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EE C247B - ME C218 Introduction to MEMS Design Spring 2019

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Lecture Module 10: Resonance Frequency

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Lecture Outline

- Reading: Senturia, Chpt. 10: §10.5, Chpt. 19
- Lecture Topics:
 - ↪ Estimating Resonance Frequency
 - ↪ Lumped Mass-Spring Approximation
 - ↪ ADXL-50 Resonance Frequency
 - ↪ Distributed Mass & Stiffness
 - ↪ Folded-Beam Resonator

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Estimating Resonance Frequency

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Clamped-Clamped Beam μ Resonator

Resonator Beam
 W_r , L_r , h

Electrode
 v_i

Voltage-to-Force Capacitive Transducer
 V_P

Sinusoidal Excitation
 $v_i = V_i \cos[\omega_o t]$

Sinusoidal Forcing Function
 $f_i = F_i \cos[\omega_o t]$

$\frac{i_o}{v_i}$

$Q \sim 10,000$

ω_o , ω

- $\omega \neq \omega_o$: small amplitude
- $\omega = \omega_o$: maximum amplitude \rightarrow beam reaches its maximum potential and kinetic energies

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Folded-Beam Suspension

Comb-Driven Folded Beam Actuator

$$\hat{x}(y) = \frac{F_x}{48EI_z} (3Ly^2 - 2y^3) \quad 0 \leq y \leq L$$

Case: $y=0 \Rightarrow \hat{x}(y=0) = 0 \checkmark$

Case: $y=L \Rightarrow \hat{x}(y=L) = \frac{F_x}{48EI_z} L^3 \rightarrow k = \frac{F_x/L}{\hat{x}} = \frac{12EI_z}{L^3} = \frac{k_c}{2} \checkmark$

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Brute Force Methods for Resonance Frequency Determination

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Basic Concept: Scaling Guitar Strings

Guitar String

Vib. Amplitude vs Freq. (110 Hz)

Vibrating "A" String (110 Hz)

Stiffness

Mass

Freq. Equation:

$$f_o = \frac{1}{2\pi} \sqrt{\frac{k_r}{m_r}}$$

μMechanical Resonator

Metallized Electrode, Anchor, Polysilicon Clamped-Clamped Beam, W_r , L_r , h_r

[Bannon 1996]

Performance:

- $L_r = 40.8 \mu\text{m}$
- $m_r \sim 10^{-13} \text{ kg}$
- $W_r = 8 \mu\text{m}$, $h_r = 2 \mu\text{m}$
- $d = 1000 \text{ \AA}$, $V_p = 5 \text{ V}$
- Press. = 70 mTorr

Transmission [dB] vs Frequency [MHz]

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Anchor Losses

Fixed-Fixed Beam Resonator

Elastic Wave Radiation

Anchor, Electrode, Gap, Anchor

Problem: direct anchoring to the substrate \Rightarrow anchor radiation into the substrate \Rightarrow lower Q

Solution: support at motionless nodal points \Rightarrow isolate resonator from anchors \Rightarrow less energy loss \Rightarrow higher Q

Free-Free Beam Resonator

Supporting Beams, Anchor, Free-Free Beam

$Q = 15,000$ at 92MHz

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92 MHz Free-Free Beam μ Resonator

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- Free-free beam μ mechanical resonator with non-intrusive supports \Rightarrow reduce anchor dissipation \Rightarrow higher Q

Design/Performance:
 $L_f = 13.1 \mu\text{m}$, $W_f = 6 \mu\text{m}$
 $h = 2 \mu\text{m}$, $d = 1000 \text{\AA}$
 $V_p = 28\text{--}76 \text{V}$, $W_e = 2.8 \mu\text{m}$
 $f_o \sim 92.25 \text{MHz}$
 $Q \sim 7,450 @ 10 \text{mTorr}$

[Wang, Yu, Nguyen 1998]

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Higher Order Modes for Higher Freq.

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2nd Mode Free-Free Beam

3rd Mode Free Free Beam

Distinct Mode Shapes

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Flexural-Mode Beam Wave Equation

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$\rho A dx \frac{\partial^2 u}{\partial t^2} = ma$ (inertial action)

$M + \frac{\partial M}{\partial x} dx$

$F + \frac{\partial F}{\partial x} dx$

Free Body Diagram

internal actions (shear force & moments)

- Derive the wave equation for transverse vibration:

Dynamic Equilibrium Condition for forces in the y-direction:

$$F - (F + \frac{\partial F}{\partial x} dx) - \rho A dx \frac{\partial^2 u}{\partial t^2} = 0 \quad (1)$$

and the moment equilibrium condition: $-F dx + \frac{\partial M}{\partial x} dx \approx 0 \quad (2)$

Combining (1) & (2):

$$\frac{\partial^2 M}{\partial x^2} dx = -\rho A dx \frac{\partial^2 u}{\partial t^2} \rightarrow \frac{\partial^2}{\partial x^2} (-EI \frac{\partial^2 u}{\partial x^2}) = -\rho A \frac{\partial^2 u}{\partial t^2} \rightarrow \frac{\partial^2 u}{\partial t^2} = \left(\frac{EI}{\rho A} \right) \frac{\partial^4 u}{\partial x^4}$$

$I_y = \frac{Wh^3}{12}$

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Example: Free-Free Beam

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- Determine the resonance frequency of the beam
- Specify the lumped parameter mechanical equivalent circuit
- Transform to a lumped parameter electrical equivalent circuit
- Start with the flexural-mode beam equation:

$$\frac{\partial^2 u}{\partial t^2} = \left(\frac{EI}{\rho A} \right) \frac{\partial^4 u}{\partial x^4}$$

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Free-Free Beam Frequency

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- Substitute $u = u_1 e^{j\omega t}$ into the wave equation:

$$\frac{\partial^4 u}{\partial x^4} = \left(\omega^2 \frac{\rho A}{EI} \right) u \quad (1)$$
- This is a 4th order differential equation with solution:

$$u(x) = \mathcal{A} \cosh kx + \mathcal{B} \sinh kx + \mathcal{C} \cos kx + \mathcal{D} \sin kx \quad (2)$$

Give the mode shape during resonance vibration.
- Boundary Conditions:

At $x = 0$	At $x = \ell$	
$\frac{\partial^2 u}{\partial x^2} = 0$	$\frac{\partial^2 u}{\partial x^2} = 0$	$M = 0$ (Bending moment)
$\frac{\partial^3 u}{\partial x^3} = 0$	$\frac{\partial^3 u}{\partial x^3} = 0$	$\frac{\partial M}{\partial x} = 0$ (Shearing force)

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Free-Free Beam Frequency (cont)

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- Applying B.C.'s, get $A=C$ and $B=D$, and

$$\begin{bmatrix} (\cosh k\ell - \cos k\ell) & (\sinh k\ell - \sin k\ell) \\ (\sinh k\ell + \sin k\ell) & (\cosh k\ell - \cos k\ell) \end{bmatrix} \begin{bmatrix} \mathcal{A} \\ \mathcal{B} \end{bmatrix} = 0 \quad (3)$$
- Setting the determinant = 0 yields

$$\cos k\ell = \frac{1}{\cosh k\ell}$$
- Which has roots at

$$k_1\ell = 4.730 \quad k_2\ell = 7.853 \quad k_3\ell = 10.996$$

These values of $k_n\ell$ correspond to the different modes of vibration!
- Substituting (2) into (1) finally yields:

$$k^4 = \frac{\rho A}{EI} \omega^2 \rightarrow f_n = \frac{(k_n\ell)^2}{2\pi\ell^2} \sqrt{\frac{EI}{\rho A}} \quad \left[\text{Free-Free Beam Frequency Equation} \right]$$

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Higher Order Free-Free Beam Modes

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Mode	n	Nodal Points	$k_n\ell$	f_n/f_1
Fundamental (f_1)	1	2	4.730	1.000
1st Harmonic	2	3	7.853	2.757
2nd Harmonic	3	4	10.996	5.404
3rd Harmonic	4	5	14.137	8.932
4th Harmonic	5	6	17.279	13.344

← More than 10x increase

Fundamental Mode (n=1)

1st Harmonic (n=2)

2nd Harmonic (n=3)

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Mode Shape Expression

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- The mode shape expression can be obtained by using the fact that $A=C$ and $B=D$ into (2), yielding

$$u_x = \mathcal{B} \left[\left(\frac{\mathcal{A}}{\mathcal{B}} \right) (\cosh kx + \cos kx) + (\sinh kx + \sin kx) \right]$$
- Get the amplitude ratio by expanding (3) [the matrix] and solving, which yields

$$\frac{\mathcal{A}}{\mathcal{B}} = \frac{\sin k\ell - \sinh k\ell}{\cosh k\ell - \cos k\ell}$$
- Then just substitute the roots for each mode to get the expression for mode shape

Fundamental Mode (n=1)
[Substitute $k_1\ell = 4.730$]

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