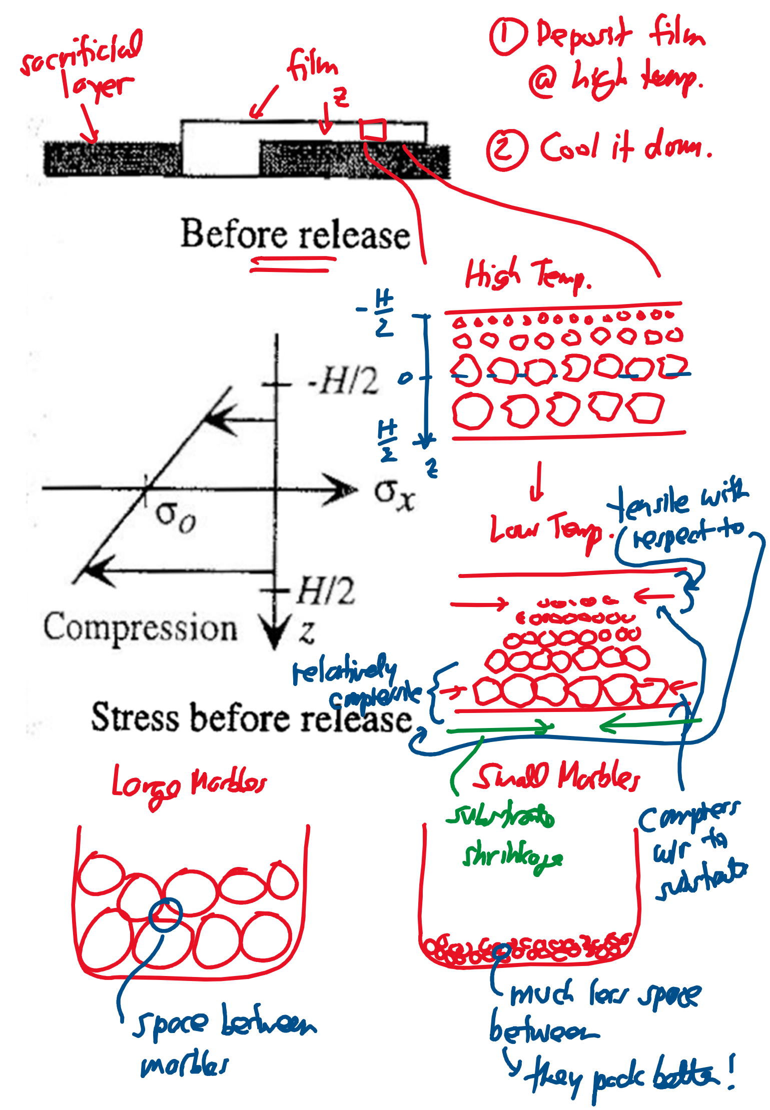
Lecture 14: Beam Combos I

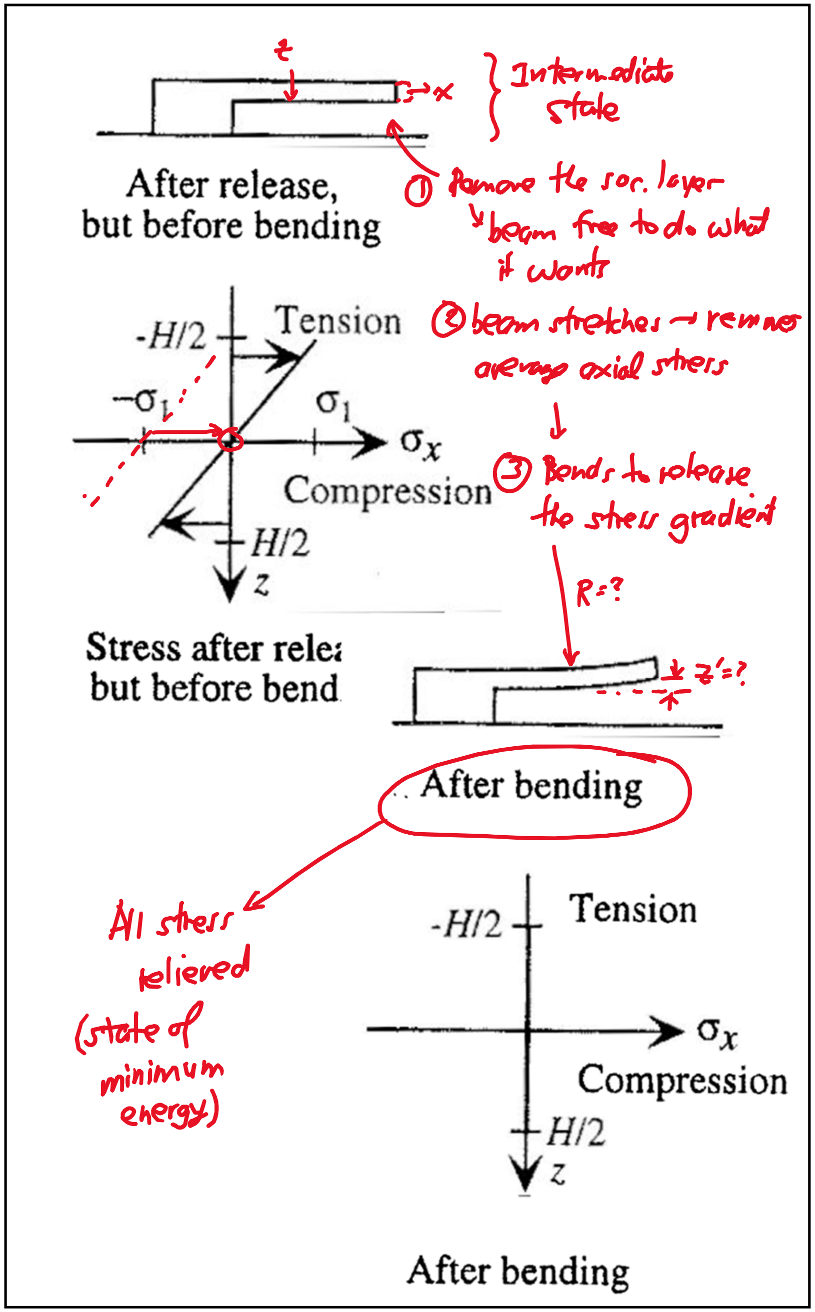
- Announcements:
- HW#4 online, due Tuesday, 3/19, 9 a.m.
- Midterm Exam about 2 weeks away, Thursday, March 21, 11-12:30 p.m., 293 Cory (right here)
- ------------------------------------
- Reading: Senturia, Chpt. 9
- Lecture Topics:
  - Bending of beams
  - Cantilever beam under small deflections
  - Combining cantilevers in series and parallel
  - Folded suspensions
  - Design implications of residual stress and stress gradients
- ------------------------------------
- Last Time:
- Working through stress gradients
- Continue with this

over

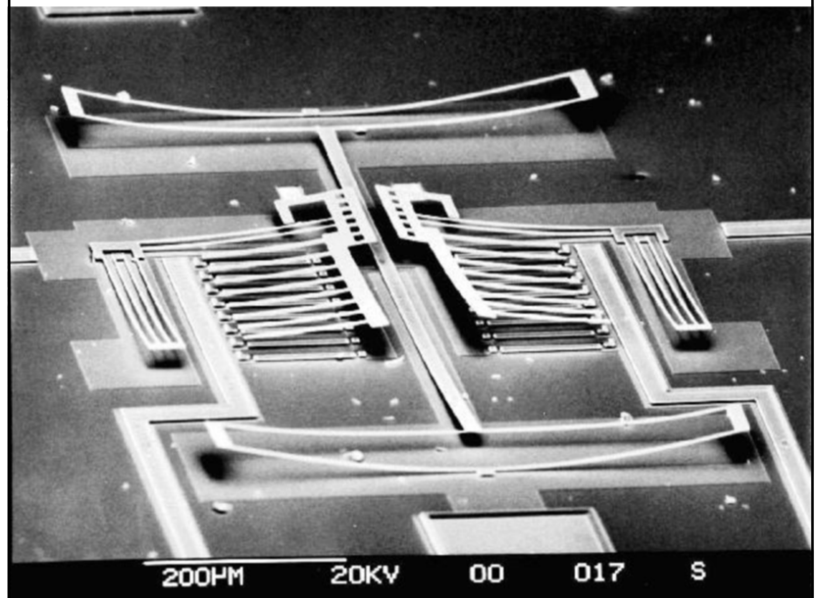
Stress Gradient in a Cantilever

⇒ study beam bending due to a stress gradient





- … and the result:



- Need to quantify this

Bending Due to Stress Gradients

Goal: Find the radius of curvature → z'

Prior to release, axial stress: $\sigma = \sigma_0 - \frac{\sigma_1}{(H/2)} z$

The internal moment:

$$M_x = \int_{-\frac{H}{2}}^{\frac{H}{2}} \left[(W dz)\cdot\sigma\right]\cdot z = \int_{-\frac{H}{2}}^{\frac{H}{2}} \left(z\sigma_0 - \frac{\sigma_1 z^2}{(H/2)}\right) dz$$

$$= W\left(\frac{1}{2}\sigma_0 z^2 - \frac{2\sigma_1 z^3}{3H}\right)\Bigg|_{-\frac{H}{2}}^{\frac{H}{2}}$$

$$= W\left(\cancel{\frac{1}{2}\sigma_0\frac{H^2}{4}} - \frac{2}{3}\sigma_1\frac{H^2}{8} - \cancel{\frac{1}{2}\sigma_0\frac{H^2}{4}} - \frac{2}{3}\sigma_1\frac{H^2}{8}\right)$$

average stress cancels out

$$\boxed{M_x = -\frac{1}{6}\sigma_1 W H^2}$$

Thus, the radius of curvature:

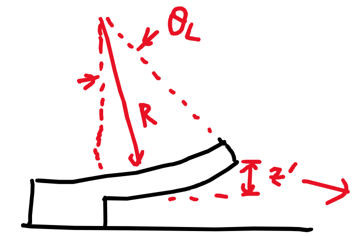
$$\frac{1}{R} = -\frac{M_x}{E'I} \longrightarrow R = -\frac{E'I}{M_x} = \frac{1}{2}\frac{E'H}{\sigma_1}$$

E': Biaxial Modulus $\qquad \left[I = \frac{1}{12}WH^3\right]$

$$\boxed{R = \frac{1}{2}\frac{E}{1-\nu}\frac{H}{\sigma_1}}$$

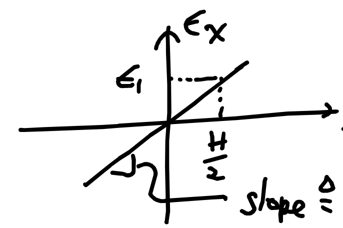
Radius of Curvature for a Cantilever w/ a Stress Gradient

Radius of Curvature → z'



→ integrate over θ_L to get z'
↓
Do in homework...

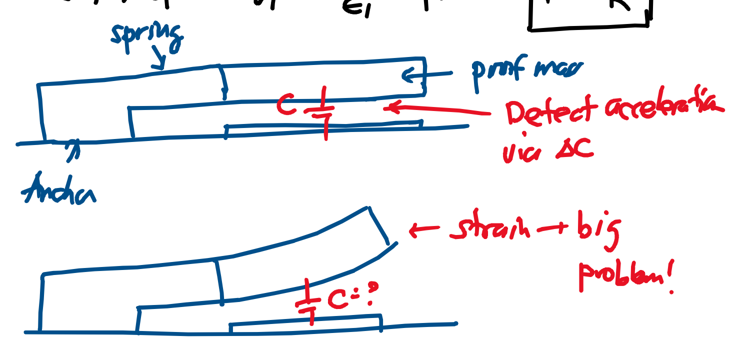
Definition: Strain Gradient



slope ≜ strain gradient = Γ

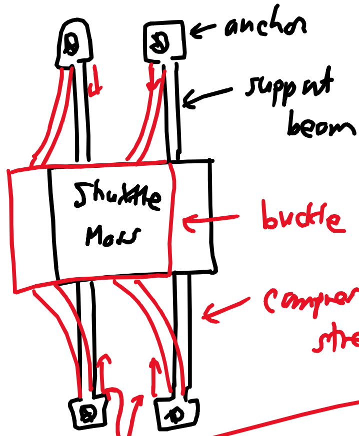
$$\Gamma = \frac{\epsilon_1}{(H/2)}$$

$$R = \frac{1}{2}\frac{E}{1-\nu}\frac{H}{\sigma_1} = \frac{H}{2}\frac{E'}{\sigma_1} = \frac{(H/2)}{\epsilon_1} = \frac{1}{\Gamma} \rightarrow \boxed{\Gamma = \frac{1}{R}}\ ✓$$



Folded Beam Suspension

⇒ Module 8, slide 22:



① Deposit @ high T. → stress free

② Cool to RT → stress!

Folded Beams eliminate these buckling forces. to 1st order!

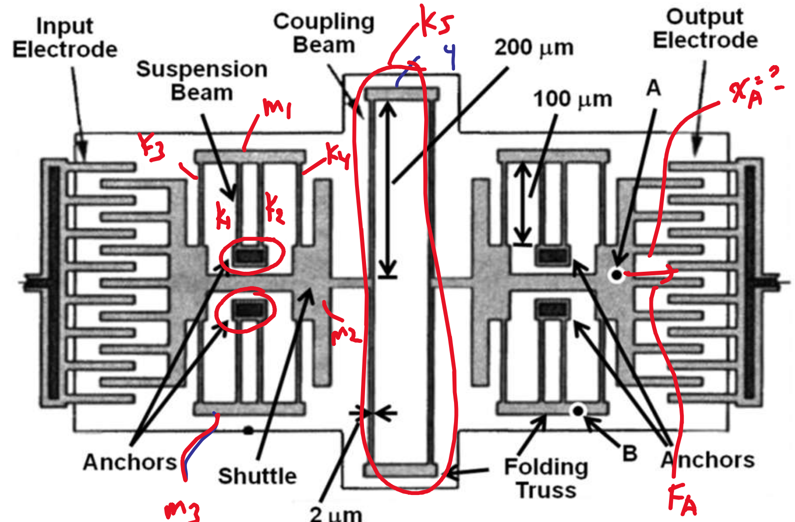
How to Defend Against the Above:

① Δ process parameters for deposition of structural material

↓ never perfect

② Solution: folded-beams!

Analyzing an Interconnected Ensemble of Beams (Springs) & Masses



Typical Questions: → all demand knowledge of $x = f(F)$ → in turn requires that we know stiffness!

① How does the structure move in response to a force at a specific location.

② What is the frequency response to an AC force applied at a specific location.

③ Noise?

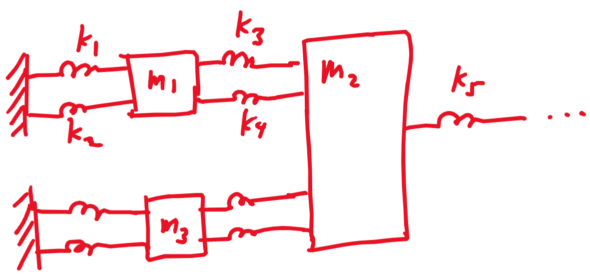
④ Response to environmental stimuli? (e.g., rotation)

⑤ How does stress affect the behavior of the structure?

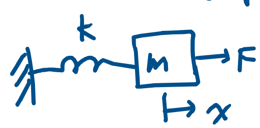
Procedure:

① Build the ckt. (extract the ckt.)
→ in the x-direction (for this example)

Anchor



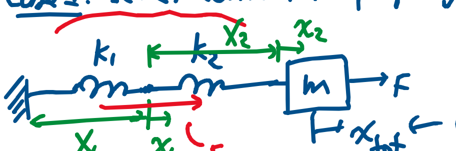
② Analyze to get $x = f(F)$ ← force
↑ displacement



$\Rightarrow F = kx \rightarrow x = \frac{F}{k} = \left(\frac{1}{k}\right)F$
↑ compliance

series because one must go thru both k_1 & k_2 to get from anchor to forcing pt.

(a) Case 1: series connection of springs



← want this ∴ need k_{tot}

F ← thru variable

$x_{tot} = \left(\frac{1}{k_{tot}}\right)F$

original spring length

Δx increase as response to force

$x_{tot} = x_1 + x_2 = F\left(\frac{1}{k_1} + \frac{1}{k_2}\right) = \frac{F}{(k_1 \| k_2)} = \frac{F}{k_{tot}}$

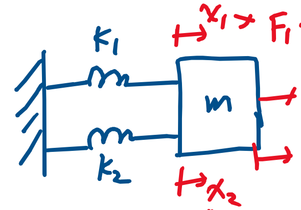
$\left[x_1 = \frac{F}{k_1},\ x_2 = \frac{F}{k_2}\right]$

$\left[\text{"}\|\text{" operator} \triangleq A \| B : \frac{1}{\frac{1}{A}+\frac{1}{B}} = \frac{AB}{A+B}\right]$

$\boxed{k_{tot} = k_1 \| k_2}$ (for k_1 & k_2 in series)

For EE's: $C_1\ C_2 \equiv C_{tot} = C_1 \| C_2$
→ springs combine like capacitors

(b) Case 2: parallel springs ← parallel if can go from anchor to forcing pt. via only one of the springs



$F_1 = k_1 x_{tot}$

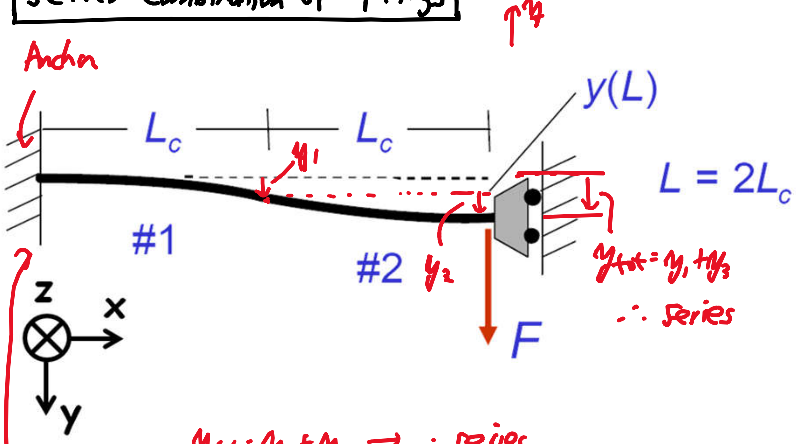
$x_{tot} = x_1 = x_2$ ← another way to tell that springs are in parallel!

$F_2 = k_2 x_{tot}$

$F = F_1 + F_2 = \underbrace{(k_1+k_2)}_{k_{tot}} x_{tot}$

$\boxed{k_{tot} = k_1 + k_2}$ (for k_1 & k_2 in parallel)

Series Combination of Springs



$y_{tot} = y_1 + y_2 \rightarrow$ ∴ series

or must go thru both springs → series to get from anchor to forcing pt.

∴ $\boxed{k_{tot} = k_1 \| k_2}$