Lecture 26: Noise II

- Announcements:
- HW#7 online and due Friday, 5/10, at 9 a.m.
- Module 17 is online (on Noise and MDS)
- Project sign-up sheet will be outside my door later
  - Thursday, 5/9, 11 a.m.? ✗
  - Monday, 5/13, 11 a.m.? ←
  - Tuesday, 5/14, 11 a.m.? ✗
- ------------------------------------
- Reading: Senturia Chpt. 14, 15
- Lecture Topics:
  - Ideal Op Amps
  - Op Amp Non-Idealities
  - MEMS-Transistor Integration
    - Mixed
    - MEMS-First
    - MEMS-Last
- Reading: Senturia Chpt. 16
- Lecture Topics:
  - Minimum Detectable Signal
  - Noise
    - Circuit Noise Calculations
    - Noise Sources
    - Equivalent Input-Referred Noise
  - Gyro MDS
    - Equivalent Noise Circuit
    - Example ARW Determination
- ------------------------------------
- Last Time:
- Started with noise … now continue …

Circuit Noise Calculations

Deterministic

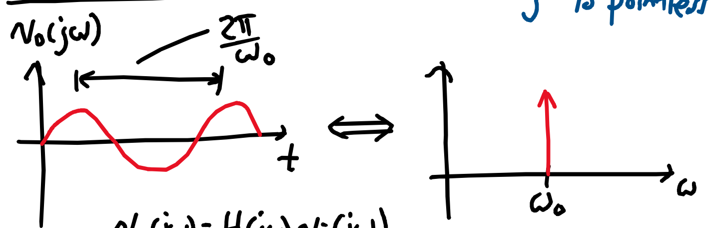
Inputs: $\mathcal{N}_i(j\omega)$, $S_i(\omega)$

Outputs: $\mathcal{N}_o(j\omega)$, $S_o(\omega)$

Random

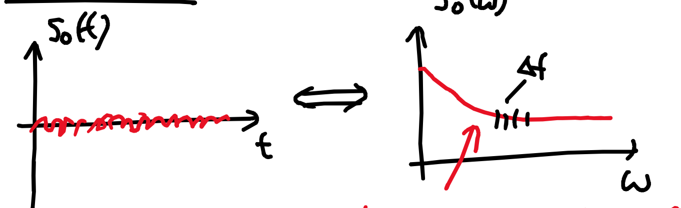
No j → noise has random phase, so "j" is pointless!

Deterministic Signals:



$\mathcal{N}_o(j\omega) = H(j\omega)\mathcal{N}_i(j\omega)$

Random Signals:



Mean-Square Spectral Density

$$S_o(\omega) = [H(j\omega)H^*(j\omega)]\,S_i(\omega) = |H(j\omega)|^2 S_i(\omega)$$

$$\sqrt{S_o(\omega)} = |H(j\omega)|\sqrt{S_i(\omega)}$$

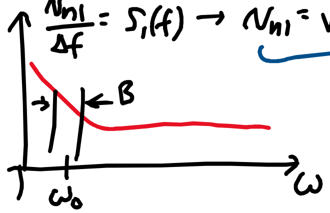
How is it we can do this?

root-mean square amplitudes

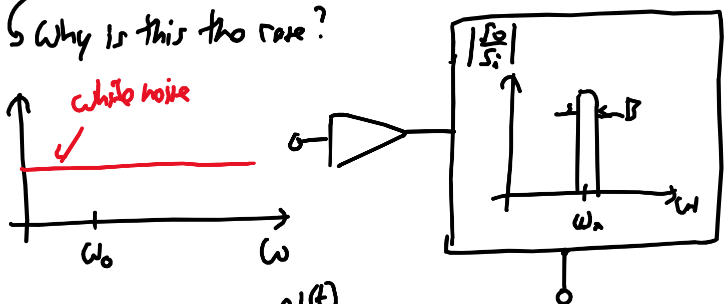
Handling Noise Deterministically

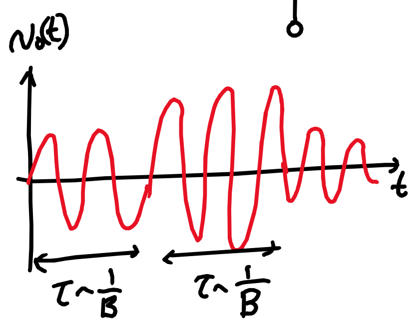
$\frac{\overline{v_{n1}^2}}{\Delta f} = S_i(f) \rightarrow v_{n1} = \sqrt{S_i(f) B}$ ← bandwidth

Can approximate this by a sinusoidal voltage generator (esp. when B is small, say 1Hz)



Why is this the case?

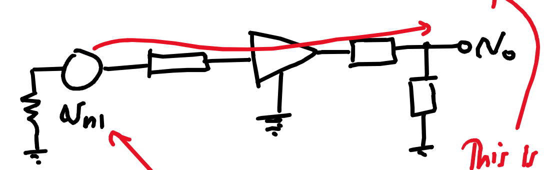




Neither the amplitude nor the phase of a signal can change appreciably within a time period $\frac{1}{B}$!

Noise Source Correlation

Case ①: Single Noise Source

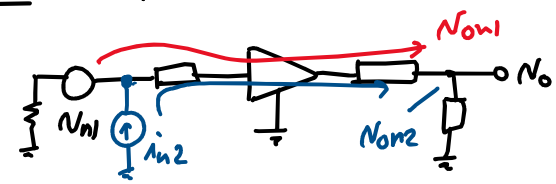


This is correlated w/ this, since it derives from it.

Thus, can write:

$v_{on1} = H(j\omega) v_{n1}$

Case ②: Multiple Noise Sources



Can write: $v_{on1} = H_1(j\omega) v_{n1}$

$v_{on2} = H_2(j\omega) i_{n2}$ ← not correlated

so

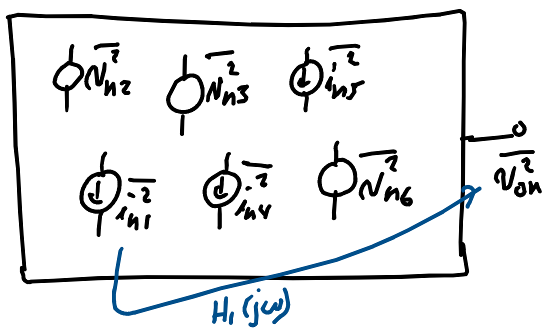
But $v_{oT} \neq v_{on1} + v_{on2}$

Rather $\overline{v_{oT}^2} = \overline{v_{on1}^2} + \overline{v_{on2}^2}$

(must add powers!)

Systematic Noise Calculation Procedure

General Ckt. w/ Several Noise Sources



Assume noise sources are uncorrelated.

1. For $\overline{i_{n1}^2}$, replace w/ a source i_{n1}
2. Calculate $v_{on1}(\omega) = i_{n1}(\omega) H_1(j\omega)$ (treating it like a deterministic signal)
3. Determine $\overline{v_{on1}^2} = \overline{i_{n1}^2} \cdot |H_1(j\omega)|^2$
4. Repeat for each noise source: $\overline{v_{n2}^2}, \overline{v_{n3}^2}, \overline{i_{n4}^2}, \ldots \longrightarrow$ output
   $\overline{v_{on2}^2} = f(\overline{v_{n2}^2}) = \overline{v_{n2}^2} \cdot |H_2(j\omega)|^2$
5. Add noise power (mean-square values)

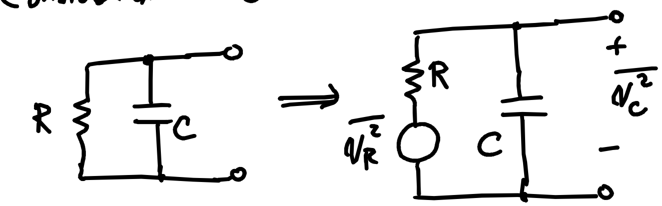
$$\overline{v_{onTOT}^2} = \overline{v_{on1}^2} + \overline{v_{on2}^2} + \overline{v_{on3}^2} + \overline{v_{on4}^2} + \cdots$$

$$\left. v_{onTOT} = \sqrt{\overline{v_{on1}^2} + \overline{v_{on2}^2} + \overline{v_{on3}^2} + \overline{v_{on4}^2} + \cdots} \right\}$$ total rms value

- Go through Module 17, slides 12-16

$\boxed{\text{Why } \frac{\overline{v_R^2}}{\Delta f} = 4kTR \text{ ?}}$ (a heuristic argument)

Consider an RC ckt:



From the Equipartition Thm:

$$E = \frac{1}{2}kT = \frac{1}{2}C\overline{v_c^2}$$

$$\therefore \boxed{\overline{v_c^2} = \frac{kT}{C}}$$ ← integrated noise over all freqs. (total mean-square voltage integrated over all freqs.)

Question: What value of $\frac{\overline{v_R^2}}{\Delta f}$ gives us this (assuming white noise) — $\frac{\overline{v_R^2}}{\Delta f}$ is constant

$$\overline{v_c^2} = \int_0^\infty \left|\frac{1}{1+j\omega RC}\right|^2 \frac{\overline{v_R^2}}{\Delta f} df$$

[noise is white] [$\omega_b = \frac{1}{RC}$]

$$= \frac{1}{2\pi}\frac{\overline{v_R^2}}{\Delta f}\int_0^\infty \frac{\omega_b^2}{\omega_b^2+\omega^2} d\omega$$

*

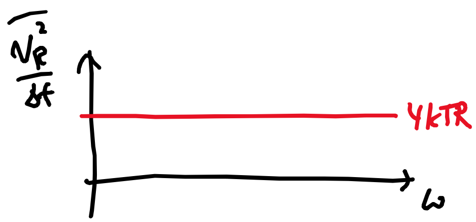
$$\left[\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)\right]$$

$$* \;= \frac{1}{2\pi}\frac{\overline{v_R^2}}{\Delta f}\frac{\omega_b^2}{\omega_b}\tan^{-1}\left(\frac{\omega}{\omega_b}\right)\Big|_0^{\infty}$$

$$= \frac{1}{2\pi}\frac{\overline{v_R^2}}{\Delta f}\left(\frac{\pi}{2}\omega_b - 0\right) = \frac{1}{4}\omega_b\frac{\overline{v_R^2}}{\Delta f} = \frac{kT}{C}$$

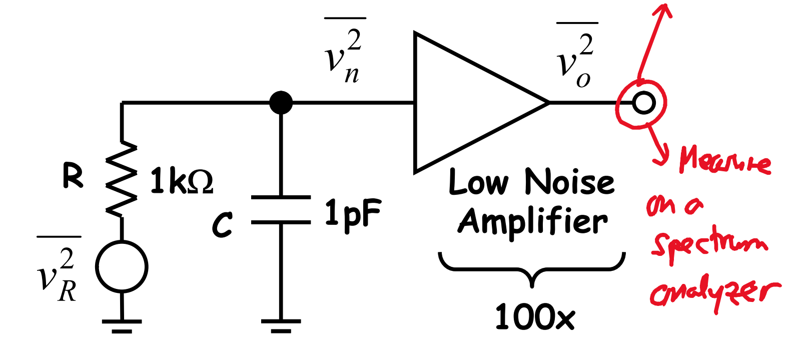
$$\frac{\overline{v_R^2}}{\Delta f} = 4kT\left(\frac{\omega_b}{C}\right) \Rightarrow \boxed{\frac{\overline{v_R^2}}{\Delta f} = 4kTR} \;\checkmark$$

R

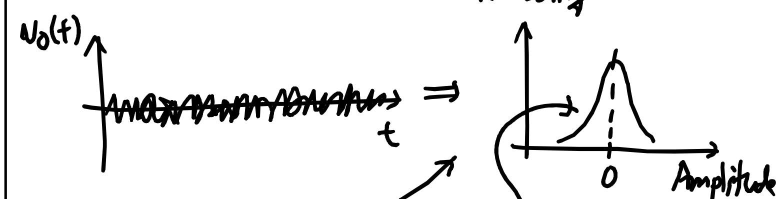


- Go through Module 17, slides 19-20

Example. Typical Noise Numbers



Time Domain:

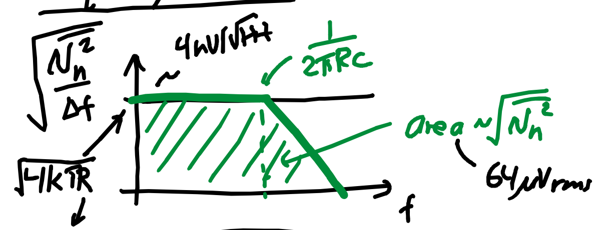


Get Gaussian amplitude distribution

68% within $\pm\sigma$

99.7% within $\pm 3\sigma$

Frequency Domain:



$R = 1k\Omega \rightarrow \sqrt{(1.66\times10^{-20})(1k)} \rightarrow *$

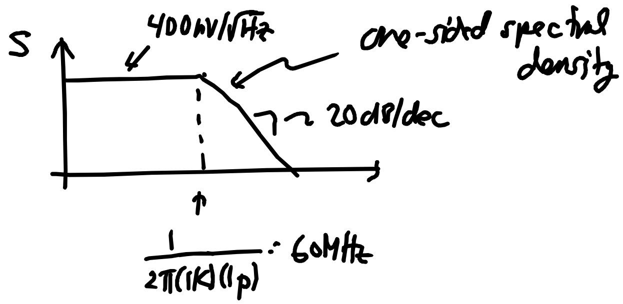
*) $1k\Omega$: $4nV/\sqrt{Hz}$ (for every $1k\Omega$ of R)

1pF : $\sqrt{\frac{kT}{C}} = 64\mu V_{rms}$

Case: AC voltmeter

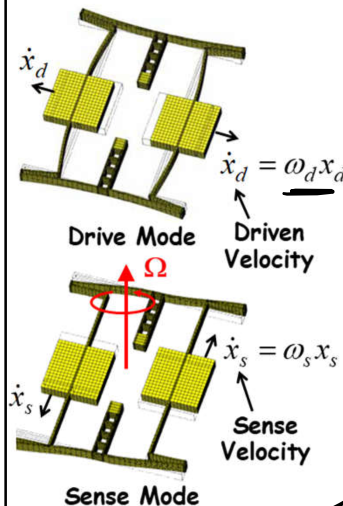
$\sqrt{\overline{v_o^2}} = (100)(64\mu V_{rms}) = 6.4mV_{rms}$

Case: Spectrum Analyzer

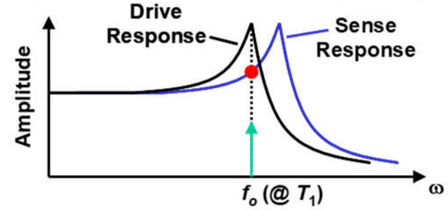


- Go through Module 17, slides 23-29

Gyroscope Drive-to-Sense Xfer Fcn

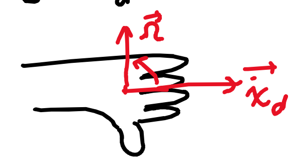


Drive/Sense Response Spectra:



Rotation-Induced Coriolis Force:

$\vec{a}_c = 2\dot{\vec{x}}_d \times \vec{\Omega}$



$\vec{a}_c = 2\omega_d \vec{x}_d \times \vec{\Omega}$ → acts in the sense direction ↓ $\vec{a}_c$

$a_s = a_c = 2\omega_d x_d \Omega \sin 90°$

$\boxed{a_s = 2\omega_d x_d \Omega}$

rotation rate

drive resonance frequency

drive displacement amplitude

$F_s = m_s a_s = m_s a_c$