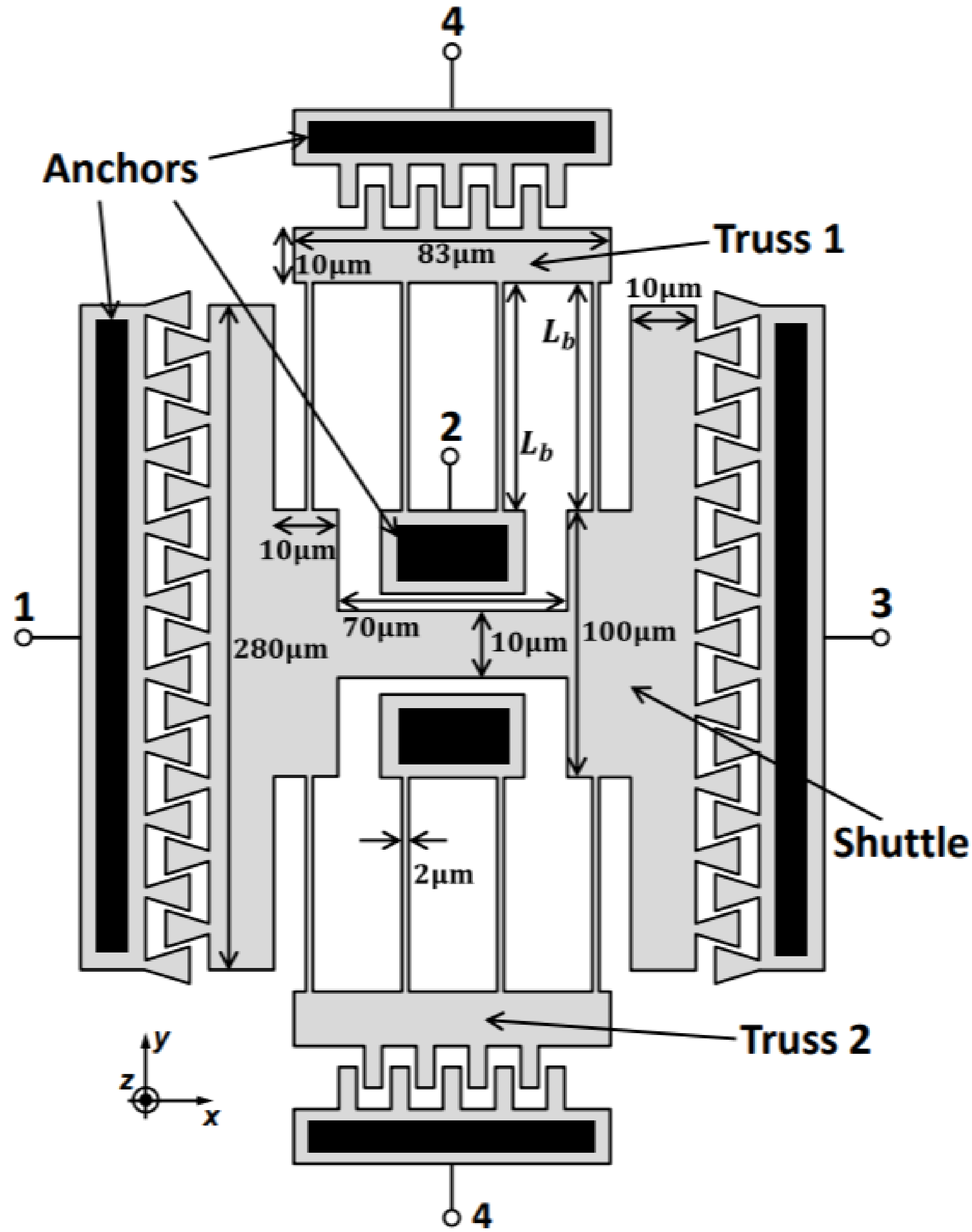
PROBLEM SET #6

Issued: Tuesday, April 9, 2019

Due: Tuesday, April 23, 2019 at 9:00 a.m. on Gradescope.

Figure PS.6-1 presents a capacitive transduced micromechanical comb-drive resonator with numerous ports, constructed entirely of doped polysilicon. Here, all lightly shaded regions are suspended 2μm above a substrate; while all darkly shaded regions are firmly anchored to the substrate. Assume that all structures are rigid, except for the thin beams in the folded beam suspensions. The device is symmetric in both the x and y directions. Dimensions for the structure are given in both Figure PS.6-1 and Figure PS.6-2.

1. Determine the length of L_b so that the x-directed natural resonant frequency of the suspended comb-drive structure is 20 kHz.
2. For the rest of this problem, assume that $L_b = 100\mu$m. (Note that this is not necessarily the correct answer to part (1).) Find the static displacement of the shuttle in the x-direction when Port 3 is at a voltage of 10 V and Port 1, 2 & 4 are at 0 V. Is this static displacement positive or negative?
3. Suppose the angle θ of the comb fingers are reduced to 10° while all other labeled geometric dimensions remain the same. Again, find the static displacement x_0 of the shuttle in the x-direction when Port 3 is at a voltage of 10 V and Port 1, 2 & 4 are at 0 V. Is this static displacement positive or negative?
4. Suppose you apply a voltage V_{PI} at Port 3 while maintaining Port 1, 2 & 4 at 0 V. Calculate the minimum voltage V_{PI} that will cause the structure to pull-in to Port 3.
5. Calculate the x-directed resonant frequency of the structure at room temperature (20°C), assuming Port 2 is biased at 5 V while Port 1, 3 and 4 are biased at 0 V.
6. The suspended comb-drive structure's resonant frequency varies with environmental temperature fluctuations due to temperature dependent Young's Modulus, which causes issues in applications such as timing. The electrical stiffness actually can help stabilize the resonant frequency when temperature changes. For example, an electric potential V_{tune} can be applied at Port 4 to cancel the intrinsic frequency variation due to temperature change. Figure PS.6-3 presents a test configuration that drives the suspended comb-drive structure into resonance, with Port 1 driven by a small-signal ac voltage v_i and Port 2, 3 & 4 biased at 5 V, 0 V, and V_{tune}, respectively. Derive the expression V_{tune} of in terms of temperature T that can stabilize the resonant frequency and plot V_{tune} vs. T over a temperature range of 0°C to 70°C. What value of V_{tune} is needed at 0°C and 70°C? (Assume the temperature coefficient of the structure's resonant frequency (TCF) due to intrinsic Young's Modulus change is -30ppm/°C over the whole temperature range.)



Structural Material Properties
$E = 150$ GPa, $\rho = 2300$ kg/m^3, Poisson ratio, $\nu = 0.226$
Geometric Dimensions:
All beams widths, $w = 2$μm
All finger gaps, $g_o = 1$μm
Thickness, $t = 2$μm

Fig. PS.6-1

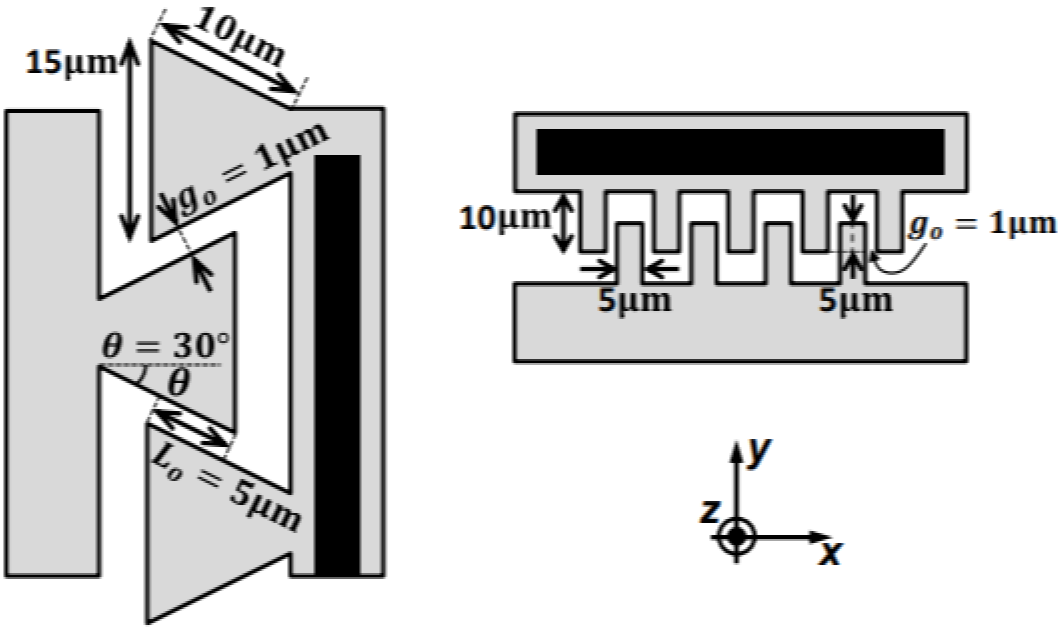


Fig. PS.6-2

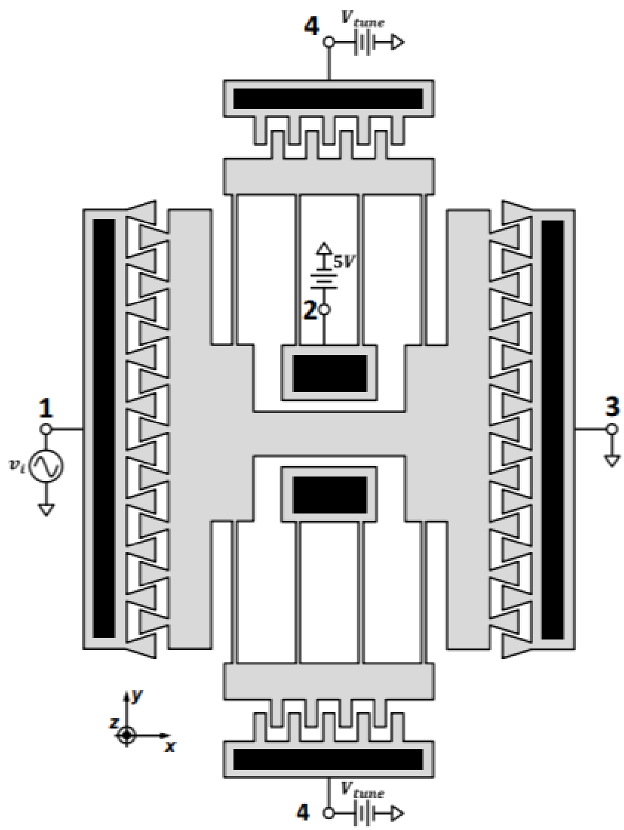


Fig. PS.6-3