Lecture Module 13: Equivalent Circuits II

Electromechanical Analogies

Input Modeling

Electromechanical Analogies

Equation of Motion:
\[ m_{eq} \ddot{x} + c_{eq} \dot{x} + k_{eq} x = F(t) \]

\[ \Rightarrow \text{by analogy:} \]
\[ F = N \quad m_{eq} \rightarrow \ell_x \quad c_{eq} \rightarrow r_x \quad k_{eq} \rightarrow \frac{l_x}{c_x} \]

\[ N(t) \cos(\omega t) \rightarrow x(t) \rightarrow \text{current change} \]
\[ I_x \]

\[ \text{Impedance, \; } N = \frac{\ell_x \sin(\omega t)}{j \omega} + r_x \]

\[ \text{Parameter Relationships in the Current Analogy} \]

\[ \text{Change:} \]
\[ m_{eq} \rightarrow \ell_x \quad c_{eq} \rightarrow r_x \quad k_{eq} \rightarrow \frac{l_x}{c_x} \]
**Bandpass Biquad Transfer Function**

\[
F(s) = \frac{k_{eq} + \omega_c^2 s^2 + j \omega_c s}{\omega_c^2 + (\omega_c/Q)s + 1}
\]

\[
X(s) = \frac{1}{k_{eq}} \left( \omega_c^2 s^2 + j \omega_c s + 1 \right)^{-1}
\]

\[
F(s) = \frac{k_{eq} + \omega_c^2 s^2 + j \omega_c s}{\omega_c^2 + (\omega_c/Q)s + 1}
\]

**Force-to-Velocity Relationship**

- The relationship between input voltage \( v_1 \) and force \( F_{dl} \):
  \[
  F_{dl} \approx -V_p \frac{dC}{dx} v_1
  \]

- When displacement \( x \) is the mechanical output variable:
  \[
  X(s) = \frac{1}{k} \left( \omega_c^2 s^2 + j \omega_c s + 1 \right)^{-1}
  \]

- When velocity \( v \) is the mechanical output variable:
  \[
  v(s) = \frac{sX(s)}{F_{dl}(s)} = \frac{1}{k} \left( \omega_c^2 s^2 + j \omega_c s + 1 \right)^{-1}
  \]

**Force-to-Velocity Equiv. Ckt.**

- Combine the previous lumped LCR mechanical equivalent circuit with a circuit modeling the capacitive transducer \( \rightarrow \) circuit model for voltage-to-velocity

**Equiv. Circuit for a Linear Transducer**

- A transducer ...
  \( \square \) converts energy from one domain (e.g., electrical) to another (e.g., mechanical)
  \( \square \) has at least two ports
  \( \square \) is not generally linear, but is virtually linear when operated with small signals (i.e., small displacements)
Equiv. Circuit for a Linear Transducer

For physical consistency, use a transformer equivalent circuit to model the energy conversion from the electrical domain to mechanical domain.

\[ \begin{align*}
\begin{bmatrix} e_2 \\ f_2 \end{bmatrix} &= \begin{bmatrix} \eta & 0 \\ 0 & -1/\eta \end{bmatrix} \begin{bmatrix} e_1 \\ f_1 \end{bmatrix} \\
\end{align*} \]

Describing Matrix