.854 \times 10^{-12} F/m$$

Assume electrode covers the whole sidewall.

Then, get noise:

$$\frac{\overline{i_{eq}^2}}{\Delta f} = \frac{4kT}{R_x} |\Theta_s(j\omega_d)|^2 + \frac{4kT}{R_f} + \frac{\overline{i_{ia}^2}}{\Delta f} + \frac{\overline{N_{ie}^2}}{\Delta f} \left(\frac{1}{R_f} \right)$$

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Example ARW Calculation (cont)

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$$R_x = \frac{\omega_s m}{Q_s \eta_e^2} = \frac{2\pi(15K)(4.6 \times 10^{-10})}{(50K)(8.854 \times 10^{-8})^2} = 110.6 k\Omega$$

$$\frac{\overline{i_{eqTOT}^2}}{\Delta f} = \frac{(1.66 \times 10^{-29})}{(110.6K)} (0.000024)^2 + \frac{(1.66 \times 10^{-29})}{1M} + (0.01p)^2 + \frac{(12n)^2}{(1M)^2}$$

$8.64 \times 10^{-35} A^2/Hz$ $1.66 \times 10^{-26} A^2/Hz$ $1 \times 10^{-28} A^2/Hz$ $1.44 \times 10^{-28} A^2/Hz$

sensor element noise insignificant Noise from R_f dominates!

$$\therefore \frac{\overline{i_{eqTOT}^2}}{\Delta f} = 1.68 \times 10^{-26} A^2/Hz \rightarrow i_{eqTOT} = \sqrt{\frac{\overline{i_{eqTOT}^2}}{\Delta f}} = 1.30 \times 10^{-13} A/\sqrt{Hz}$$

$$\therefore \Omega_{min} = \frac{i_{eqTOT}}{A} \left(\frac{3600s}{hr} \right) \left(\frac{180^\circ}{\pi} \right) = \frac{1.30 \times 10^{-13}}{2.83 \times 10^{-12}} (3600) \left(\frac{180}{\pi} \right) = 9448 (\%/hr)/\sqrt{Hz}$$

And finally:

$$ARW = \frac{1}{60} \Omega_{min} = \frac{1}{60} (9448) = 157 \%/hr = ARW \Rightarrow \text{Almost turned around in 1 hour!}$$

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What if $\omega_d = \omega_s$?

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If $\omega_d = \omega_s = 15\text{kHz}$, then $|\Theta(j\omega_d)| = 1$ and

$$A = 2 \frac{\omega_d}{\omega_s} Q_s \chi_d \eta_e |\Theta(j\omega_d)| = 2 Q_s \chi_d \eta_e = 2(50k)(20\mu)(5)(2000\epsilon_0) = 1.77 \times 10^{-7} \text{C}$$

$$\frac{\dot{i}_{eqTOT}^2}{\Delta f} = \frac{(1.66 \times 10^{-29})^2}{(110.6k)^2} + \frac{(1.66 \times 10^{-29})^2}{1M} + (0.01p)^2 + \frac{(12n)^2}{(1M)^2}$$

$\swarrow 1.51 \times 10^{-25} \text{A}^2/\text{Hz}$
 $\swarrow 1.66 \times 10^{-26} \text{A}^2/\text{Hz}$
 $\swarrow 1 \times 10^{-28} \text{A}^2/\text{Hz}$
 $\swarrow 1.44 \times 10^{-28} \text{A}^2/\text{Hz}$

Now, the sensor element dominates!

$$\therefore \frac{\dot{i}_{eqTOT}^2}{\Delta f} = 1.67 \times 10^{-25} \text{A}^2/\text{Hz} \rightarrow \dot{i}_{eqTOT} = \sqrt{\frac{\dot{i}_{eqTOT}^2}{\Delta f}} = 4.08 \times 10^{-13} \text{A}/\sqrt{\text{Hz}}$$

$$\therefore \Sigma_{min} = \frac{\dot{i}_{eqTOT}}{A} \left(\frac{3600s}{hr} \right) \left(\frac{180^\circ}{\pi} \right) = \frac{4.08 \times 10^{-13}}{1.77 \times 10^{-7}} (3600) \left(\frac{180}{\pi} \right) = 0.476 (\%/hr)/\sqrt{\text{Hz}}$$

And finally:

$$ARW = \frac{1}{60} \Sigma_{min} = \frac{1}{60} (0.476) = 0.0079 \%/hr = ARW \Rightarrow \text{Navigation grade!}$$

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