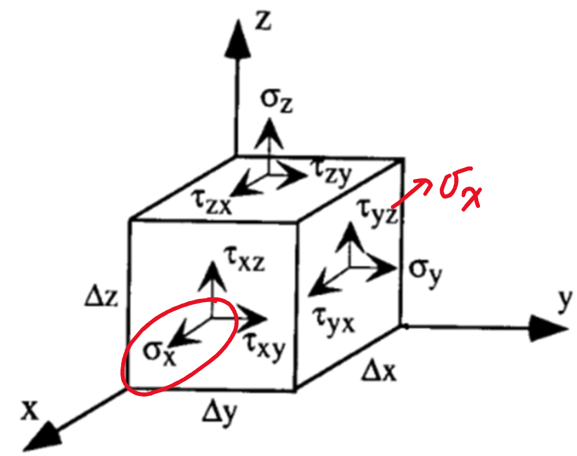
Lecture 11: Mechanics of Materials I

- Announcements:
- Module 7 on "Mechanics of Materials" online
- HW#3 due Tuesday, 3/5, at 9 a.m.
- Note: The hdmi projector interface was broken
  - Had to spend time setting up the VGA interface
  - Lost about 15 minutes, which we'll make up in a video lecture the next time I travel
- ----------------------------------
- Reading: Senturia, Chpt. 8
- Lecture Topics:
  - Stress, strain, etc., for isotropic materials
  - Thin films: thermal stress, residual stress, and stress gradients
  - Internal dissipation
  - MEMS material properties and performance metrics
- ----------------------------------
- Last Time:
- Started Module 7 on "Mechanics of Materials"
- Now, continue with this …

Example. Exercise the "terms"

⇒ determine the volume change ΔV for a uniaxial stress (along the x-direction)



Upon application of σ_x, what is the volume change ΔV?

Before σ_x → After σ_x

$$\left.\begin{aligned} \Delta x &\longrightarrow \Delta x(1+\epsilon_x) \\ \Delta y &\longrightarrow \Delta y(1-\nu\epsilon_x) \\ \Delta z &\longrightarrow \Delta z(1-\nu\epsilon_x) \end{aligned}\right\}$$

assuming isotropic material

same ν along x & y

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The resulting volume change:

$$\Delta V = \underbrace{\Delta x \Delta y \Delta z (1+\epsilon_x)(1-\nu\epsilon_x)^2}_{\text{volume after applying } \sigma_x} - \Delta x \Delta y \Delta z$$

$$= \Delta x \Delta y \Delta z \left[(1+\epsilon_x)(1-\nu\epsilon_x)^2 - 1 \right]$$

[Assume small strains] $\Rightarrow \underbrace{(1+mx)^n \approx 1+nmx}_{\text{Binomial Theorem}}$

$$\Delta V = \Delta x \Delta y \Delta z \left[(1+\epsilon_x)(1-2\nu\epsilon_x) - 1 \right]$$

$$\boxed{\Delta V = \Delta x \Delta y \Delta z (1-2\nu)\epsilon_x}$$

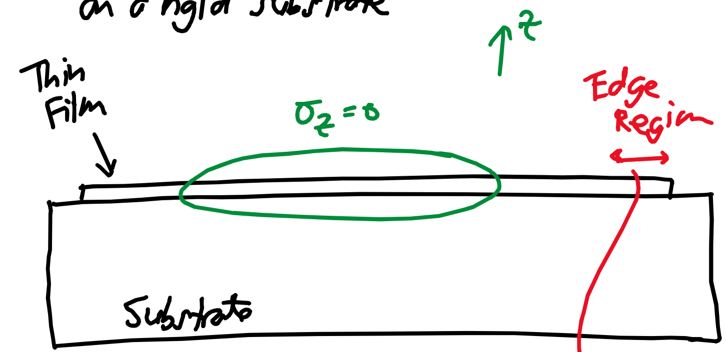
For $\nu = 0.5$ (rubber) $\rightarrow \Delta V = 0$, no volume change! ✓

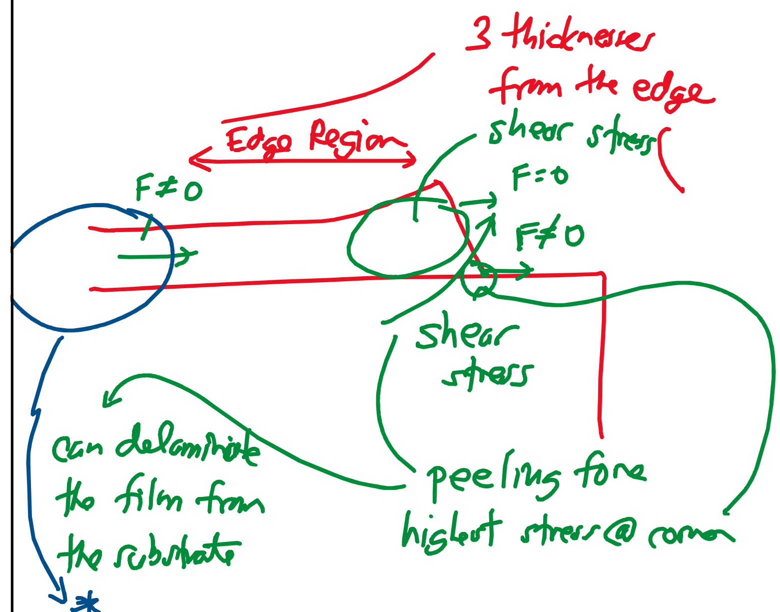
$\nu < 0.5 \rightarrow$ finite ΔV

For isotropic materials $\rightarrow$ Module 7 (last 2 slides)

$\boxed{\text{Important Case: Plane Stress}}$

$\Rightarrow$ common case for a thin-film coating on a rigid substrate





* Take a closer look @ this region: $\sigma_z = 0$

Get two components of stress (& strain)

$$\epsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + 0)]$$

$$\epsilon_y = \frac{1}{E}[\sigma_y - \nu(\sigma_x - 0)]$$

Assume: plane stress → isotropic: $\sigma_x = \sigma_y = \sigma$
(symmetry in the xy-plane)

$\epsilon_x = \epsilon_y = \epsilon$

$$\epsilon_x = \frac{1}{E}[\sigma - \nu\sigma]$$

$$= \frac{\sigma}{\left(\frac{E}{1-\nu}\right)} \Rightarrow \epsilon_x = \frac{\sigma}{E'}$$

where $\boxed{E' \triangleq \text{Biaxial Modulus} = \frac{E}{1-\nu}}$

Linear Thermal Expansion

temperature ↑ → solids expand in volume

Definition: linear thermal expansion coefficient

$$\boxed{\left.\begin{matrix}\text{Linear Thermal}\\ \text{Exp. Coefficient}\end{matrix}\right\} \triangleq \alpha_T = \frac{d\epsilon_x}{dT} \quad [\text{Kelvin}^{-1}]}$$

Remarks:

1. α_T values tend to be in the 10^{-6} to 10^{-7} range
2. $10^{-6}K^{-1} = 1\mu\text{strain}/K$
3. In 3D, get a volume thermal exp. coefficient: $\frac{\Delta V}{V} = 3\alpha_T \Delta T$
4. For moderate ΔT's → $\alpha_T \approx$ constant
   for larger ΔT, then $\alpha_T = f(T)$
   See Module 7 slide 18

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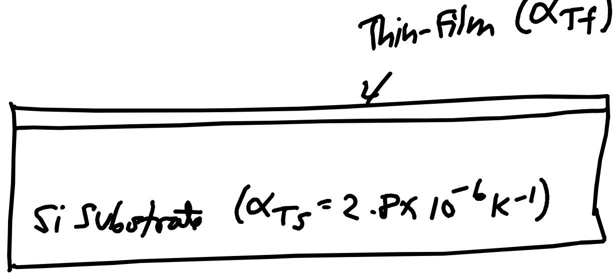
Ex. Thin-Film Thermal Stress



Assume.

1. Substrate is much thicker than the film.
2. The film deposits stress-free @ T_d ← deposition temperature
3. Then, the whole thing cools to room temperature, T_r.

Thermal strain of the Substrate:
(in one plane dimension)

$$\epsilon_s = -\alpha_{Ts}\Delta T, \text{ where } \Delta T = T_d - T_r$$

If the film were not attached to substrate

$$\epsilon_{f,free} = -\alpha_{Tf}\Delta T$$

But the film is attached to the substrate
→ thickness of substrate >> film thickness
∴ substrate wins!

Therefore, the actual strain experienced by the film is that of the substrate:

$$\epsilon_{f,attached} = -\alpha_{Ts}\Delta T$$

Thus:

$$\left.\begin{array}{l}\text{Thermal}\\ \text{Mismatch}\\ \text{Strain}\end{array}\right\} = \epsilon_{f,mismatch} = (\alpha_{Tf} - \alpha_{Ts})\Delta T$$

→ Note this is biaxial strain (assuming the film deposits isotropically on the substrate)

$$\sigma_{f,mismatch} = \underbrace{\left(\frac{E}{1-\nu}\right)}_{E'} \epsilon_{f,mismatch}$$

Ex. Thin-film is polyimide

$\alpha_{Tf} = 70 \times 10^{-6}\ K^{-1}$

$E' = 4GPa$

$\underbrace{\text{deposited @ } 250^\circ C\text{, then cooled to } RT = 25^\circ C}_{\Delta T = 225K}$

$\epsilon_{f,mismatch} = (70 - 2.8)\mu\,(225) = 1.5 \times 10^{-2}$

[$\mu = 10^{-6}$, $m = 10^{-3}$, $k = 10^{3}$, $G = 10^{9}$]

$\sigma_{f,mismatch} = (4G)(1.5 \times 10^{-2}) = 60.5\ MPa$

stress is (+) → tensile

[if (−), then compressive]

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