Noise Sources

Thermal Noise

- **Thermal Noise in Electronics:** (Johnson noise, Nyquist noise)
  - Produced as a result of the thermally excited random motion of free $e^-$'s in a conducting medium
  - Path of $e^-$'s randomly oriented due to collisions

- **Thermal Noise in Mechanics:** (Brownian motion noise)
  - Thermal noise is associated with all dissipative processes that couple to the thermal domain
  - Any damping generates thermal noise, including gas damping, internal losses, etc.

- **Properties:**
  - Thermal noise is white (i.e., constant w/ frequency)
  - Proportional to temperature
  - Not associated with current
  - Present in any real physical resistor

Circuit Representation of Thermal Noise

- Thermal Noise can be shown to be represented by a series voltage generator $\frac{V_R}{R}$ or a shunt current generator $\frac{I_R}{R}$

\[
\frac{I_R}{\Delta f} = \frac{4kT}{R} \quad \frac{V_R}{\Delta f} = 4kTR
\]

where $4kT = 1.66 \times 10^{-20} V \cdot C$ and where these are spectral densities.

Note: These are one-sided mean-square spectral densities! To make them 2-sided, must divide by 2.

Noise in Capacitors and Inductors?

- Resistors generate thermal noise
- Capacitors and inductors are noiseless $\rightarrow$ why?

Ladder

- Now, add a resistor:
  - Can oscillate forever
  - Decays to zero

**But this violates the laws of thermodynamics, which require that things be in constant motion at finite temperature**

Need to add a forcing function, like a noise voltage $\frac{V_n}{\Delta f}$ to keep the motion going $\rightarrow$ and this noise source is associated with $R$
Why 4kTR?

- Why is $\overline{v^2} = 4kT\Delta f$ (a heuristic argument)
- The Equipartition Theorem of Statistical Thermodynamics says that there is a mean energy $1/2kT$ associated with each degree of freedom in a given system
- An electronic circuit possesses two degrees of freedom:
  - Current, $i$, and voltage, $v$
  - Thus, we can write:
    $$\frac{1}{2}Li^2 = \frac{1}{2}k_BT , \quad \frac{1}{2}Cv^2 = \frac{1}{2}k_BT$$
- Similar expressions can be written for mechanical systems
  - For example: for displacement, $x$
    $$\frac{1}{2}kx^2 = \frac{1}{2}k_BT$$

Flicker (1/f) Noise

- In general, associated with random trapping & release of carriers from "slow" states
- Time constant associated with this process gives rise to noise signal with energy concentrated at low frequencies
- Often, get a mean-square noise spectral density that looks like this:

Shot Noise

- Associated with direct current flow in diodes and bipolar junction transistors
- Arises from the random nature by which $e^-$'s and $h^+$'s surmount the potential barrier at a pn junction
- The DC current in a forward-biased diode is composed of $h^+$'s from the p-region and $e^-$'s from the n-region that have sufficient energy to overcome the potential barrier at the junction
- Noise process should be proportional to DC current
- Attributes:
  - Related to DC current over a barrier
  - Independent of temperature
  - White (i.e., const. w/ frequency)
  - Noise power $\sim I_D$ & bandwidth

Back to Determining Sensor Resolution
MEMS-Based Tuning Fork Gyroscope

Drive Axis Equivalent Circuit

Gyro Readout Equivalent Circuit (for a single tine)

\[ F_c = m \ddot{\alpha} = m \cdot (2 \dddot{x}_d \times \Omega) \]

\[ \dddot{x}_d = \omega_j x_d \]

\[ \dddot{x}_s = \omega_j x_s \]

\[ \Omega \]

\[ \omega_j \]

\[ \dddot{x}_d = \omega_j x_d \]

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Minimum Detectable Signal (MDS)

- Minimum Detectable Signal (MDS): Input signal level when the signal-to-noise ratio (SNR) is equal to unity

- The sensor scale factor is governed by the sensor type
- The effect of noise is best determined via analysis of the equivalent circuit for the system

Move Noise Sources to a Common Point

- Move noise sources so that all sum at the input to the amplifier circuit (i.e., at the output of the sense element)
- Then, can compare the output of the sensed signal directly to the noise at this node to get the MDS