Lecture Outline

- Reading: Senturia, Chpt. 5
- Lecture Topics:
  - Lumped Mass
  - Lumped Stiffness
  - Lumped Damping
  - Lumped Mechanical Equivalent Circuits
  - Electromechanical Analogies

Lumped Parameter Mechanical Equivalent Circuit

Once the mode shape is known, the lumped parameter equivalent circuit can then be specified.

Determine the equivalent mass at a specific location \( x \) using knowledge of kinetic energy and velocity:

\[
M_{eq} = \frac{1}{2} \rho A \int_0^L V^2(x) \, dx
\]

\[
C_1
\]

Maximum Kinetic Energy

Equivalent Mass

Maximum Velocity @ location \( x \)

Maximum Velocity Function
Equivalent Dynamic Mass

* For the folded-beam structure, we've already determined the maximum kinetic energy.
* And in our resonance frequency analysis, we've already determined expressions for velocity.

Equivalent Dynamic Stiffness & Damping

* Stiffness then follows directly from knowledge of mass and resonance frequency.
  \[ \omega_r \sqrt{\frac{K_{eq}(m)}{M_{eq}(m)}} \rightarrow K_{eq}(m) = \omega_r^2 M_{eq}(m) \]  
* And damping also follows readily from knowledge of Q or other loss measurands.
  \[ Q = \frac{\omega_r M_{eq}(m)}{C_{eq}(m)} = \frac{\sqrt{K_{eq}(m) M_{eq}(m)}}{C_{eq}(m)} \]
* With mass, stiffness, and damping \( \Rightarrow \) lumped parameter equivalent circuit.

Get Potential Energy & Frequency

- Folded-beam suspension
- Shuttle w/ mass \( M_s \)
- Folding truss w/ mass \( M_t \)

\[ M_{eq}(truss) = 8.64 \times 10^{-11} \text{ kg} \]
\[ K_{eq}(truss) = 19.2 \text{ N/m} \]
\[ C_{eq}(truss) = 4.08 \times 10^{-10} \text{ kg/s} \]

Electromechanical Analogies

- \( F(t) = F \cos(\omega t) \rightarrow x(t) = X \cos(\omega t) \)
  Equation of Motion:
  \[ m \ddot{x} + c \dot{x} + k x = F(t) \]
  \( \Rightarrow \) using phasor concepts:
  \[ F = j \omega m \dot{x} + k \omega x + c \dot{x} \]
  \( \Rightarrow \) by analogy:
  \[ F \rightarrow N \ m \dot{x} \rightarrow \frac{\dot{E}}{C_x} \ c \rightarrow \frac{1}{C_x} \ r_x \]

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Electromechanical Analogies (cont)

Mechanical to electrical correspondence in the current analogy:

<table>
<thead>
<tr>
<th>Mechanical Variable</th>
<th>Electrical Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Damping, $c$</td>
<td>Resistance, $R$</td>
</tr>
<tr>
<td>Stiffness, $k^{-1}$</td>
<td>Capacitance, $C$</td>
</tr>
<tr>
<td>Mass, $m$</td>
<td>Inductance, $L$</td>
</tr>
<tr>
<td>Force, $f$</td>
<td>Voltage, $V$</td>
</tr>
<tr>
<td>Velocity, $v$</td>
<td>Current, $I$</td>
</tr>
</tbody>
</table>

Bandpass Biquad Transfer Function

\[
\begin{align*}
F(s) &= \frac{k_{eq}}{s^2 + \frac{1}{k_{eq}}s + \frac{1}{k_m} + \frac{1}{k_c} + \frac{1}{k_b}} \\
X_{eq}(s) &= \frac{k_{eq}}{s^2 + \frac{1}{k_{eq}}s + 1} + j\frac{\omega_0}{k_{eq}}X_{eq}(s) \\
V_o(s) &= \frac{s^2 + \frac{1}{k_{eq}}s + 1 + j\frac{\omega_0}{k_{eq}}}{s^2 + \frac{1}{k_{eq}}s + 1} V_i(s) \\
\end{align*}
\]

3CC 3$\lambda$/4 Bridged μMechanical Filter

Performance:
- $f_0 = 9$ MHz, $BW = 20$ kHz, $PBW = 0.2$
- IL = 2.79 dB, Stop. Rej = 51 dB
- 20 dB S.F. = 1.95, 40 dB S.F. = 6.45

Design:
- $L_r = 40 \mu m$
- $W_c = 6.5 \mu m$
- $h_b = 2 \mu m$
- $L_r = 3.5 \mu m$
- $L_o = 1.6 \mu m$
- $V_f = 10.47 V$
- $P_o = 5$ dBm
- $R_o = R_{eq} = 12 k\Omega$

Micromechanical Filter Circuit

[Li, et al., UFFCS’04]
Micromechanical Filter Circuit

Equivalent Circuits I

3CC 3λ/4 Bridged μMechanical Filter

Micromechanical Filter Circuit

Micromechanical Filter Circuit

C. Nguyen

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Beam Resonator Equivalent Circuits
(Pretty Much the Same Stuff)

Equivalent Dynamic Mass

- Once the mode shape is known, the lumped parameter equivalent circuit can then be specified
- Determine the equivalent mass at a specific location \( x \) using knowledge of kinetic energy and velocity

\[
M_{eq}(x) = \frac{K.E.}{\frac{1}{2}V_x^2} = \frac{\frac{1}{2} \rho A \int V^2(x) \, dx}{\frac{1}{2}V_x^2}
\]

Maximum Kinetic Energy

Equivalent Mass = \( M_{eq}(x) \)

Maximum Velocity @ location \( x \)

Maximum Velocity Function

Equivalent Dynamic Stiffness & Damping

- Stiffness then follows directly from knowledge of mass and resonance frequency

\[
\omega_s = \sqrt{\frac{k_{eq}(x)}{M_{eq}(x)}} \rightarrow k_{eq}(x) = \omega_s^2 M_{eq}(x)
\]

- And damping also follows readily

\[
Q = \frac{\omega_s M_{eq}(x)}{C_{eq}(x)} \rightarrow C_{eq}(x) = \frac{\omega_s M_{eq}(x)}{Q}
\]
Equivalent Lumped Mechanical Circuit

\[ K_{eq}(x) = \omega_0^2 M_{eq}(x) \]

\[ M_{eq}(x) = \frac{\rho A}{\omega_0} \int_{A} \left[ u(x') \right]^2 dx' \]

\[ C_{eq}(x) = \frac{\omega_0 M_{eq}(x)}{Q} \]

Example: Polysilicon with \( l=14.9 \mu m, W=6 \mu m, h=2 \mu m \rightarrow 70 \text{ MHz} \)

\[ K_{eq}(0) = 19,927 \text{ N/m} \]
\[ M_{eq}(0) = 1.03 \times 10^{-13} \text{ kg} \]
\[ C_{eq}(0) = 5.66 \times 10^{-9} \text{ kg/s} \]

\[ K_{eq}(l/2) = 53,938 \text{ N/m} \]
\[ M_{eq}(l/2) = 2.78 \times 10^{-13} \text{ kg} \]
\[ C_{eq}(l/2) = 1.53 \times 10^{-8} \text{ kg/s} \]