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EE C247B – ME C218
Introduction to MEMS Design
Spring 2019

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Lecture Module 11: Equivalent Circuits I

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Lecture Outline

- Reading: Senturia, Chpt. 5
- Lecture Topics:
  - Lumped Mass
  - Lumped Stiffness
  - Lumped Damping
  - Lumped Mechanical Equivalent Circuits
  - Electromechanical Analogies

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Lumped Parameter Mechanical Equivalent Circuit

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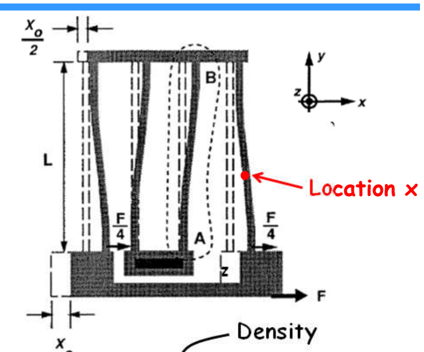
Equivalent Dynamic Mass

- Once the mode shape is known, the lumped parameter equivalent circuit can then be specified
- Determine the equivalent mass at a specific location x using knowledge of kinetic energy and velocity



Maximum Kinetic Energy
Density

$$\text{Equivalent Mass} = M_{eq\,x} = \frac{K.E.}{\frac{1}{2}V_x^2} = \frac{\frac{1}{2}\rho A\int_0^L V^2(x)\,dx}{\frac{1}{2}V_x^2}$$

Maximum Velocity @ location x
Maximum Velocity Function

Equivalent Dynamic Mass

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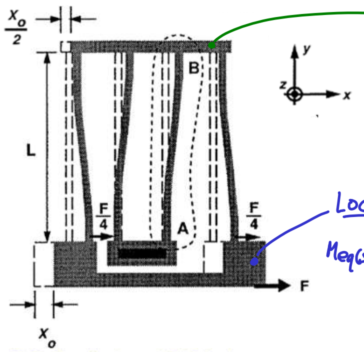
- For the folded-beam structure, we've already determined the maximum kinetic energy
- And in our resonance frequency analysis, we've already determined expressions for velocity



Location on the Truss:

$$M_{eq(truss)} = \frac{KE_{max}}{\frac{1}{2}V_{truss}^2} = \frac{\cancel{\omega_o^2 X_o^2 (\frac{1}{2})}\left[M_s + \frac{1}{4}M_t + \frac{12}{35}M_b\right]}{\cancel{\frac{1}{2}}(\frac{1}{4})\cancel{\omega_o^2 X_o^2}}$$

$$\therefore M_{eq(truss)} = 4\left[M_s + \frac{1}{4}M_t + \frac{12}{35}M_b\right]$$

Location on the Shuttle:

$$M_{eq(shuttle)} = \frac{KE_{max}}{\frac{1}{2}V_{shuttle}^2} = \frac{\cancel{\omega_o^2 X_o^2 (\frac{1}{2})}\left[M_s + \frac{1}{4}M_t + \frac{12}{35}M_b\right]}{\cancel{\frac{1}{2}\omega_o^2 X_o^2}}$$

$$\therefore M_{eq(shuttle)} = M_s + \frac{1}{4}M_t + \frac{12}{35}M_b$$

Equivalent Dynamic Stiffness & Damping

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- Stiffness then follows directly from knowledge of mass and resonance frequency

$$\omega_o = \sqrt{\frac{k_{eq}(x)}{M_{eq}(x)}} \longrightarrow \boxed{k_{eq}(x) = \omega_o^2 M_{eq}(x)}$$

→ large equiv. mass & large stiffness go hand-in-hand

- And damping also follows readily from knowledge of Q or other loss measurands

$$Q = \frac{\omega_o M_{eq}(x)}{\underbrace{c_{eq}(x)}_{\text{damping}}} \longrightarrow \boxed{c_{eq}(x) = \frac{\omega_o M_{eq}(x)}{Q} = \frac{\sqrt{k_{eq}(x) M_{eq}(x)}}{Q}}$$

- With mass, stiffness, and damping ⇨ lumped parameter equivalent circuit

Get Potential Energy & Frequency

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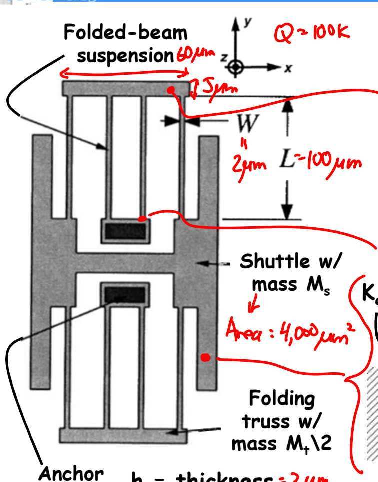
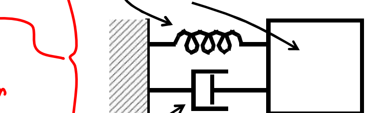


$k_{eq(truss)}$ = 19.2 N/m

$M_{eq(truss)}$ = 8.64x10⁻¹¹ kg

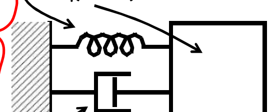


$c_{eq(truss)}$ = 4.08x10⁻¹⁰ kg/s

$k_{eq} = M_{eq} = c_{eq} = \infty$

$k_{eq(shuttle)}$ = 4.8 N/m

$M_{eq(shuttle)}$ = 2.16x10⁻¹¹ kg



$c_{eq(shuttle)}$ = 1.02x10⁻¹⁰ kg/s

h = thickness = 2μm

Electromechanical Analogies

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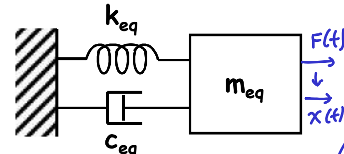
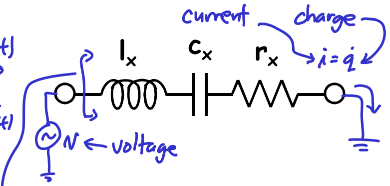




$F(t) = F\cos(\omega t) \longrightarrow x(t) = X\cos\omega t$

Equation of Motion:

$$m_{eq}\ddot{x} + c_{eq}\dot{x} + k_{eq}x = F(t)$$

⇒ using phasor concepts:

$$F = j\omega m_{eq}\dot{x} + \frac{k_{eq}}{j\omega}\dot{x} + c_{eq}\dot{x}$$

$v(t) = V\cos\omega t \longrightarrow i(t) = I\cos\omega t$

Impedance looking in:

$$\frac{v}{i} = j\omega l_x + \frac{1}{j\omega c_x} + r_x$$

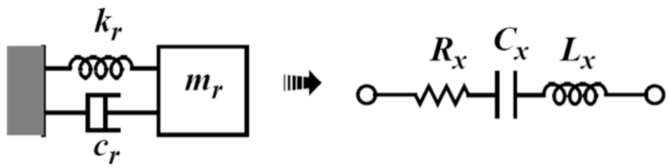
$$v = j\omega l_x i + \frac{(1/c_x)}{j\omega} i + r_x i$$

⇒ by analogy:

$$\boxed{F \to v \quad \dot{x} \to i \quad m_{eq} \to l_x \quad k_{eq} \to \frac{1}{c_x} \quad c_{eq} \to r_x}$$

[Parameter Relationships in the Current Analogy]

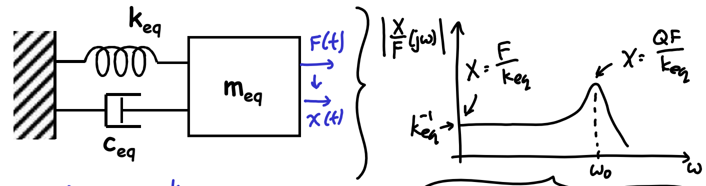
Electromechanical Analogies (cont)



- Mechanical-to-electrical correspondence in the current analogy:

Mechanical Variable	Electrical Variable
Damping, c	Resistance, R
Stiffness^{-1}, k^{-1}	Capacitance, C
Mass, m	Inductance, L
Force, f	Voltage, V
Velocity, v	Current, I

Bandpass Biquad Transfer Function



$F = j\omega m_{eq}\dot{x} + \frac{k_{eq}}{j\omega}\dot{x} + c_{eq}\dot{x}$

$\Rightarrow$ converting to full phasor form:

$F = (j\omega)(j\omega X)m_{eq} + \frac{k_{eq}}{\cancel{j\omega}}(\cancel{j\omega}X) + c_{eq}(j\omega X)$

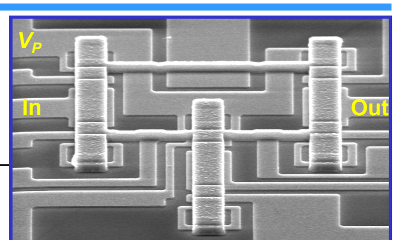
$$\frac{X}{F}(j\omega) = \frac{1}{k_{eq}}\left[-\omega^2\frac{m_{eq}}{k_{eq}} + 1 + j\frac{c_{eq}\omega}{k_{eq}}\right]^{-1} = \frac{1}{k_{eq}}\left[-\left(\frac{\omega}{\omega_o}\right)^2 + 1 + j\frac{\omega}{Q\omega_o}\right]^{-1}$$

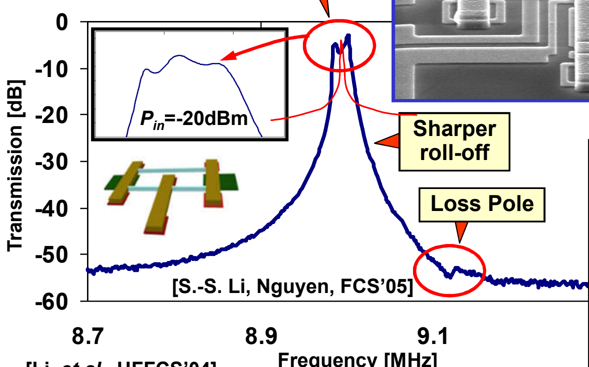
$$\left[\frac{k_{eq}}{m_{eq}} = \omega_o^2,\ Q = \frac{m_{eq}\omega_o}{c_{eq}} = \frac{k_{eq}}{\omega_o c_{eq}} \rightarrow \frac{k_{eq}}{c_{eq}} = Q\omega_o\right]$$

$$\boxed{\frac{X}{F}(j\omega) = \frac{k_{eq}^{-1}}{1 - \left(\frac{\omega}{\omega_o}\right)^2 + j\frac{\omega}{Q\omega_o}}}$$

3CC 3λ/4 Bridged μMechanical Filter

Performance:
f_o=9MHz, BW=20kHz, PBW=0.2%
I.L.=2.79dB, Stop. Rej.=51dB
20dB S.F.=1.95, 40dB S.F.=6.45

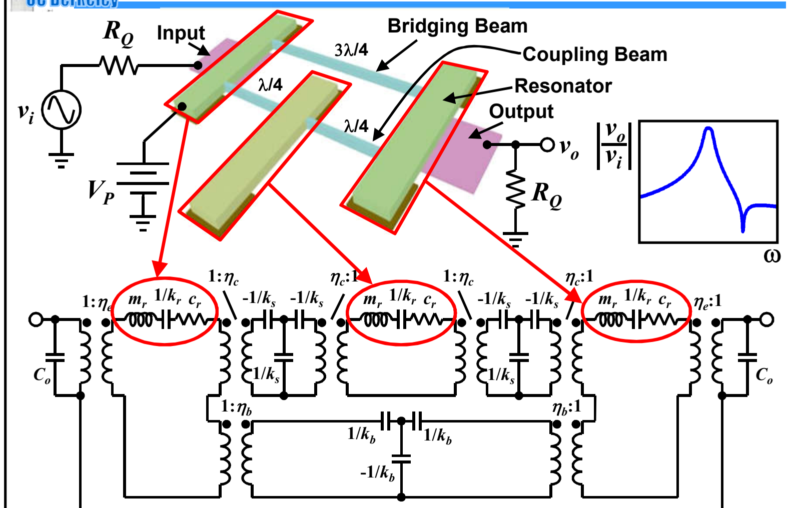




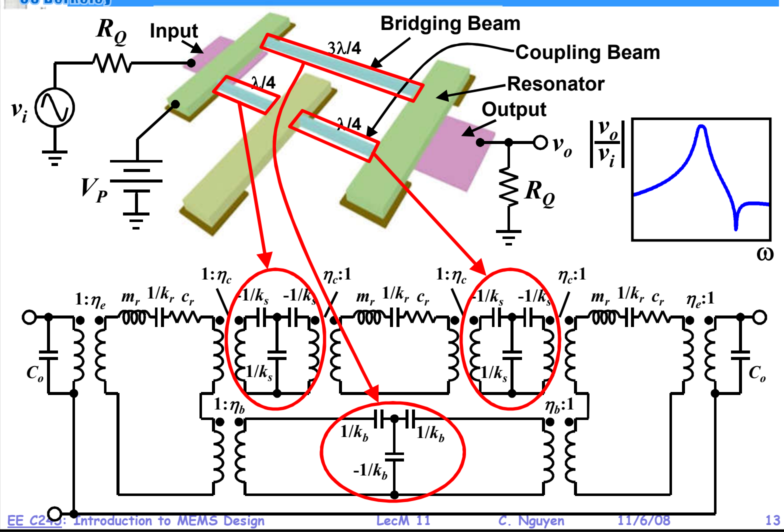
Design:
L_r=40μm
W_r=6.5μm
h_r=2μm
L_c=3.5μm
L_b=1.6μm
V_P=10.47V
P=-5dBm
R_{Qi}=R_{Qo}=12kΩ

[Li, *et al.*, UFFCS'04]

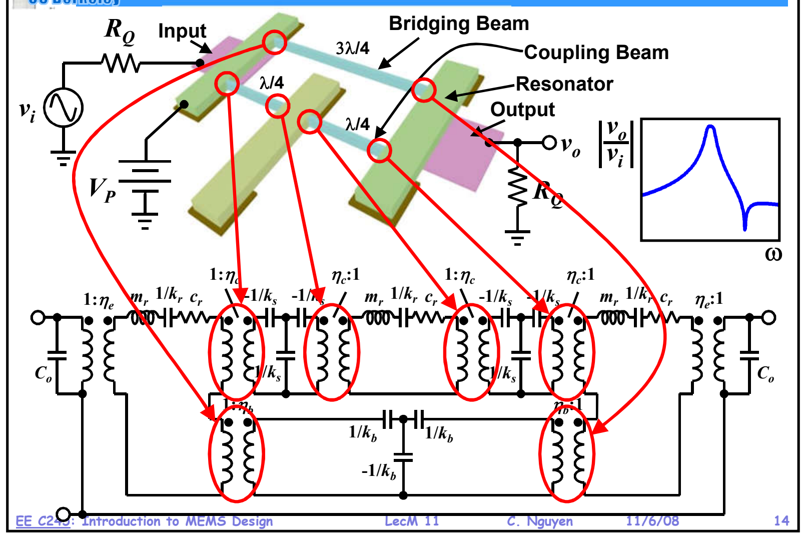
Micromechanical Filter Circuit



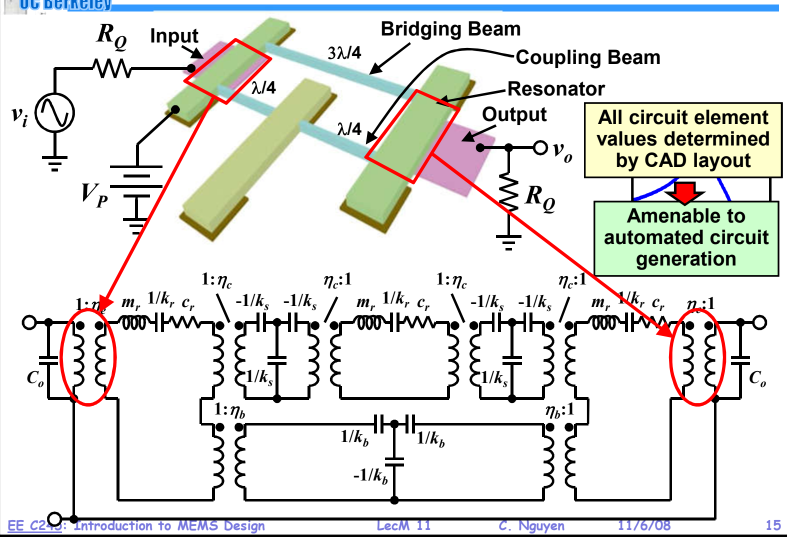
Micromechanical Filter Circuit



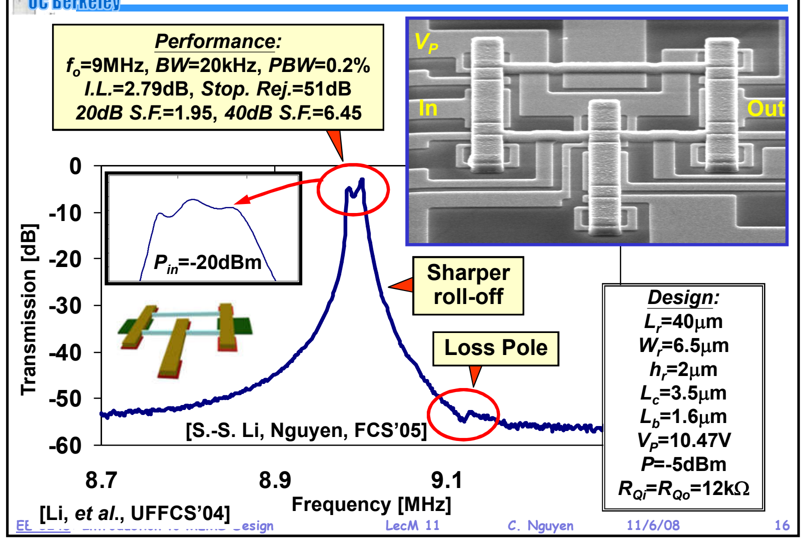
Micromechanical Filter Circuit



Micromechanical Filter Circuit



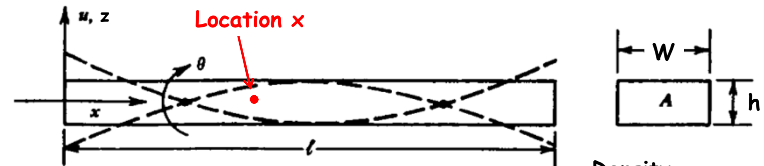
3CC 3λ/4 Bridged μMechanical Filter



Beam Resonator Equivalent Circuits (Pretty Much the Same Stuff)

Equivalent Dynamic Mass

- Once the mode shape is known, the lumped parameter equivalent circuit can then be specified
- Determine the equivalent mass at a specific location x using knowledge of kinetic energy and velocity



Maximum Kinetic Energy

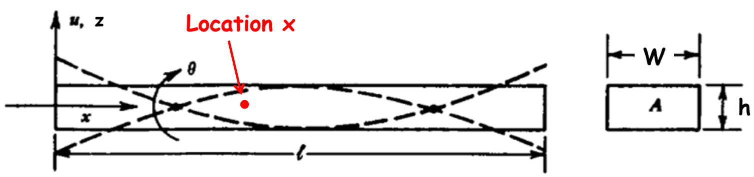
Density

$$\text{Equivalent Mass} = M_{eq\,x} = \frac{K.E.}{\frac{1}{2}V_x^2} = \frac{\frac{1}{2}\rho A\int_0^{\ell} V^2(x)\,dx}{\frac{1}{2}V_x^2}$$

Maximum Velocity @ location x

Maximum Velocity Function

Equivalent Dynamic Mass



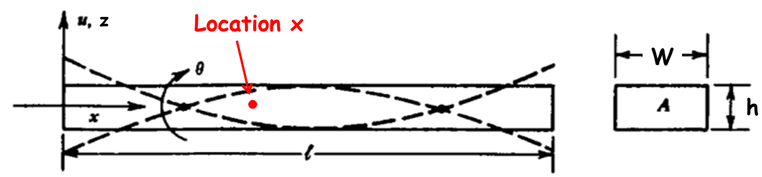
- We know the mode shape, so we can write expressions for displacement and velocity at resonance

Displacement: $u(x) = B\left[\varsigma(\cosh kx + \cos kx) + (\sinh kx + \sin kx)\right]$, $\varsigma = \frac{A}{B}$

$$[V(x) = \omega_0 u(x)] \Rightarrow M_{eq}(x) = \frac{KE_{max}}{\frac{1}{2}[V(x)]^2} = \frac{\cancel{\frac{1}{2}}\rho A\int_0^{\ell}\cancel{\omega_o^2}[u(x')]^2 dx'}{\cancel{\frac{1}{2}}\cancel{\omega_o^2}[u(x)]^2}$$

$$\boxed{M_{eq}(x) = \frac{\rho A\int_0^{\ell}\cancel{B^2}[\varsigma(\cosh kx' + \cos kx') + (\sinh kx' + \sin kx')]^2 dx'}{\cancel{B^2}[\varsigma(\cosh kx + \cos kx) + (\sinh kx + \sin kx)]^2}}$$

Equivalent Dynamic Stiffness & Damping



- Stiffness then follows directly from knowledge of mass and resonance frequency

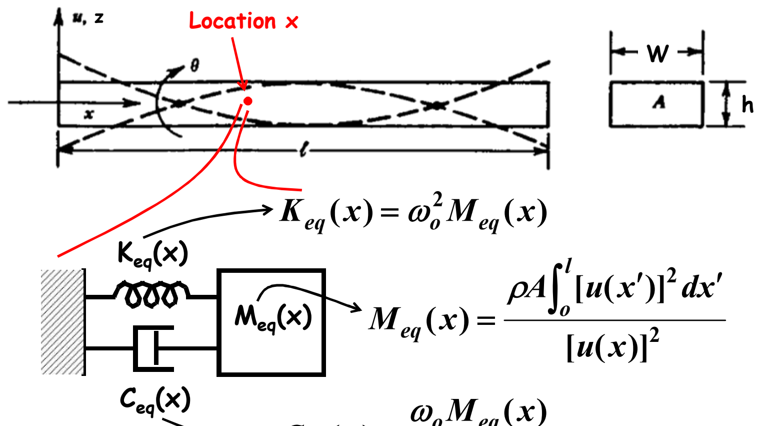
$$\omega_o = \sqrt{\frac{K_{eq}(x)}{M_{eq}(x)}} \longrightarrow \boxed{K_{eq}(x) = \omega_o^2 M_{eq}(x)}$$

- And damping also follows readily

$$Q = \frac{\omega_o M_{eq}(x)}{C_{eq}(x)} \longrightarrow \boxed{C_{eq}(x) = \frac{\omega_o M_{eq}(x)}{Q} = \frac{\sqrt{K_{eq}(x)M_{eq}(x)}}{Q}}$$

damping

Equivalent Lumped Mechanical Circuit



$$K_{eq}(x) = \omega_o^2 M_{eq}(x)$$

$$M_{eq}(x) = \frac{\rho A \int_o^l [u(x')]^2 dx'}{[u(x)]^2}$$

$$C_{eq}(x) = \frac{\omega_o M_{eq}(x)}{Q}$$

Equivalent Lumped Mechanical Circuit

Example: Polysilicon w/ ℓ=14.9μm, W=6μm, h=2μm → 70 MHz