

Lecture 14m: Microstructural Elements

Sign Conventions for Moments & Shear Forces

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Positive Negative

(+) moment leads to deformation with a (+) radius of curvature (i.e., upwards)

(-) moment leads to deformation with a (-) radius of curvature (i.e., downwards)

(+) shear forces produce clockwise rotation

(-) shear forces produce counter-clockwise rotation

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Measurement of Stress Gradient

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- Use cantilever beams
 - Strain gradient ($\Gamma = \text{slope of strain-thickness curve}$) causes beams to deflect up or down
 - Assuming linear strain gradient Γ , $z = \Gamma L^2/2$

compressive
tensile

[P. Krulevitch Ph.D.]

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Folded-Flexure Suspensions

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Folded-Beam Suspension

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- Use of folded-beam suspension brings many benefits
 - Stress relief: folding truss is free to move in y-direction, so beams can expand and contract more readily to relieve stress
 - High y-axis to x-axis stiffness ratio

Folding truss → free to move to accommodate beam expansion

Guided End

Folding Truss

like free

90°

Clamped End

Anchor

Comb-Driven Folded Beam Actuator

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Beam End Conditions

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TABLE 4.1
Types of commonly used support conditions for beams and frames

Type of support	Displacement boundary conditions	Force boundary conditions
 FREE	None	All, as specified
 PINNED	$u = 0$ $w = 0$	Moment is specified
 ROLLER (vertical)	$u = 0$	Transverse force and moment are specified
 ROLLER (horizontal)	$w = 0$	Horizontal force and bending moment are specified
 FIXED or CLAMPED	$u = 0$ $w = 0$ $dw/dx = 0$	None specified

[From Reddy, Finite Element Method]

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Common Loading & Boundary Conditions

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- Displacement equations derived for various beams with concentrated load F or distributed load f
- Gary Fedder Ph.D. Thesis, EECS, UC Berkeley, 1994

(a) cantilever beam, concentrated load.

(d) cantilever beam, distributed load.

cantilever	guided-end	fixed-fixed
$x = \frac{F_y L^2}{Eh^3 w}$	$x = \frac{F_y L}{Eh^3 w}$	$x = \frac{F_y L^2}{4Eh^3 w}$
$y = 4 \frac{F_y L^3}{Eh^3 w^3}$	$y = \frac{F_y L^3}{Eh^3 w^3}$	$y = \frac{1}{16} \frac{F_y L^3}{Eh^3 w^3}$
$z = 4 \frac{F_y L^3}{Eh^3 w^3}$	$z = \frac{F_y L^3}{Eh^3 w^3}$	$z = \frac{1}{16} \frac{F_y L^3}{Eh^3 w^3}$

(a) Concentrated load.

(b) guided-end beam, concentrated load.

(e) guided-end beam, distributed load.

cantilever	guided-end	fixed-fixed
$x = \frac{f_y L^2}{E}$	$x = \frac{f_y L^2}{E}$	$x = \frac{f_y L^2}{4E}$
$y = \frac{3}{2} \frac{f_y L^4}{Eh^3 w^3}$	$y = \frac{1}{2} \frac{f_y L^4}{Eh^3 w^3}$	$y = \frac{1}{32} \frac{f_y L^4}{Eh^3 w^3}$
$z = \frac{3}{2} \frac{f_y L^4}{Eh^3 w^3}$	$z = \frac{1}{2} \frac{f_y L^4}{Eh^3 w^3}$	$z = \frac{1}{32} \frac{f_y L^4}{Eh^3 w^3}$

(b) Distributed load.

(c) clamped-clamped beam, concentrated load.

(f) clamped-clamped beam, distributed load.

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