1. Suppose that chemical vapor deposition of a material with properties summarized in table PS4.1 yields a film with non-uniform grain sizes with the cross-sectional structure shown below (which is not quite to scale). Here, the deposition starts with small symmetric grains, but as time passes in the chamber the earlier grains cluster and deform, while newly deposited grains remain smaller.

(a) If all gaps between grains are of uniform separation \( g = 0.1 \text{nm} \), and we cool this film from a stress-free state at \( T_i = 650^\circ \text{C} \) to 25\(^\circ \text{C} \) (where the gaps fully close), what are the final stress and strain gradients? Provide formulas and assume a continuous linear gradient.

(b) Assume this film is now shaped into a cantilever beam with length \( L = 100 \mu \text{m} \) and width \( W = 5 \mu \text{m} \). As the beam relieves its internal stress gradient, it will begin to warp. What is the deflection at the tip of the beam? Which direction does it deflect?

(c) You attempt to negate this deflection by depositing a very thin stressed film on top of the beam, with properties at room temperature in the table below. Should the film itself be under tensile or compressive stress? How thick does it need to be?

<table>
<thead>
<tr>
<th>Film Properties (part a, b, c)</th>
<th>Thin Stressed Film (part c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young's Modulus ( (E) )</td>
<td>150 GPa</td>
</tr>
<tr>
<td>Linear Thermal Coefficient ( (\alpha_T) )</td>
<td>1.3 ( \mu )strain/K</td>
</tr>
<tr>
<td>Density ( (\rho) )</td>
<td>2700 kg/m(^3)</td>
</tr>
<tr>
<td>Poisson’s Ratio ( (\nu) )</td>
<td>0.3</td>
</tr>
<tr>
<td>Initial Uniform Stress ( (\sigma_0) )</td>
<td>0 MPa</td>
</tr>
</tbody>
</table>

Table PS4.1
2. Suppose you applied the following fabrication process flow to the layout shown below:

1. Deposit 2 μm of LTO.
2. Deposit 2 μm of undoped polysilicon.
3. Ion implant with phosphorous.
4. Do lithography: Spin and expose resist with the single mask layer shown below.
5. Etch the polysilicon using reactive ion etching (RIE). Assume that the photoresist sidewalls are perfectly straight and the etch is completely anisotropic with an infinite selectivity of polysilicon to oxide.
6. Remove photoresist.
7. Dip the wafer in 5:1 buffered hydrofluoric acid (BHF) for 10 minutes, where the etch rate of LTO in BHF is 700 nm/min.

Assume for this problem that the undoped polysilicon film is completely stress free after deposition and before the ion implantation. Also assume that there is no stiction.

![Diagram of the process flow](image)

**Polysilicon Material Properties:**
Young’s Modulus, $E = 150$ GPa; Density, $\rho = 2,300$ kg/m$^3$; Poisson ratio, $\nu = 0.226$

**Fig PS4.2**

Answer the following questions regarding this problem.

(a) Draw the final cross-section of the structure after release along A-A’.

(b) If the implanted phosphorous is 10 nm deep with a uniform axial compressive stress of 400 MPa, how far above the surface of the wafer will the tip of the beam be once the structure is released?
3. Fig. PS4.3 presents top view of a 2 µm-thick polysilicon structure suspended 2 µm above the substrate except for the anchoring locations indicated as the darkly shaded regions. Key dimensions for the beams and data on the structural material used in this problem are given in the box below the figure. Assume that all folding trusses and shuttles are rigid in all directions.

\[ \text{Structural Material Properties:} \]
\[ E = 150 \text{ GPa}, \rho = 2330 \text{ kg/m}^3, \text{Poisson ratio} = 0.2 \]

\[ \text{Geometric Dimensions:} \]
\[ L_1 = 30 \text{ µm}, L_2 = 10 \text{ µm}, \]
\[ \text{width of all beams} = w = 2 \text{ µm} \]

(a) Write an expression for the static spring constant in the x-direction at location A and calculate its numerical value (with units).

(b) Write an expression for the static spring constant in the x-direction at location B and calculate its numerical value (with units).

(c) If point A moves \( X_A = 1 \text{µm} \) in the x-direction due to a force applied at A, how much does point B move? Provide an expression for \( X_B \) in terms of \( X_A \) and spring constants, and calculate a numerical value.
4. Thermoelastic damping (TED) can occur in any material subject to periodic stress. It is pronounced in flexural mode resonators when heat moves from compressed parts to tensioned parts during vibration. For example, as shown in Fig. PS4.4, when a clamped-clamped beam (CC-Beam) resonator undergoes bending, the tensile and compressive parts in the structure will generate temperature gradient. Thermal conductivity in the material will allow these hot and cold regions to equilibrate, which causes heat flux and energy loss, thereby limits \( Q \) of the resonator. In most cases, for low frequency CC-Beam resonators, TED is the main loss mechanism and dominates the quality factor. The magnitude of the TED in a clamped-clamped beam is given by the equations in Module 7 of Lecture 11. Suppose you want to design a 100 MHz clamped-clamped beam resonator with very high quality factor. You have the freedom to choose either quartz or single crystalline silicon as the structure material, the properties of which are shown in Table PS4.4. Below are the design guidelines and limitations you should follow:

- The CC-beam resonator length \( (L) \) and width \( (W) \) should be no smaller than 1 \( \mu \text{m} \) due to lithography resolution limit.
- The beam thickness \( h \) should be less than 3 \( \mu \text{m} \).
- In order to have minimum mode shape distortion, the length/thickness ratio \( L/h \) should be no smaller than 5, and the beam width \( W \) should be exactly 5\( \times \) smaller compared with the beam length \( L \).

Come up with a design of 100 MHz clamped-clamped beam resonator that attains the highest \( Q \) if TED is the dominant loss. Clearly state the material you pick, the beam thickness \( h \) you choose, the key dimensions of the beam \((L, W)\) and the theoretical quality factor \( Q \) it can reach. (Assume the resonator is operating in vacuum at room temperature 300 K.)

![Fig PS4.4](image)

![Table PS4.4](image)