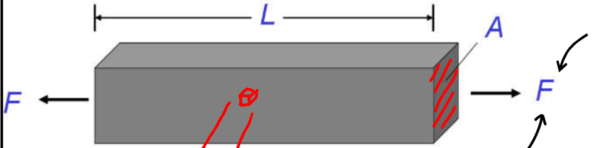
## Normal Stress (1D)

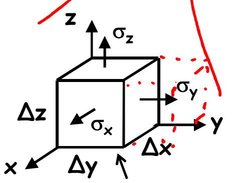



If the force acts normal to a surface, then the stress is called a **normal stress**

Force assumed uniform over the whole area A

$$\text{Stress} = \begin{Bmatrix}\text{Force per}\\ \text{unit area}\end{Bmatrix} = \sigma = \frac{F}{A} \quad [N/m^2 = Pa]$$

standard mks unit



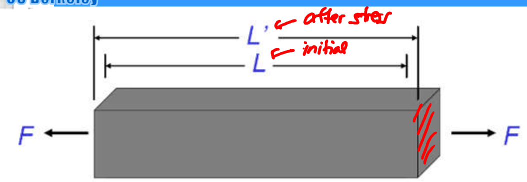

Differential volume element

⇒ Microscopic Definition: force per unit area acting on the surface of a differential volume element of a solid body

⇒ Note: assume stress acts uniformly across the entire surface of the element, not at just a point



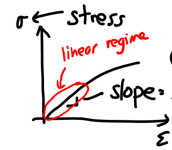
## Strain (1D)




Sometimes a unit called the "microstrain" is used, where $1\mu\varepsilon = \frac{\Delta L}{L}$ of 1 part in $10^6$

$$\text{Strain} = \begin{Bmatrix}\text{Fractional Change}\\ \text{in Length}\end{Bmatrix} = \varepsilon = \frac{L'-L}{L} = \frac{\Delta L}{L} \quad [\text{unitless}]$$

In the elastic regime (i.e., for "small" stresses at "low" temperatures), strain is found to be proportional to stress



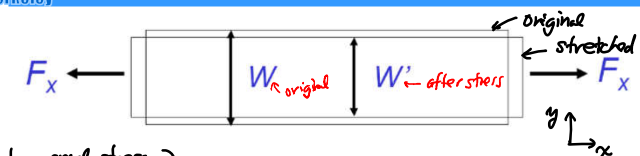

For solids: MPa → GPa

$\sigma = \varepsilon E \rightarrow$ $\boxed{\varepsilon = \frac{\sigma}{E} \quad [\text{unitless}]}$

Thus, the units of E are the same as $\sigma \rightarrow Pa$



## The Poisson Ratio




Apply normal stress to a free-standing object → uniaxial strain
→ but also get contraction in directions transverse to the uniaxial strain

⇒ contraction creates a (−) strain:

$$\varepsilon_y = \frac{W'-W}{W} = \frac{\Delta W}{W} = -\nu\varepsilon_x$$

$\nu$ = Poisson ratio [unitless]

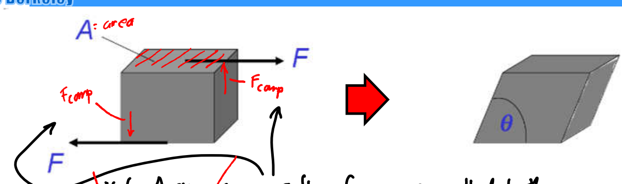
typical values: 0 → 0.5

⇒ inorganic solids: 0.2 → 0.3

⇒ elastomers (e.g., rubber): ~0.5



## Shear Stress & Strain (1D)




Note: Assume compensating forces are applied to the vertical faces to avoid a net torque. (This by convention.)

$$\text{Shear Stress} = \begin{Bmatrix}\text{Force Per Unit Area}\\ \text{Parallel to the Surfaces}\end{Bmatrix} = \tau = \frac{F}{A} \quad [Pa]$$

Generates a shear strain:

$$\text{Shear Strain} = \theta = \frac{\tau}{G}$$

$G \triangleq$ shear modulus

$$G = \frac{E}{2(1+\nu)}$$

## 2D and 3D Considerations

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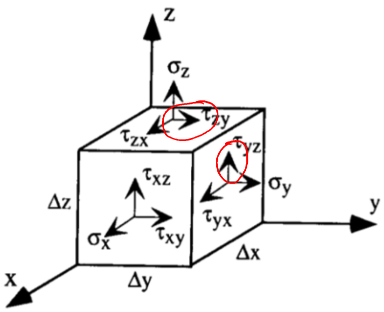
- Important assumption: the differential volume element is in static equilibrium → no net forces or torques (i.e., rotational movements)
  - Every σ must have an equal σ in the opposite direction on the other side of the element
  - For no net torque, the shear forces on different faces must also be matched as follows:




Stresses acting on a differential volume element

$$\tau_{xy} = \tau_{yx} \qquad \tau_{xz} = \tau_{zx} \qquad \tau_{yz} = \tau_{zy}$$



## 2D Strain

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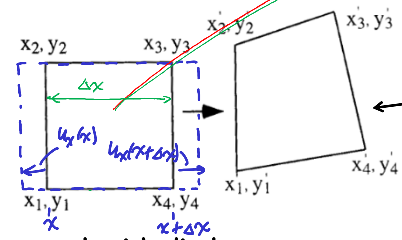
- In general, motion consists of
  - rigid-body displacement (motion of the center of mass)
  - rigid-body rotation (rotation about the center of mass)
  - Deformation relative to displacement and rotation




Area element experiences both displacement and deformation

- Must work with displacement vectors
- Differential definition of axial strain: ⟶ $\varepsilon_x = \dfrac{u_x(x+\Delta x) - u_x(x)}{\Delta x} = \dfrac{\partial u_x}{\partial x}$



## 2D Shear Strain

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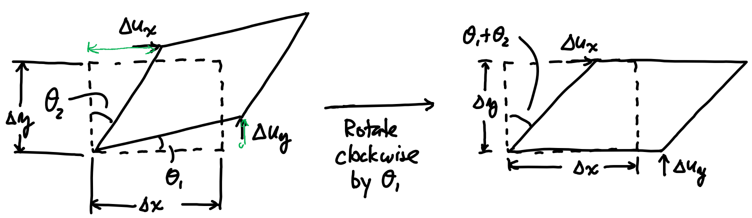


⇒ For shear strains, must remove any rigid body rotation that accompanies the deformation

↳ use a symmetric definition of shear strain:

$$\gamma_{xy} = \theta_2 + \theta_1 \approx \left(\frac{\Delta u_x}{\Delta y} + \frac{\Delta u_y}{\Delta x}\right) = \left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right)$$

For small amplitude deformations.



## Volume Change for a Uniaxial Stress

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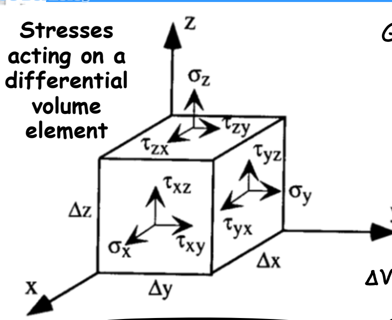
Stresses acting on a differential volume element




Given an x-directed uniaxial stress, $\sigma_x$:

$\Delta x \rightarrow \Delta x(1+\epsilon_x)$

$\Delta y \rightarrow \Delta y(1-\nu\epsilon_x)$

$\Delta z \rightarrow \Delta z(1-\nu\epsilon_x)$

↓ The resulting change in volume $\Delta V$

$$\Delta V = \Delta x \Delta y \Delta z (1+\epsilon_x)(1-\nu\epsilon_x)^2 - \Delta x \Delta y \Delta z$$
$$= \Delta x \Delta y \Delta z \left[(1+\epsilon_x)(1-\nu\epsilon_x)^2 - 1\right]$$

[Assume small strains] ⇒ $\Delta V = \Delta x \Delta y \Delta z \left[(1+\epsilon_x)(1-2\nu\epsilon_x) - 1\right]$

$[(1+mx)^n \approx 1 + nmx]$ ⇒ $\approx \Delta x \Delta y \Delta z \left[\cancel{1} + \epsilon_x - 2\nu\epsilon_x - \cancel{2\nu\epsilon_x^2} - \cancel{1}\right]$ negl.

$$\boxed{\Delta V \approx \Delta x \Delta y \Delta z (1-2\nu)\epsilon_x}$$

For $\nu = 0.5$ (rubber) → no $\Delta V$!

$\nu < 0.5$ → finite $\Delta V$

## Isotropic Elasticity in 3D

- Isotropic = same in all directions
- The complete stress-strain relations for an isotropic elastic solid in 3D: (i.e., a generalized Hooke's Law)

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right] \qquad \gamma_{xy} = \frac{1}{G}\tau_{xy}$$

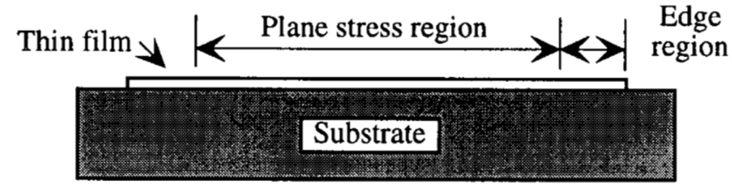
$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu\left(\sigma_z + \sigma_x\right)\right] \qquad \gamma_{yz} = \frac{1}{G}\tau_{yz}$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \underbrace{\nu\left(\sigma_x + \sigma_y\right)}_{\text{Basically, add in off-axis strains from normal stresses in other directions}}\right] \qquad \gamma_{zx} = \frac{1}{G}\tau_{zx}$$



## Important Case: Plane Stress

- Common case: very thin film coating a thin, relatively rigid substrate (e.g., a silicon wafer)




- At regions more than 3 thicknesses from edges, the top surface is stress-free → $\sigma_z = 0$
- Get two components of in-plane stress:

$$\varepsilon_x = (1/E)[\sigma_x - \nu(\sigma_y + 0)]$$

$$\varepsilon_y = (1/E)[\sigma_y - \nu(\sigma_x + 0)]$$



## Important Case: Plane Stress (cont.)

- Symmetry in the xy-plane → $\sigma_x = \sigma_y = \sigma$
- Thus, the in-plane strain components are: $\varepsilon_x = \varepsilon_y = \varepsilon$ where

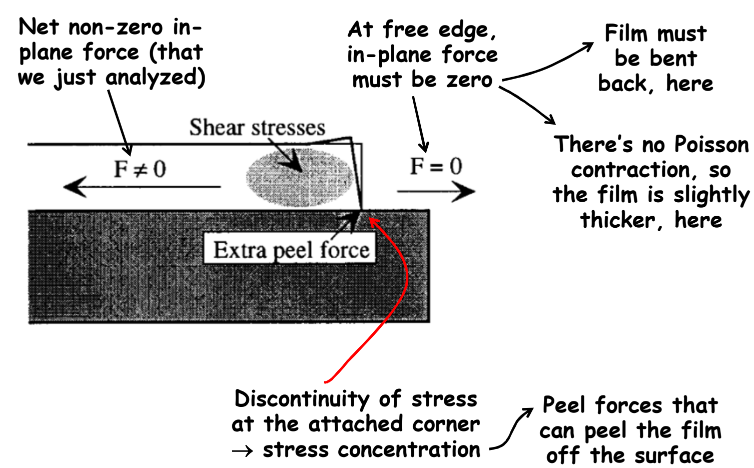
$$\varepsilon_x = (1/E)[\sigma - \nu\sigma] = \frac{\sigma}{[E/(1-\nu)]} = \frac{\sigma}{E'}$$

and where

$$\text{Biaxial Modulus} \triangleq E' = \frac{E}{1-\nu}$$



## Edge Region of a Tensile ($\sigma$>0) Film



## Linear Thermal Expansion

- As temperature increases, most solids expand in volume
- Definition: linear thermal expansion coefficient

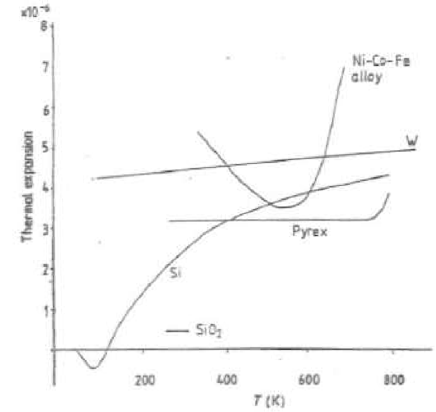
$$\left.\begin{array}{c}\text{Linear thermal}\\ \text{expansion coefficient}\end{array}\right\} \triangleq \alpha_T = \frac{d\varepsilon_x}{dT} \qquad [\text{Kelvin}^{-1}]$$

Remarks:

- $\alpha_T$ values tend to be in the $10^{-6}$ to $10^{-7}$ range
- Can capture the $10^{-6}$ by using dimensions of μstrain/K, where $10^{-6}\ K^{-1}$ = 1 μstrain/K
- In 3D, get volume thermal expansion coefficient $\longrightarrow$ $\frac{\Delta V}{V} = 3\alpha_T \Delta T$
- For moderate temperature excursions, $\alpha_T$ can be treated as a constant of the material, but in actuality, it is a function of temperature



## $\alpha_T$ As a Function of Temperature

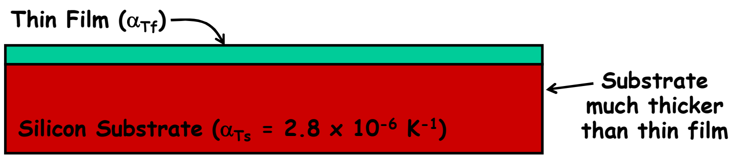



[Madou, Fundamentals of Microfabrication, CRC Press, 1998]

- Cubic symmetry implies that α is independent of direction



## Thin-Film Thermal Stress




- Assume film is deposited stress-free at a temperature $T_r$, then the whole thing is cooled to room temperature $T_r$
- Substrate much thicker than thin film → substrate dictates the amount of contraction for both it and the thin film

Thermal strain of the substrate: (in one in-plane dimension)

$$\epsilon_s = -\alpha_{Ts}\Delta T \quad , \text{ where } \Delta T = T_d - T_r$$

If the film were not attached to the substrate: $\epsilon_{f,free} = -\alpha_{Tf}\Delta T$

over



## Linear Thermal Expansion

But the film is attached to the substrate, so the actual strain in the film is the same as that in the substrate:

$$\epsilon_{f,attached} = -\alpha_{Ts}\Delta T$$

Thus:

$$\boxed{\text{Thermal Mismatch Strain} = \epsilon_{f,mismatch} = (\alpha_{Tf} - \alpha_{Ts})\Delta T}$$

Note that this is biaxial strain

it can only be developed by an in-plane biaxial stress:

$$\sigma_{f,mismatch} = \left(\frac{E}{1-\nu}\right)\epsilon_{f,mismatch}$$

Ex. Thin-film is polyimide → $\alpha_{Tf} = 70\times10^{-6}\ K^{-1}$, $E' = 4GPa$

deposited @ 250°C, then cooled to RT = 25°C → $\Delta T = 225K$

e.g., $SiO_2$

$$\epsilon_{f,mismatch} = (70-2.8)\mu(225) = 1.5\times10^{-2}$$

$$\sigma_{f,mismatch} = (4G)(1.5\times10^{-2}) = \underline{\underline{60.5 MPa}}$$

stress is (+), ∴ tensile [(-) would be compressive]

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## MEMS Material Properties



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## Material Properties for MEMS

| Material | Density, ρ, Kg/m³ | Modulus, E, GPa | E/ ρ GN/kg-m |
|---|---|---|---|
| Silicon | 2330 | 165 | 72 |
| Silicon Oxide | 2200 | 73 | 36 |
| Silicon Nitride | 3300 | 304 | 92 |
| Nickel | 8900 | 207 | 23 |
| Aluminum | 2710 | 69 | 25 |
| Aluminum Oxide | 3970 | 393 | 99 |
| Silicon Carbide | 3300 | 430 | 130 |
| Diamond | 3510 | 1035 | 295 |

Units: $(m/s)^2$

$\sqrt{(E/\rho)}$ is acoustic velocity

units of m/s

[Mark Spearing, MIT]



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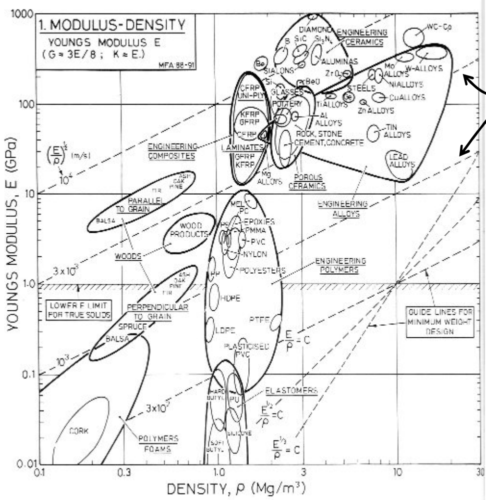
## Young's Modulus Versus Density




Lines of constant acoustic velocity

[Ashby, Mechanics of Materials, Pergamon, 1992]