Lecture 14: Beam Combos I

- Announcements:
  - HW#4 online, due Tuesday, 3/19, 9 a.m.
  - Midterm Exam about 2 weeks away, Thursday, March 21, 11-12:30 p.m., 293 Cory (right here)

- Reading: Senturia, Chpt. 9

Lecture Topics:
- Bending of beams
- Cantilever beam under small deflections
- Combining cantilevers in series and parallel
- Folded suspensions
- Design implications of residual stress and stress gradients

Last Time:
- Working through stress gradients
- Continue with this

\[ \sigma(x) = \begin{cases} \sigma_0 & \text{compression} \\ \sigma_x & \text{tension} \end{cases} \]

\[ \sigma_{\text{before release}} = \begin{cases} \sigma_0 & \text{long range} \\ \sigma_x & \text{strain range} \end{cases} \]

\[ \sigma_{\text{after release}} = \begin{cases} \sigma_0 & \text{tensile with respect to} \\ \sigma_x & \text{compress in substrate} \end{cases} \]
... and the result:

- Need to quantify this
Bending Due to Stress Gradient

Goal: Find the radius of curvature \( R \)

Prior to release, axial stress: \( \sigma_0 = \sigma_0 \frac{H}{(H/2)} \)

The internal moment:

\[
M_x = \int_{-H/2}^{H/2} (Wdx)z^2 = \int_{-H/2}^{H/2} (z^2 - \frac{\sigma_0 z^3}{(H/2)}) dz
\]

\[
= W \left( \frac{1}{2} \sigma_0 \frac{H^2}{4} - \frac{2}{3} \sigma_0 \frac{H^3}{8} - \frac{1}{2} \sigma_0 \frac{H^2}{4} - \frac{2}{3} \sigma_0 \frac{H^3}{8} \right)
\]

\[
= W \left( \frac{1}{2} \sigma_0 \frac{H^2}{4} - \frac{2}{3} \sigma_0 \frac{H^3}{8} - \frac{1}{2} \sigma_0 \frac{H^2}{4} - \frac{2}{3} \sigma_0 \frac{H^3}{8} \right)
\]

Thus, the radius of curvature:

\[
\frac{1}{R} = -\frac{M_x}{E'I} \rightarrow R = -\frac{E'I}{M_x} = -\frac{E'I}{\frac{1}{2} \sigma_0 \frac{H^2}{4}}
\]

Biaxial Modulus

\[
[ I = \frac{1}{12} Wh^3 ]
\]

\[
R = \frac{E'}{1-\nu} \frac{H}{\sigma_0}
\]

Radius of Curvature

for a Cantilever \( w \) with Stress Gradient

\[
\frac{1}{R} \frac{E'}{1-\nu} \frac{H}{\sigma_0}
\]

Definition: Strain Gradient

\[
\Gamma = \frac{\varepsilon_1}{(H/2)}
\]

\[
R = \frac{1}{2} \frac{E'}{1-\nu} \frac{H}{\sigma_0} = \frac{H}{2} \frac{E'}{\sigma_0} \left( \frac{H/2}{(H/2)} \right) = \frac{1}{\Gamma} \rightarrow \Gamma = \frac{1}{R}
\]

Detect accelerations via AC
deflect

problem!
**Folded Beam Suspension**

- Module 9, slide 24:
  1. Deposit @ high T. Stress free
  2. Cool to RT → stress!

**Analyzing an Interconnected Ensemble of Beams (Springs) + Masses**

- Typical Questions: all demand knowledge of $x = f(t)$
  1. How does the structure move in response to a force at a specific location?
  2. What is the frequency response to an AC force that applied at a specific location?
  3. Noise?
  4. Response to environmental stimuli? (e.g., rotation)
  5. How does stress affect the behavior of the structure?
Procedure:

1. Build the clt. (Extract the clt.)
   - in the x-direction (for this example)

2. Analyze to get $x_1, f(x)$, force - displacement

   $F = kx \Rightarrow x = \frac{F}{k}$

   - Series because one must go compliance thru both $k_1$ and $k_2$, to get from anchor to forcing pt.

   (a) Case 1: series connection of springs forcing pt.

   $x_1 = \frac{1}{k_1} \cdot x$
   $x_2 = \frac{1}{k_2} \cdot x$  \textit{want this: need $k_{tot}$}

   Original spring length as response to force

   $\Rightarrow K_{tot} = \frac{1}{k_{tot}} F$

   - Parallel if can go from anchor to $x_1$ forcing pt. via only one of the springs

   $x_2 = \frac{1}{k_2} \cdot x$

   $F_2 = k_2 \cdot x_{tot}$

   Another way to tell that springs are in parallel:

   $F = F_1 + F_2 = (k_1 + k_2) x_{tot}$

   $F_{tot} = k_{tot} x_{tot}$

   $k_{tot} = k_1 + k_2$  \textit{(for $k_1$ + $k_2$ in parallel)}
Series Combination of Springs

\[ y_{\text{tot}} = y_1 + y_2 \rightarrow \text{series or must go thru both springs} \rightarrow \text{series to get from anchor to forcing pt.} \]

\[ k_{\text{tot}} = k_1 + k_2 \]