Energy Methods

More General Geometries

• Euler-Bernoulli beam theory works well for simple geometries
• But how can we handle more complicated ones?
• Example: tapered cantilever beam
• Objective: Find an expression for displacement as a function of location \( x \) under a point load \( F \) applied at the tip of the free end of a cantilever with tapered width \( W(x) \)

\[
W(x) = W(1 - \frac{x}{2L_c})
\]
Solution: Use Principle of Virtual Work

* In an energy-conserving system (i.e., elastic materials), the energy stored in a body due to the quasi-static (i.e., slow) action of surface and body forces is equal to the work done by these forces ...

* Implication: if we can formulate stored energy as a function of the deformation of a mechanical object, then we can determine how an object responds to a force by determining the shape the object must take in order to minimize the difference $U$ between the stored energy and the work done by the forces:

$$U = \text{Stored Energy} - \text{Work Done}$$

* Key idea: we don't have to reach $U = 0$ to produce a very useful, approximate analytical result for load-deflection