


EE C247B - ME C218 Introduction to MEMS Design Spring 2018

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Lecture Module 7: Mechanics of Materials


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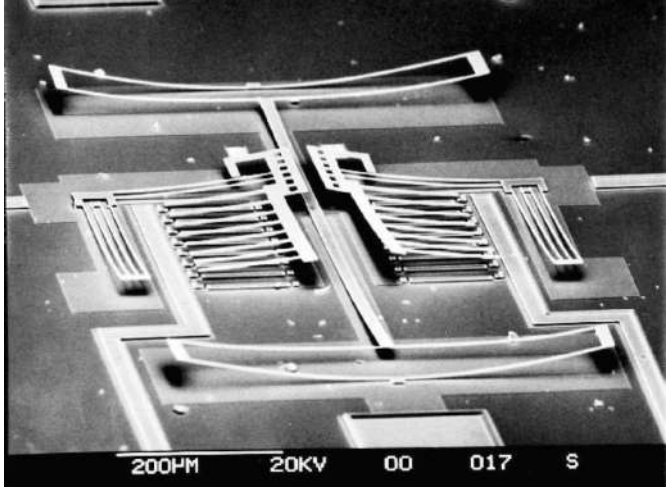
Outline

- Reading: Senturia, Chpt. 8
- Lecture Topics:
 - ↪ Stress, strain, etc., for isotropic materials
 - ↪ Thin films: thermal stress, residual stress, and stress gradients
 - ↪ Internal dissipation
 - ↪ MEMS material properties and performance metrics


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 **Vertical Stress Gradients**

- Variation of residual stress in the direction of film growth
- Can warp released structures in z-direction



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 **Elasticity**

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Normal Stress (1D)

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If the force acts normal to a surface, then the stress is called a **normal stress**

Force assumed uniform over the whole area A

Stress = $\left\{ \begin{array}{l} \text{Force per} \\ \text{unit area} \end{array} \right\} = \sigma = \frac{F}{A} \quad [N/m^2 = Pa]$
 ↗ standard mks unit

⇒ Microscopic Definition: force per unit area acting on the surface of a differential volume element of a solid body

⇒ Note: assume stress acts uniformly across the entire surface of the element, not at just a point

Differential volume element

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Strain (1D)

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Sometimes a unit called the "microstrain" is used, where $1 \mu\epsilon = \frac{\Delta L}{L}$ of 1 part in 10^6

Strain = $\left\{ \begin{array}{l} \text{Fractional Change} \\ \text{in Length} \end{array} \right\} = \epsilon = \frac{L' - L}{L} = \frac{\Delta L}{L} \quad [\text{unitless}]$

In the elastic regime (i.e., for "small" stresses at "low" temperatures), strain is found to be proportional to stress

σ ← stress For solids: MPa → GPa

σ = Eε → $\epsilon = \frac{\sigma}{E} \quad [\text{unitless}]$

Thus, the units of E are the same as σ → Pa

— slope = E = Young's modulus of elasticity

ε ← strain

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The Poisson Ratio

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Apply normal stress to a free-standing object } uniaxial strain
 but also get contraction in directions transverse to the uniaxial strain

⇒ contraction creates a (-) strain:

$$\epsilon_y = \frac{W' - W}{W} = \frac{\Delta W}{W} = -\nu \epsilon_x$$

↳ ν = Poisson ratio [unitless]
 ↳ typical values: 0 → 0.5
 ⇒ inorganic solids: 0.2 → 0.3
 ⇒ elastomers (e.g., rubber): ~ 0.5

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Shear Stress & Strain (1D)

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Note: Assume compensating forces are applied to the vertical faces to avoid a net torque. (This by convention.)

Shear Stress = $\left\{ \begin{array}{l} \text{Force per Unit Area} \\ \text{Parallel to the Surfaces} \end{array} \right\} = \tau = \frac{F}{A} \quad [\text{Pa}]$

↳ Generates a shear strain:

$$\text{Shear Strain} = \theta = \frac{\tau}{G} \leftarrow G \triangleq \text{shear modulus}$$

$$G = \frac{E}{2(1+\nu)}$$

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2D and 3D Considerations

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- **Important assumption:** the differential volume element is in static equilibrium \rightarrow no net forces or torques (i.e., rotational movements)
 - \hookrightarrow Every σ must have an equal σ in the opposite direction on the other side of the element
 - \hookrightarrow For no net torque, the shear forces on different faces must also be matched as follows:

Stresses acting on a differential volume element

$$\tau_{xy} = \tau_{yx} \quad \tau_{xz} = \tau_{zx} \quad \tau_{yz} = \tau_{zy}$$

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2D Strain

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- In general, motion consists of
 - \hookrightarrow rigid-body displacement (motion of the center of mass)
 - \hookrightarrow rigid-body rotation (rotation about the center of mass)
 - \hookrightarrow Deformation relative to displacement and rotation

- Must work with displacement vectors
- Differential definition of axial strain: $\longrightarrow \epsilon_x = \frac{u_x(x + \Delta x) - u_x(x)}{\Delta x} = \frac{\partial u_x}{\partial x}$

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2D Shear Strain

Rotate clockwise by θ_1

⇒ For shear strains, must remove any rigid body rotation that accompanies the deformation
 ↳ use a symmetric definition of shear strain:

$$\tau_{xy} = \theta_2 + \theta_1 \approx \left(\frac{\Delta u_x}{\Delta y} + \frac{\Delta u_y}{\Delta x} \right) = \left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} \right)$$

↑
For small amplitude deformations.

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Volume Change for a Uniaxial Stress

Stresses acting on a differential volume element

Given an x-directed uniaxial stress, σ_x :

$$\begin{aligned} \Delta x &\rightarrow \Delta x (1 + \epsilon_x) \\ \Delta y &\rightarrow \Delta y (1 - \nu \epsilon_x) \\ \Delta z &\rightarrow \Delta z (1 - \nu \epsilon_x) \end{aligned}$$

↓ The resulting change in volume ΔV

$$\begin{aligned} \Delta V &= \Delta x \Delta y \Delta z (1 + \epsilon_x)(1 - \nu \epsilon_x)^2 - \Delta x \Delta y \Delta z \\ &= \Delta x \Delta y \Delta z [(1 + \epsilon_x)(1 - \nu \epsilon_x)^2 - 1] \end{aligned}$$

{Assume small strains} ⇒ $\Delta V = \Delta x \Delta y \Delta z [(1 + \epsilon_x)(1 - 2\nu \epsilon_x) - 1]$

$(1 + m\epsilon)^n \approx 1 + nm\epsilon$ ⇒ $\approx \Delta x \Delta y \Delta z [\nu \epsilon_x - 2\nu \epsilon_x - \nu^2 \epsilon_x^2 - 1]$

$\Delta V \approx \Delta x \Delta y \Delta z (1 - 2\nu) \epsilon_x$

For $\nu = 0.5$ (rubber) → no ΔV !
 $\nu < 0.5$ → finite ΔV

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Isotropic Elasticity in 3D

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- Isotropic = same in all directions
- The complete stress-strain relations for an isotropic elastic solid in 3D: (i.e., a generalized Hooke's Law)

$$\varepsilon_x = \frac{1}{E} [\sigma_x - \nu(\sigma_y + \sigma_z)] \quad \gamma_{xy} = \frac{1}{G} \tau_{xy}$$

$$\varepsilon_y = \frac{1}{E} [\sigma_y - \nu(\sigma_z + \sigma_x)] \quad \gamma_{yz} = \frac{1}{G} \tau_{yz}$$

$$\varepsilon_z = \frac{1}{E} [\sigma_z - \nu(\sigma_x + \sigma_y)] \quad \gamma_{zx} = \frac{1}{G} \tau_{zx}$$

Basically, add in off-axis strains from normal stresses in other directions

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Important Case: Plane Stress

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- Common case: very thin film coating a thin, relatively rigid substrate (e.g., a silicon wafer)

- At regions more than 3 thicknesses from edges, the top surface is stress-free $\rightarrow \sigma_z = 0$
- Get two components of in-plane stress:

$$\varepsilon_x = (1/E)[\sigma_x - \nu(\sigma_y + 0)]$$

$$\varepsilon_y = (1/E)[\sigma_y - \nu(\sigma_x + 0)]$$

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Important Case: Plane Stress (cont.)

- Symmetry in the xy-plane $\rightarrow \sigma_x = \sigma_y = \sigma$
- Thus, the in-plane strain components are: $\varepsilon_x = \varepsilon_y = \varepsilon$
 where

$$\varepsilon_x = (1/E)[\sigma - \nu\sigma] = \frac{\sigma}{[E/(1-\nu)]} = \frac{\sigma}{E'}$$

and where

$$\text{Biaxial Modulus} \triangleq E' = \frac{E}{1-\nu}$$

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Edge Region of a Tensile ($\sigma > 0$) Film

Net non-zero in-plane force (that we just analyzed) $F \neq 0$

At free edge, in-plane force must be zero $F = 0$

Film must be bent back, here

There's no Poisson contraction, so the film is slightly thicker, here


Shear stresses

Extra peel force

Discontinuity of stress at the attached corner \rightarrow stress concentration

Peel forces that can peel the film off the surface

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Linear Thermal Expansion


- As temperature increases, most solids expand in volume
- Definition: linear thermal expansion coefficient

$$\left. \begin{array}{l} \text{Linear thermal} \\ \text{expansion coefficient} \end{array} \right\} \triangleq \alpha_T = \frac{d\varepsilon_x}{dT} \quad [\text{Kelvin}^{-1}]$$

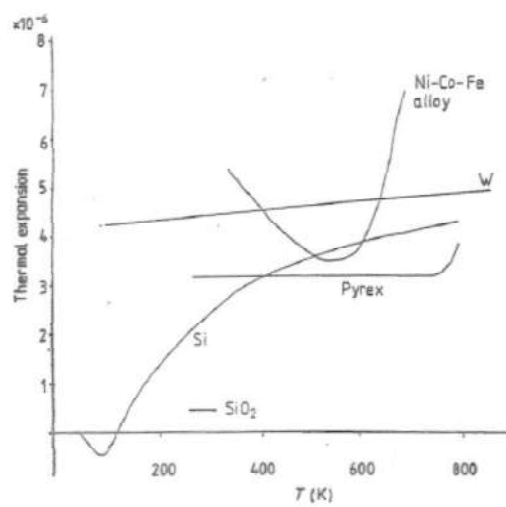
Remarks:

- α_T values tend to be in the 10^{-6} to 10^{-7} range
- Can capture the 10^{-6} by using dimensions of $\mu\text{strain/K}$, where $10^{-6} \text{ K}^{-1} = 1 \mu\text{strain/K}$
- In 3D, get volume thermal expansion coefficient $\longrightarrow \frac{\Delta V}{V} = 3\alpha_T \Delta T$
- For moderate temperature excursions, α_T can be treated as a constant of the material, but in actuality, it is a function of temperature

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α_T As a Function of Temperature



[Madou, Fundamentals of Microfabrication, CRC Press, 1998]

- Cubic symmetry implies that α is independent of direction

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Thin-Film Thermal Stress

- Assume film is deposited stress-free at a temperature T_d , then the whole thing is cooled to room temperature T_r
- Substrate much thicker than thin film \rightarrow substrate dictates the amount of contraction for both it and the thin film

Thermal strain of the substrate: (in one in-plane dimension)
 $\epsilon_s = -\alpha_{Ts} \Delta T$, where $\Delta T = T_d - T_r$

If the film were not attached to the substrate: $\epsilon_{f,free} = -\alpha_{Tf} \Delta T$ \curvearrowright over

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Linear Thermal Expansion

But the film is attached to the substrate, so the actual strain in the film is the same as that in the substrate:

$$\epsilon_{f,attached} = -\alpha_{Ts} \Delta T$$

Thus:

$$\text{Thermal Mismatch Strain} = \epsilon_{f,mismatch} = (\alpha_{Tf} - \alpha_{Ts}) \Delta T$$

\hookrightarrow Note that this is biaxial strain
 \hookrightarrow it can only be developed by an in-plane biaxial stress:

$$\sigma_{f,mismatch} = \left(\frac{E}{1-\nu} \right) \epsilon_{f,mismatch}$$

Ex. Thin-film is polyimide $\rightarrow \alpha_{Tf} = 70 \times 10^{-6} \text{ K}^{-1}$, $E = 4.6 \text{ GPa}$
 deposited @ 250°C , then cooled to RT: $25^\circ\text{C} \rightarrow \Delta T = 225 \text{ K}$ e.g., SiO_2

$$\epsilon_{f,mismatch} = (70 - 2.8) \mu (225) = 1.5 \times 10^{-2}$$

$$\sigma_{f,mismatch} = (46) (1.5 \times 10^{-2}) = \underline{\underline{60.5 \text{ MPa}}}$$


\leftarrow stress is (+), \therefore tensile
 [-] would be compressive

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MEMS Material Properties

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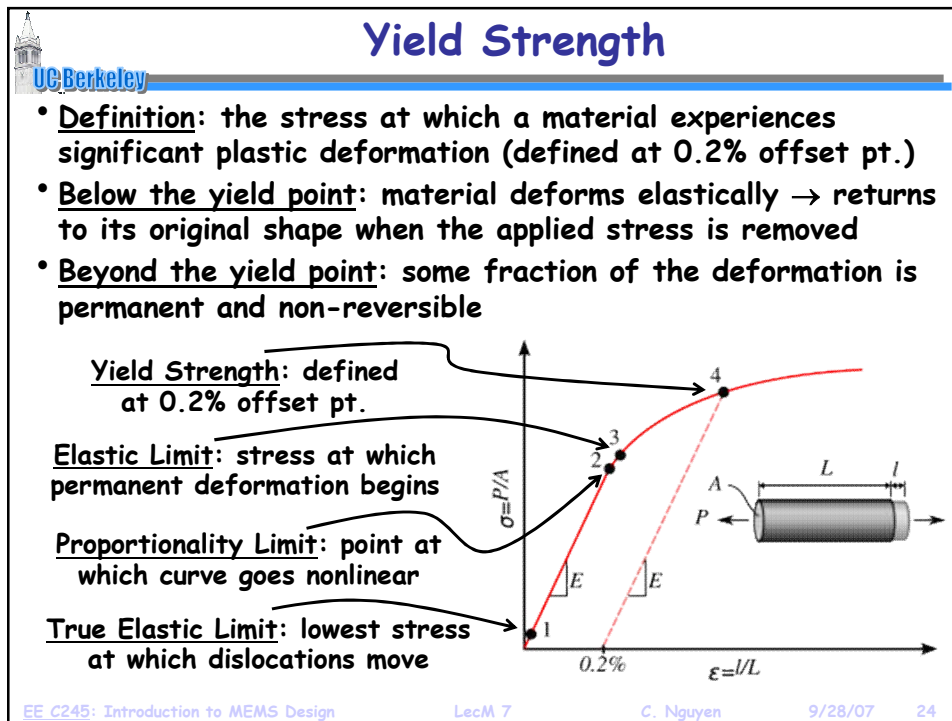
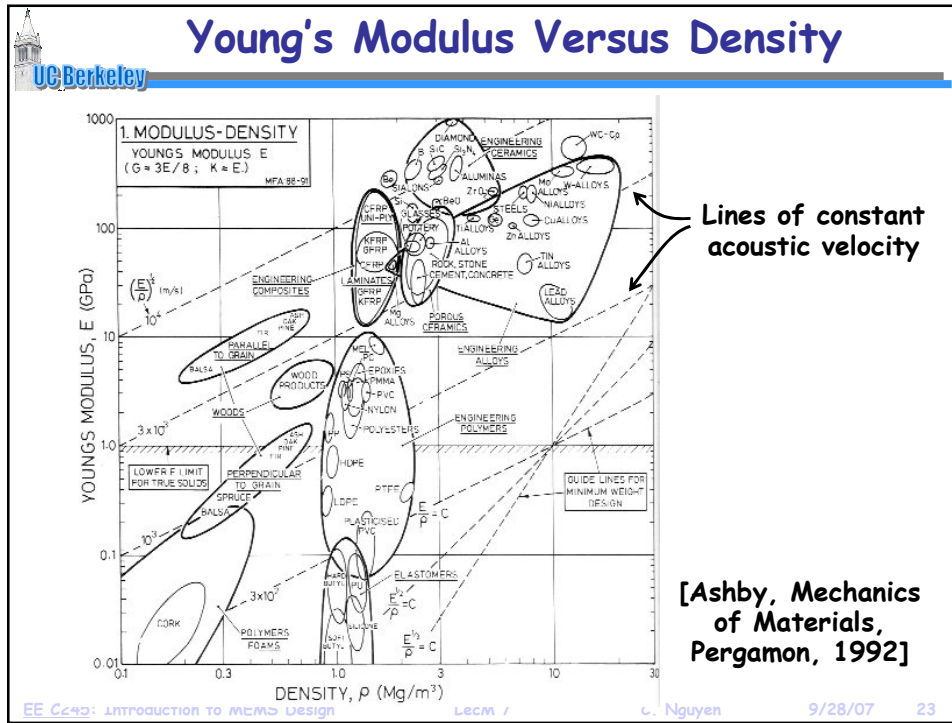
Material Properties for MEMS

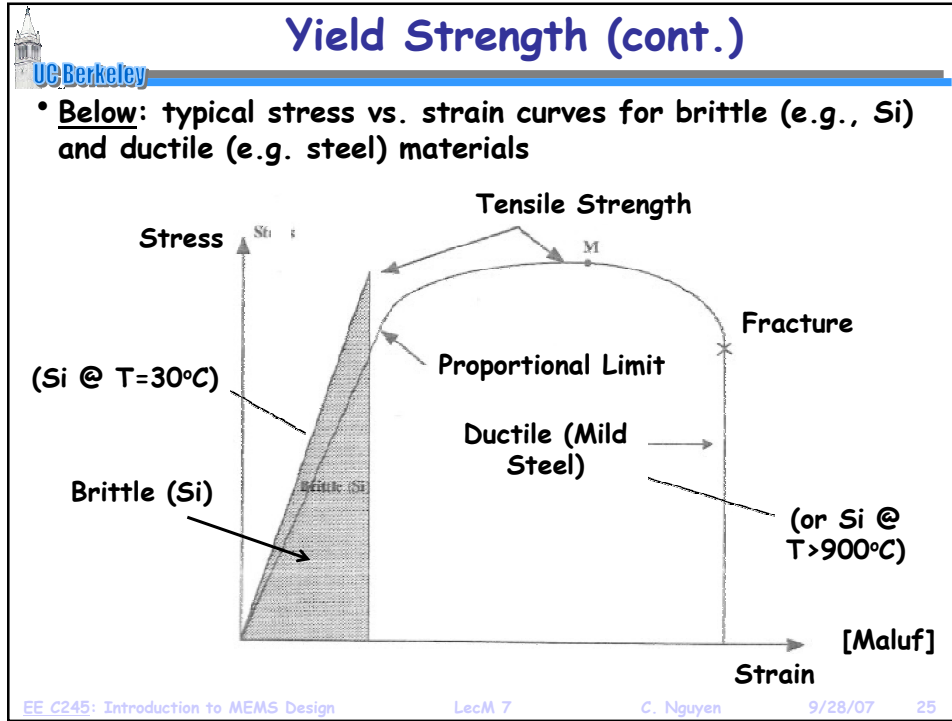
Material	Density, ρ , Kg/m ³	Modulus, E, GPa	E/ρ GN/kg-m
Silicon	2330	165	72
Silicon Oxide	2200	73	36
Silicon Nitride	3300	304	92
Nickel	8900	207	23
Aluminum	2710	69	25
Aluminum Oxide	3970	393	99
Silicon Carbide	3300	430	130
Diamond	3510	1035	295

Units: (m/s)²
 ↓
 $\sqrt{E/\rho}$ is acoustic velocity

[Mark Spearing, MIT]

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Young's Modulus and Useful Strength

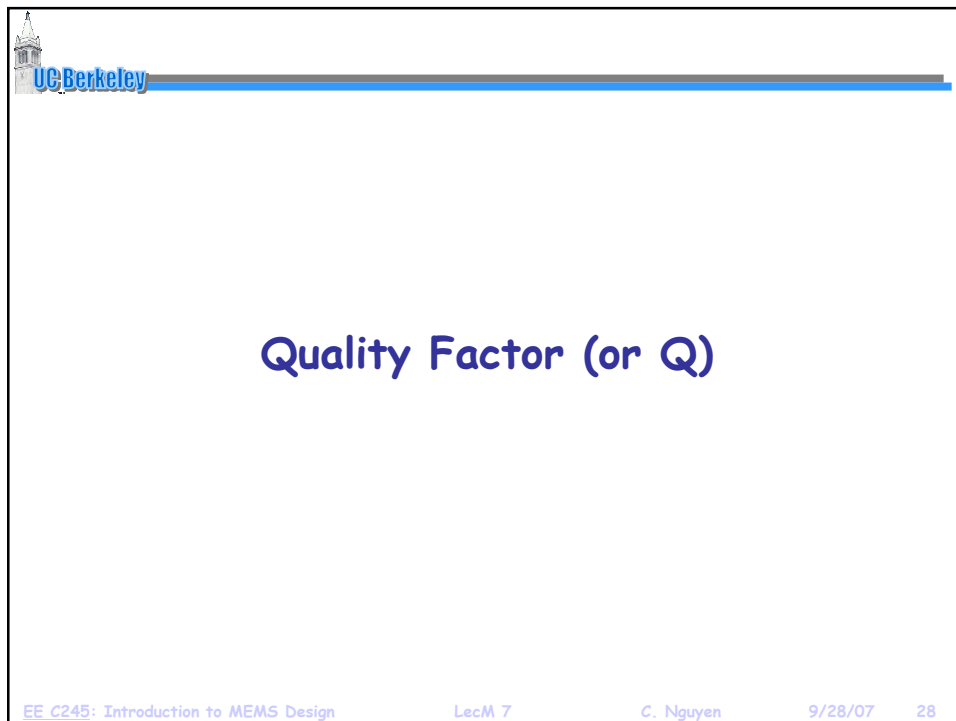
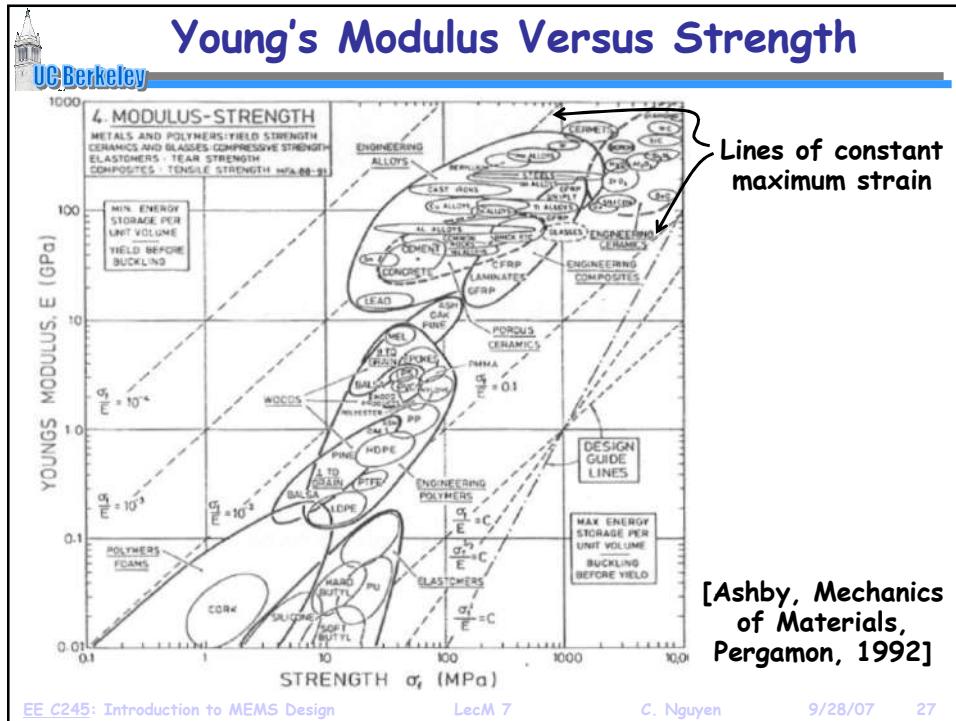
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Stored mechanical energy

Material	Modulus, E, GPa	Useful Strength*, σ_f , MPa	$\frac{\sigma_f}{E}$ (-) x 10 ⁻³	$\frac{\sigma_f^2}{E}$ MJ/m ³
Silicon	165	4000	24	97
Silicon Oxide	73	1000	13	14
Silicon Nitride	304	1000	3	4
Nickel	207	500	2	1.2
Aluminum	69	300	4	1.3
Aluminum Oxide	393	2000	5	10
Silicon Carbide	430	2000	4	9.3
Diamond	1035	1000	1	0.9

From Mark Spearing, MIT, *Future of MEMS Workshop*, Cambridge, England, May 2003

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Clamped-Clamped Beam μ Resonator

Frequency:

Stiffness k_r Young's Modulus E

$$f_o = \frac{1}{2\pi} \sqrt{\frac{k_r}{m_r}} = 1.03 \sqrt{\frac{E h}{\rho L_r^2}}$$

Density ρ

Mass m_r (e.g., $m_r = 10^{-13}$ kg) \Rightarrow Smaller mass \Rightarrow higher freq. range and lower series R_x

Note: If $V_P = 0V \Rightarrow$ device off

$i_o = V_P \frac{dC}{dt}$

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Quality Factor (or Q)

- Measure of the frequency selectivity of a tuned circuit
- **Definition:**

$$Q = \frac{\text{Total Energy Per Cycle}}{\text{Energy Lost Per Cycle}} = \frac{f_o}{BW_{3dB}}$$
- **Example:** series LCR circuit

$$Q = \frac{\text{Im}(Z)}{\text{Re}(Z)} = \frac{\omega_o L}{R} = \frac{1}{\omega_o CR}$$
- **Example:** parallel LCR circuit

$$Q = \frac{\text{Im}(Y)}{\text{Re}(Y)} = \frac{\omega_o C}{G} = \frac{1}{\omega_o LG}$$

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Selective Low-Loss Filters: Need Q

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Resonator Tank

Coupler

Resonator Tank

Coupler

Resonator Tank

General BPF Implementation

Typical LC implementation:

- In resonator-based filters: high tank Q \leftrightarrow low insertion loss
- At right: a 0.1% bandwidth, 3-res filter @ 1 GHz (simulated)
 - \rightarrow heavy insertion loss for resonator Q < 10,000

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Oscillator: Need for High Q

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- **Main Function:** provide a stable output frequency
- **Difficulty:** superposed noise degrades frequency stability

Ideal Sinusoid: $v_o(t) = V_o \sin(2\pi f_o t)$

Real Sinusoid: $v_o(t) = (V_o + \epsilon(t)) \sin(2\pi f_o t + \theta(t))$

Higher Q \rightarrow **Tighter Spectrum**

Zero-Crossing Point

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Attaining High Q

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- Problem:** IC's cannot achieve Q's in the thousands
 - transistors \Rightarrow consume too much power to get Q
 - on-chip spiral inductors \Rightarrow Q's no higher than ~ 10
 - off-chip inductors \Rightarrow Q's in the range of 100's
- Observation:** vibrating mechanical resonances \Rightarrow $Q > 1,000$
- Example:** quartz crystal resonators (e.g., in wristwatches)
 - extremely high Q's $\sim 10,000$ or higher ($Q \sim 10^6$ possible)
 - mechanically vibrates at a distinct frequency in a thickness-shear mode

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Energy Dissipation and Resonator Q

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Material Defect Losses

Gas Damping

$$\frac{1}{Q} = \frac{1}{Q_{\text{defects}}} + \frac{1}{Q_{\text{TED}}} + \frac{1}{Q_{\text{viscous}}} + \frac{1}{Q_{\text{support}}}$$

Thermoelastic Damping (TED)

Anchor Losses

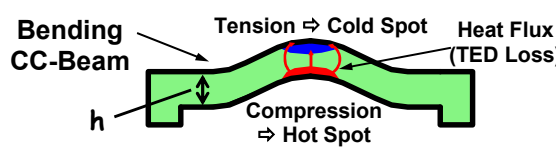
At high frequency, this is our big problem!

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Thermoelastic Damping (TED)

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- Occurs when heat moves from compressed parts to tensioned parts → heat flux = energy loss



$$\zeta = \Gamma(T)\Omega(f) = \frac{1}{2Q}$$

$$\Gamma(T) = \frac{\alpha^2 TE}{4\rho C_p}$$

$$\Omega(f_o) = 2 \left[\frac{f_{TED} f}{f_{TED}^2 + f^2} \right]$$

$$f_{TED} = \frac{\pi K}{2\rho C_p h^2}$$

ζ = thermoelastic damping factor
 α = thermal expansion coefficient
 T = beam temperature
 E = elastic modulus
 ρ = material density
 C_p = heat capacity at const. pressure
 K = thermal conductivity
 f = beam frequency
 h = beam thickness
 f_{TED} = characteristic TED frequency

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TED Characteristic Frequency

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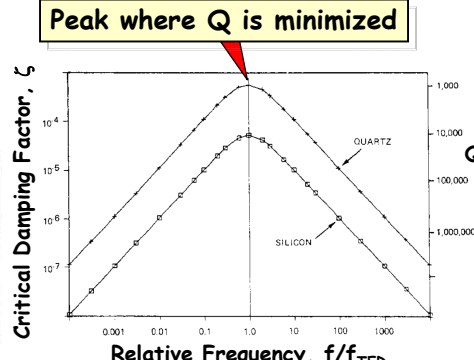
$$f_{TED} = \frac{\pi K}{2\rho C_p h^2}$$

ρ = material density
 C_p = heat capacity at const. pressure
 K = thermal conductivity
 h = beam thickness
 f_{TED} = characteristic TED frequency

- Governed by
 - Resonator dimensions
 - Material properties

TABLE 1. MATERIAL PROPERTIES

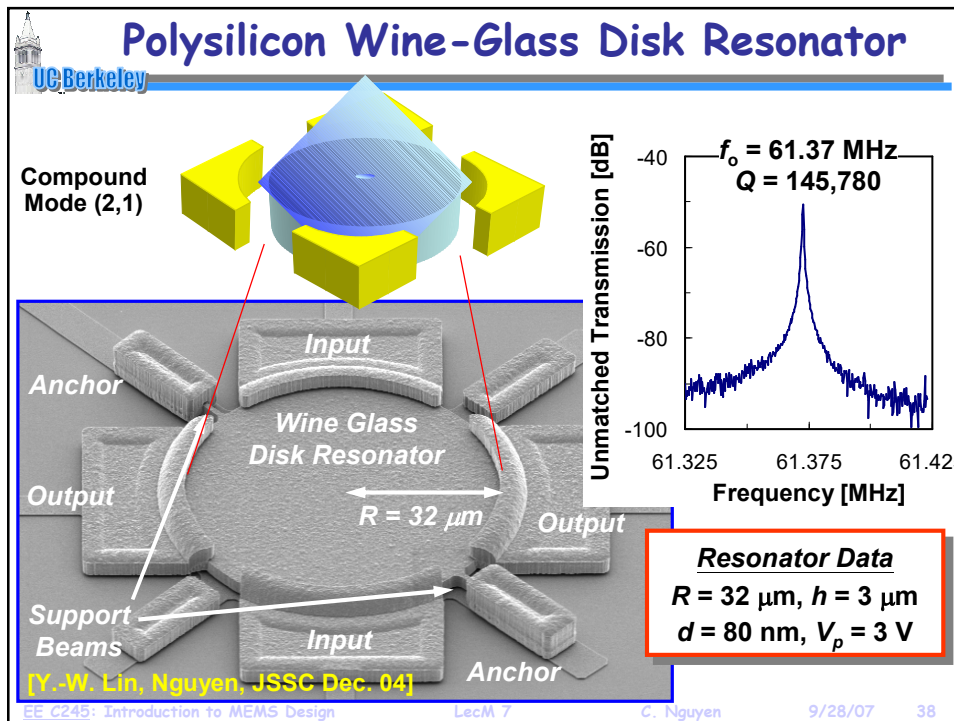
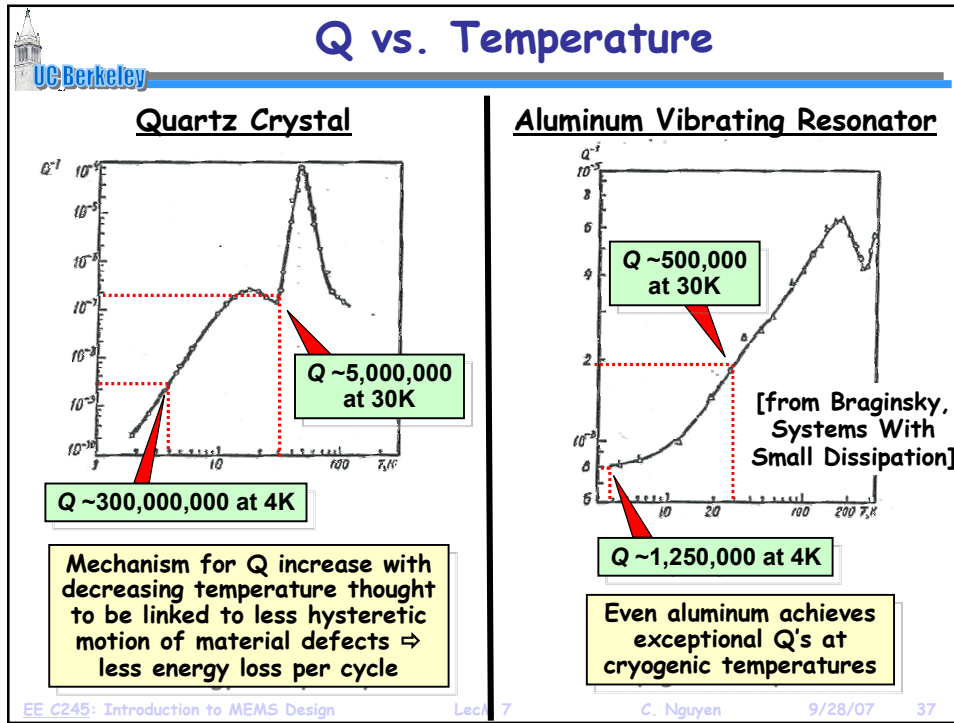
Property	Silicon	Quartz	Units
Thermal expansion	2.60	13.70	ppm/°K
Elastic modulus	1.70	0.78	10 ¹² dyne/cm ²
Material density	2.33	2.60	g/cm ³
Heat capacity	0.70	0.75	J/g/°K
Thermal conductivity	1.50	0.10	10 ⁷ dyne/°K/s
Peak damping @ 300°K	1.06	11.34	10 ⁻⁴

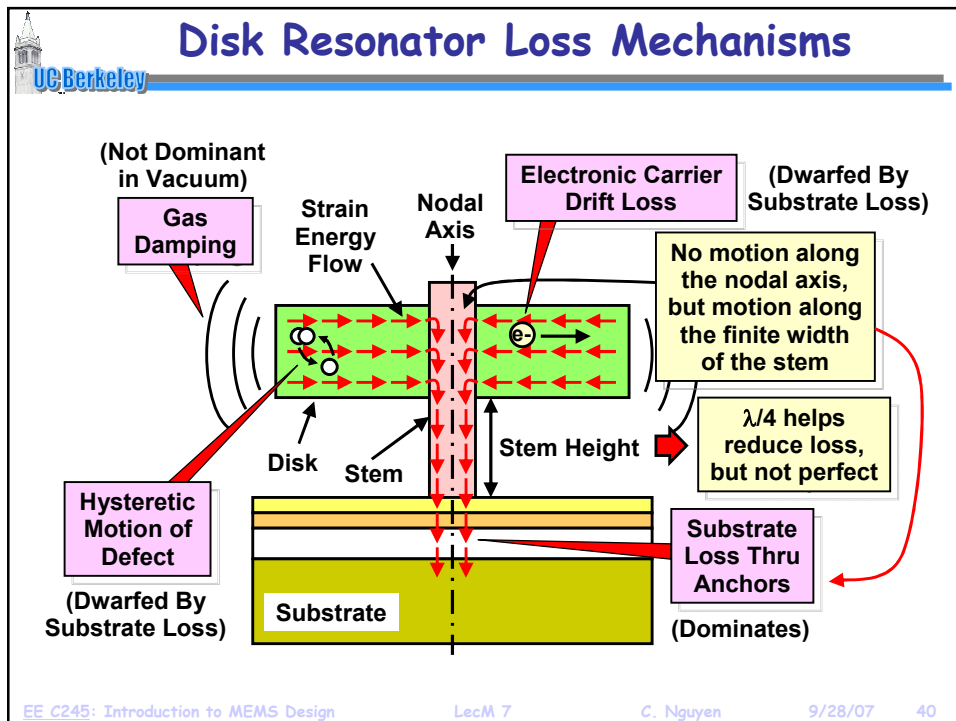
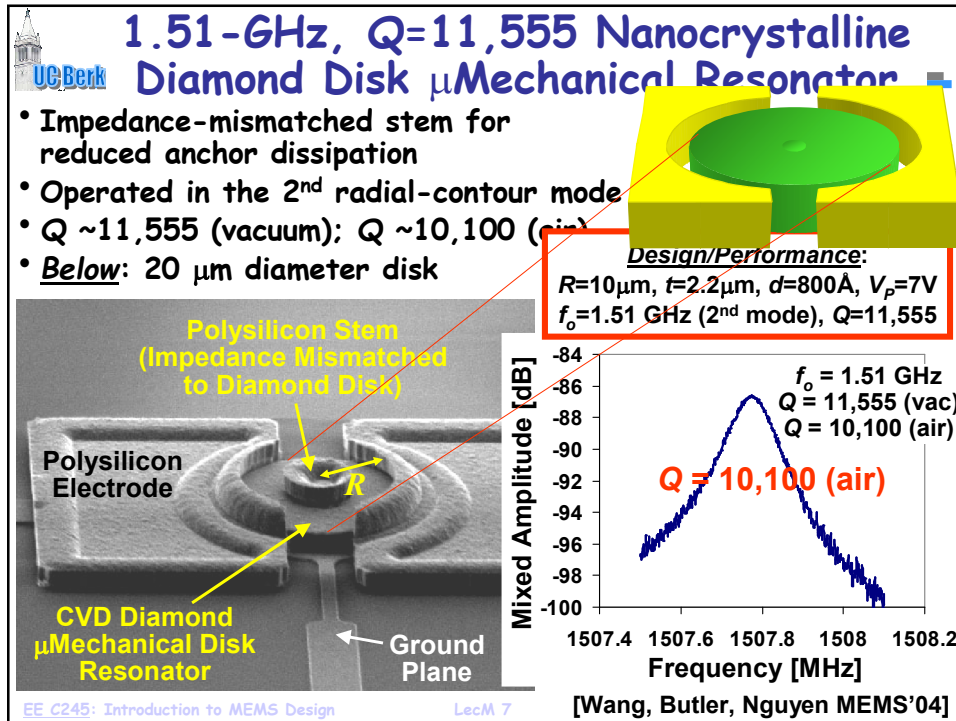


Peak where Q is minimized

[from Roszhart, Hilton Head 1990]

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MEMS Material Property Test Structures

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Stress Measurement Via Wafer Curvature

- Compressively stressed film → bends a wafer into a convex shape
- Tensile stressed film → bends a wafer into a concave shape
- Can optically measure the deflection of the wafer before and after the film is deposited
- Determine the radius of curvature R , then apply:

$$\sigma = \frac{E'h^2}{6Rt}$$

σ = film stress [Pa]
 E' = $E/(1-\nu)$ = biaxial elastic modulus [Pa]
 h = substrate thickness [m]
 t = film thickness
 R = substrate radius of curvature [m]

Slope = $2/R$
 Detector
 Laser
 Scan the mirror
 Mirror
 Si-substrate
 h
 t
 R
 x
 θ

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MEMS Stress Test Structure

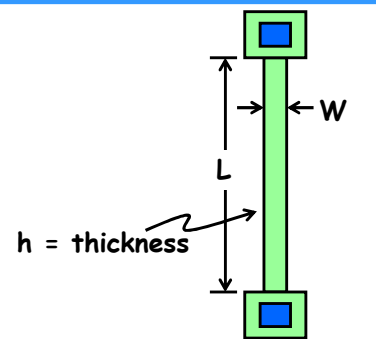
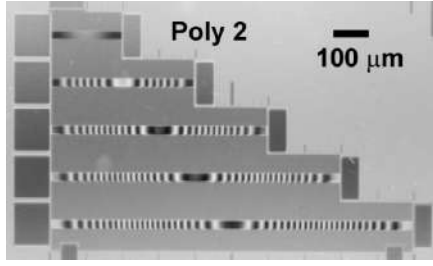
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- **Simple Approach:** use a clamped-clamped beam
 - ↳ Compressive stress causes buckling
 - ↳ Arrays with increasing length are used to determine the critical buckling load, where

$$\sigma_{critical} = -\frac{\pi^2 E h^2}{3 L^2}$$

E = Young's modulus [Pa]
 I = (1/12)Wh³ = moment of inertia
 L, W, h indicated in the figure

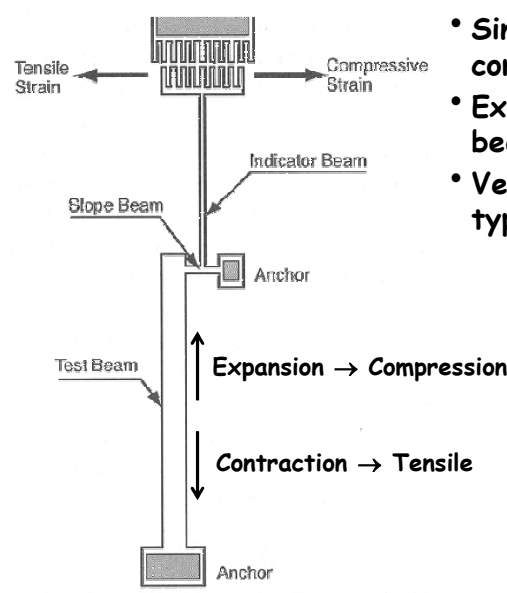
- ↳ **Limitation:** Only compressive stress is measurable

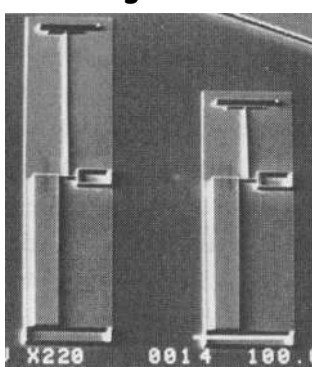
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More Effective Stress Diagnostic

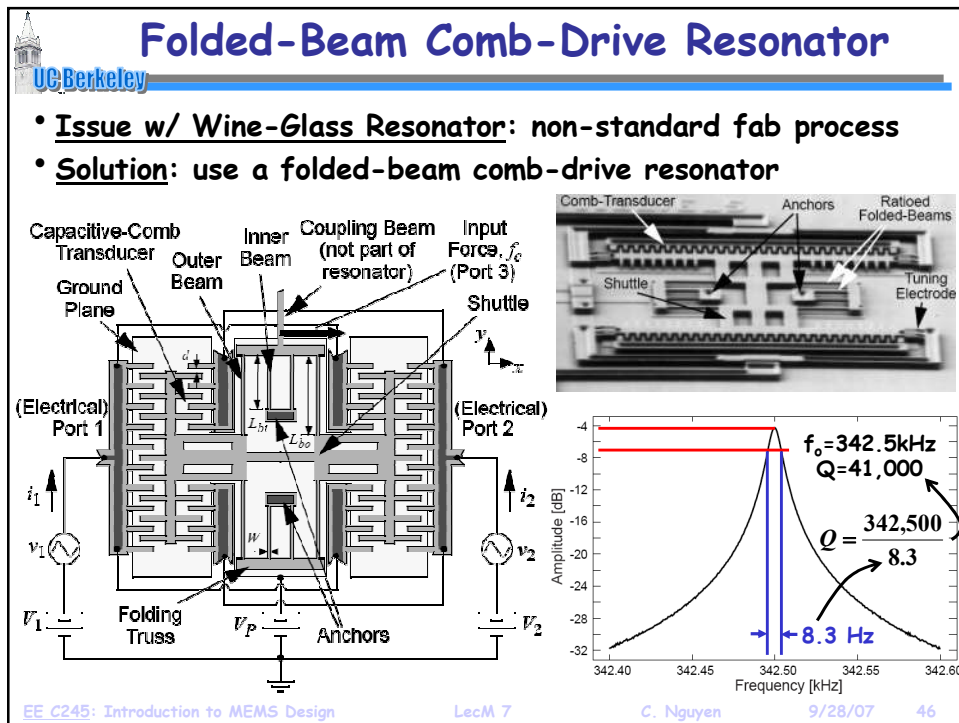
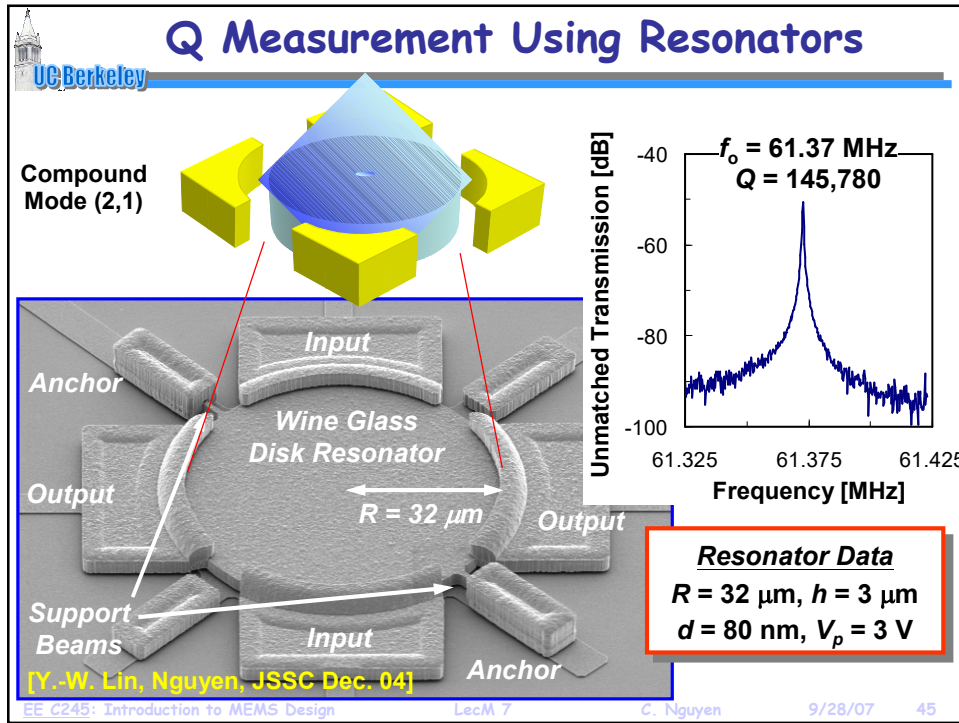
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- Single structure measures both compressive and tensile stress
- Expansion or contraction of test beam → deflection of pointer
- Vernier movement indicates type and magnitude of stress



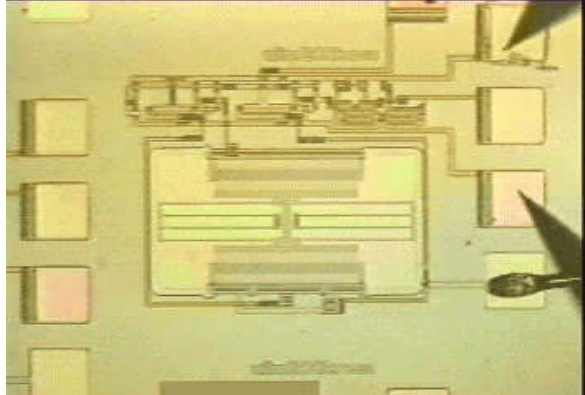
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Comb-Drive Resonator in Action

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- Below: fully integrated micromechanical resonator oscillator using a MEMS-last integration approach

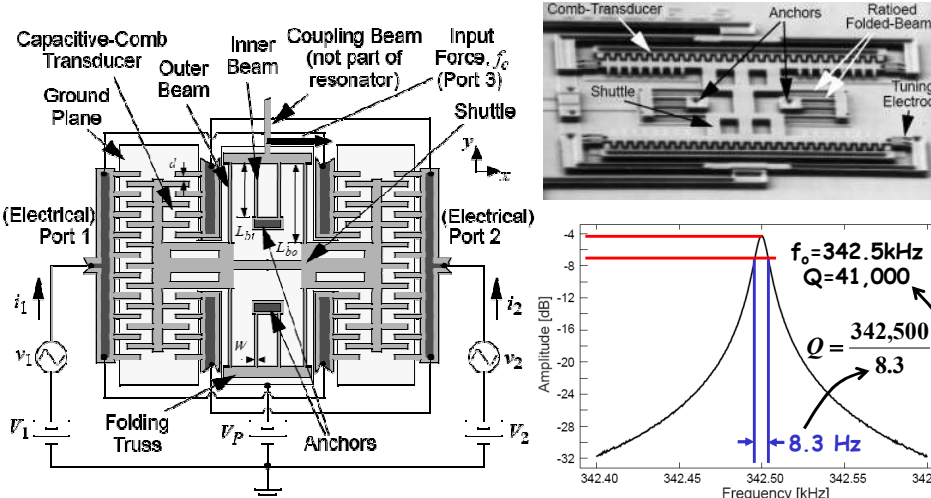


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Folded-Beam Comb-Drive Resonator

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- Issue w/ Wine-Glass Resonator: non-standard fab process
- Solution: use a folded-beam comb-drive resonator



Capacitive-Comb Transducer

Ground Plane

(Electrical) Port 1

i_1

v_1

V_1

Folding Truss

V_P

Anchors

V_2

(Electrical) Port 2

i_2

v_2

Comb-Transducer

Anchors

Ratioed Folded-Beams

Shuttle

Tuning Electrode

Input Force, f_c (Port 3)

Shuttle

Inner Beam

Coupling Beam (not part of resonator)

Outer Beam

L_{bi}

L_{bo}

w

Amplitude [dB]

Frequency [kHz]

$f_o = 342.5\text{kHz}$

$Q = 41,000$

$Q = \frac{342,500}{8.3}$

8.3 Hz

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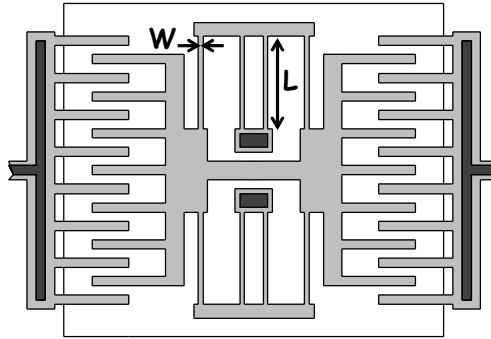
Measurement of Young's Modulus

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- Use micromechanical resonators
 - ↗ Resonance frequency depends on E
 - ↗ For a folded-beam resonator:

$$\text{Resonance Frequency} = f_o = \left[\frac{4Eh(W/L)^3}{M_{eq}} \right]^{1/2}$$

h = thickness



Young's modulus
 ↓
 $4Eh(W/L)^3$
 ↑
 Equivalent mass
 M_{eq}

- Extract E from measured frequency f_o .
- Measure f_o for several resonators with varying dimensions
- Use multiple data points to remove uncertainty in some parameters

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Anisotropic Materials

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Elastic Constants in Crystalline Materials

- Get different elastic constants in different crystallographic directions → 81 of them in all
 - ↳ Cubic symmetries make 60 of these terms zero, leaving 21 of them remaining that need be accounted for
- Thus, describe stress-strain relations using a 6x6 matrix

$$\begin{matrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{zx} \\ \tau_{xy} \end{matrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{12} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{13} & C_{23} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{14} & C_{24} & C_{34} & C_{44} & C_{45} & C_{46} \\ C_{15} & C_{25} & C_{35} & C_{45} & C_{55} & C_{56} \\ C_{16} & C_{26} & C_{36} & C_{46} & C_{56} & C_{66} \end{bmatrix} \begin{matrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{zx} \\ \gamma_{xy} \end{matrix}$$

↑ Stresses Stiffness Coefficients ↑ Strains

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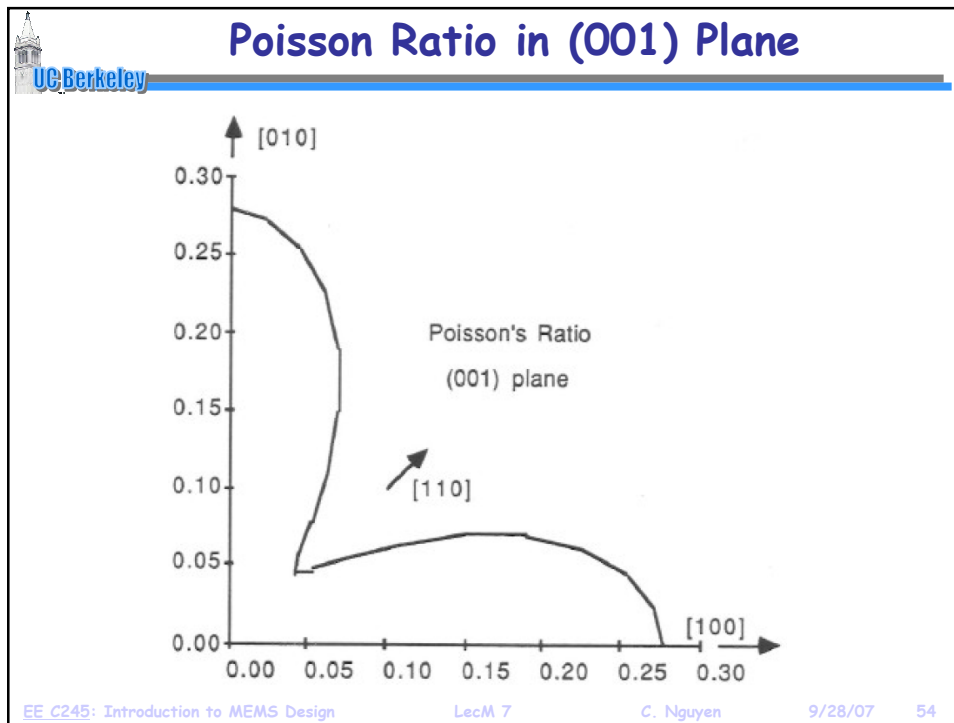
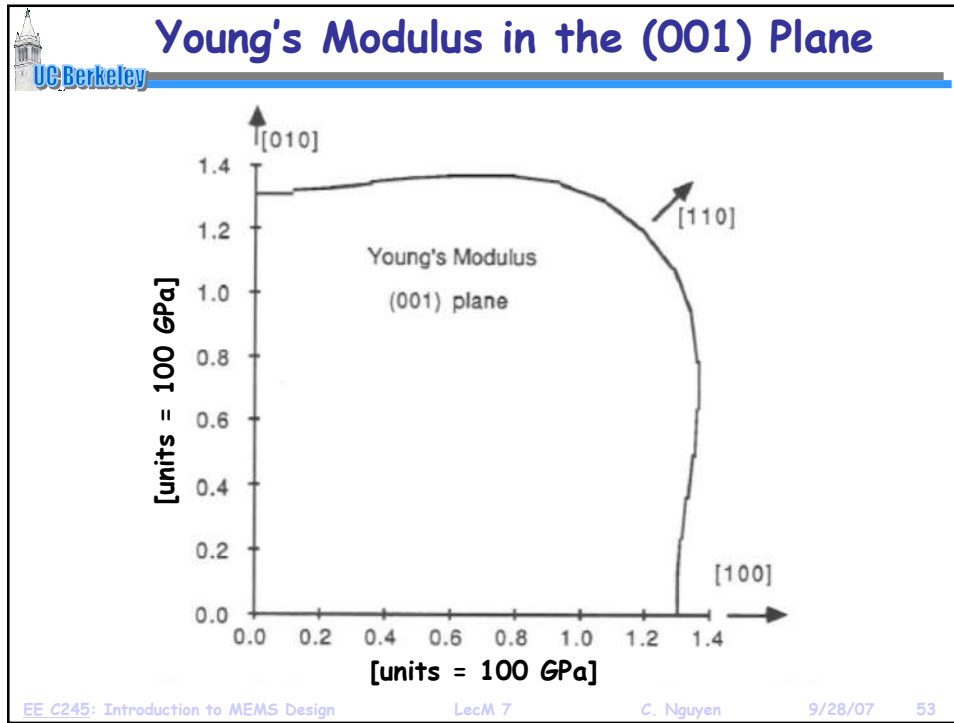
Stiffness Coefficients of Silicon


- Due to symmetry, only a few of the 21 coefficients are non-zero
- With cubic symmetry, silicon has only 3 independent components, and its stiffness matrix can be written as:

$$\begin{matrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{zx} \\ \tau_{xy} \end{matrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{matrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{zx} \\ \gamma_{xy} \end{matrix}$$

where $\begin{cases} C_{11} = 165.7 \text{ GPa} \\ C_{12} = 63.9 \text{ GPa} \\ C_{44} = 79.6 \text{ GPa} \end{cases}$

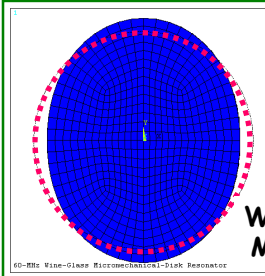
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Anisotropic Design Implications

- Young's modulus and Poisson ratio variations in anisotropic materials can pose problems in the design of certain structures
- E.g., disk or ring resonators, which rely on isotropic properties in the radial directions
 - ↳ Okay to ignore variation in RF resonators, although some Q hit is probably being taken
- E.g., ring vibratory rate gyroscopes
 - ↳ Mode matching is required, where frequencies along different axes of a ring must be the same
 - ↳ Not okay to ignore anisotropic variations, here

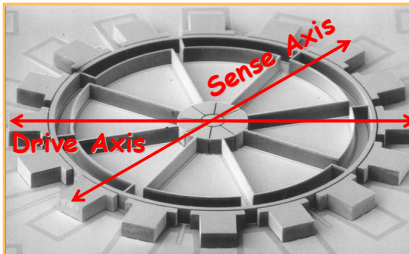


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Wine-Glass Mode Disk

60-MHz Wine-Glass Micromechanical Disk Resonator



Ring Gyroscope

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