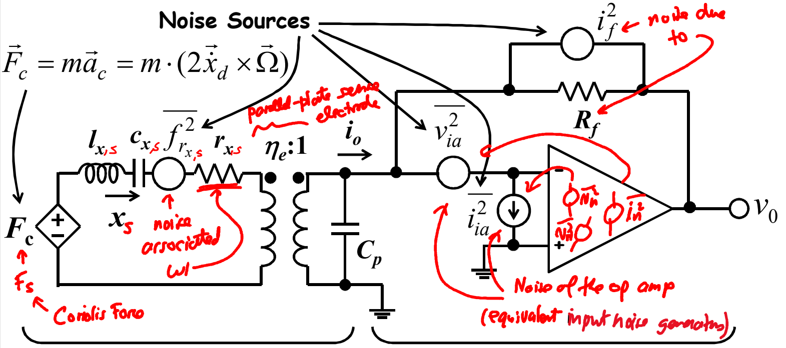
Gyro Readout Equivalent Circuit (for a single tine)



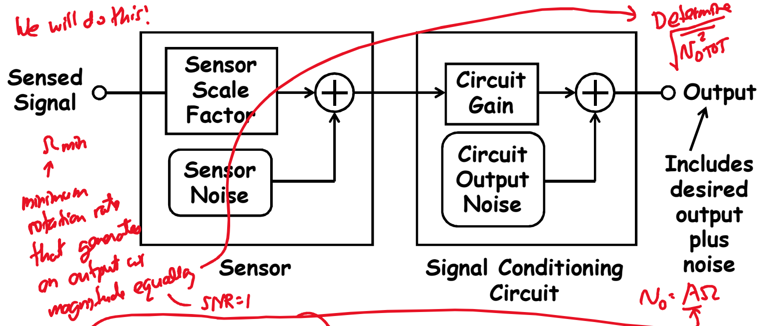
Gyro Sense Element Output Circuit

Signal Conditioning Circuit (Transresistance Amplifier)

- Easiest to analyze if all noise sources are summed at a common node

Minimum Detectable Signal (MDS)

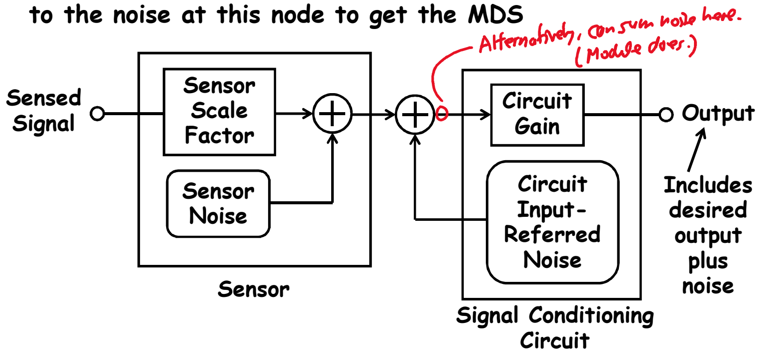
- **Minimum Detectable Signal** (MDS): Input signal level when the signal-to-noise ratio (SNR) is equal to unity



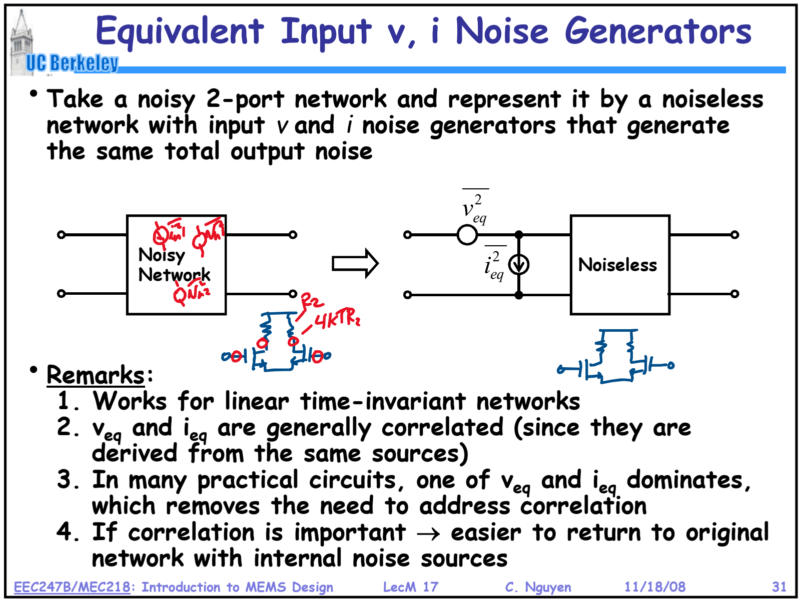
- The sensor scale factor is governed by the sensor type
- The effect of noise is best determined via analysis of the equivalent circuit for the system

Move Noise Sources to a Common Point

- Move noise sources so that all sum at the input to the amplifier circuit (i.e., at the output of the sense element)
- Then, can compare the output of the sensed signal directly to the noise at this node to get the MDS



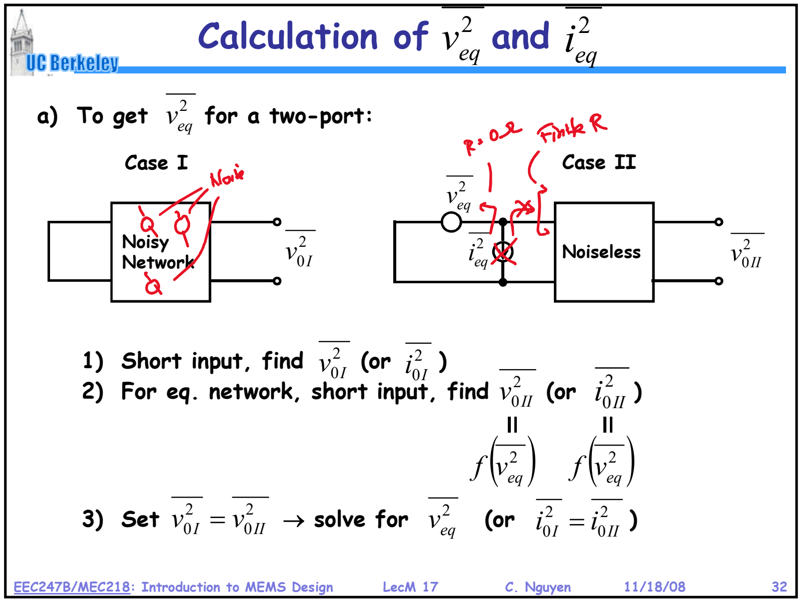
Equivalent Input-Referred Voltage and Current Noise Sources



Equivalent Input v, i Noise Generators

- Take a noisy 2-port network and represent it by a noiseless network with input *v* and *i* noise generators that generate the same total output noise

- Remarks:
  1. Works for linear time-invariant networks
  2. v_{eq} and i_{eq} are generally correlated (since they are derived from the same sources)
  3. In many practical circuits, one of v_{eq} and i_{eq} dominates, which removes the need to address correlation
  4. If correlation is important → easier to return to original network with internal noise sources



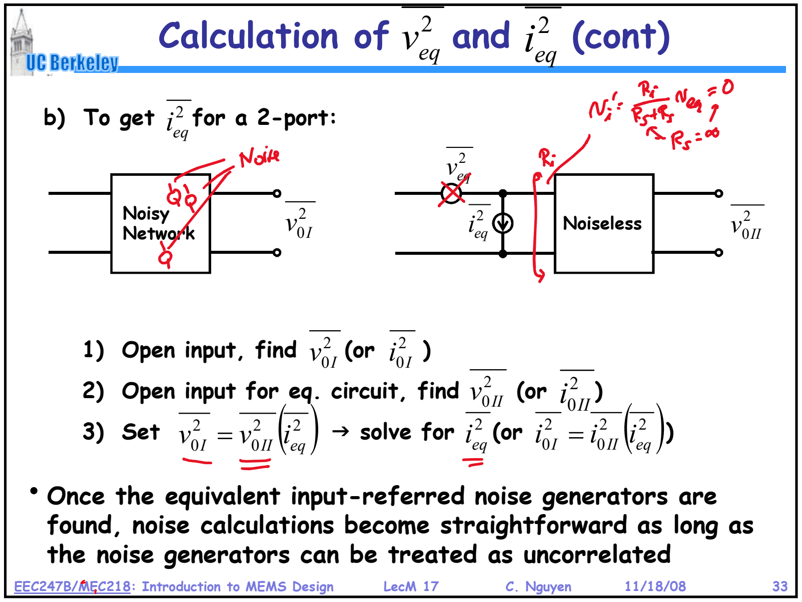
Calculation of $\overline{v_{eq}^2}$ and $\overline{i_{eq}^2}$

a) To get $\overline{v_{eq}^2}$ for a two-port:

1) Short input, find $\overline{v_{0I}^2}$ (or $\overline{i_{0I}^2}$)
2) For eq. network, short input, find $\overline{v_{0II}^2}$ (or $\overline{i_{0II}^2}$)

$$\overline{v_{0II}^2} = f\left(\overline{v_{eq}^2}\right) \qquad \overline{i_{0II}^2} = f\left(\overline{v_{eq}^2}\right)$$

3) Set $\overline{v_{0I}^2} = \overline{v_{0II}^2}$ → solve for $\overline{v_{eq}^2}$ (or $\overline{i_{0I}^2} = \overline{i_{0II}^2}$)

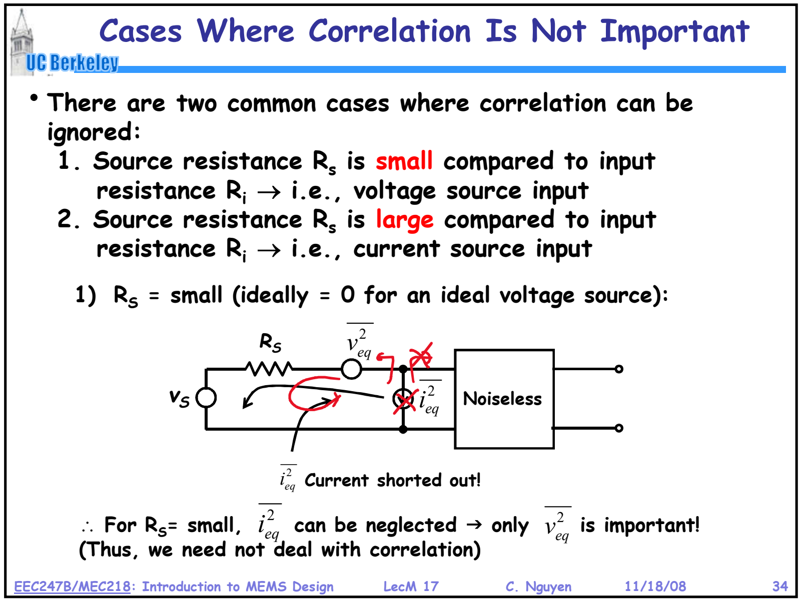


Calculation of $\overline{v_{eq}^2}$ and $\overline{i_{eq}^2}$ (cont)

b) To get $\overline{i_{eq}^2}$ for a 2-port:

1) Open input, find $\overline{v_{0I}^2}$ (or $\overline{i_{0I}^2}$)
2) Open input for eq. circuit, find $\overline{v_{0II}^2}$ (or $\overline{i_{0II}^2}$)
3) Set $\underline{\overline{v_{0I}^2}} = \underline{\underline{\overline{v_{0II}^2}}}\left(\overline{i_{eq}^2}\right)$ → solve for $\underline{\overline{i_{eq}^2}}$ (or $\overline{i_{0I}^2} = \overline{i_{0II}^2}\left(\overline{i_{eq}^2}\right)$)

- Once the equivalent input-referred noise generators are found, noise calculations become straightforward as long as the noise generators can be treated as uncorrelated



Cases Where Correlation Is Not Important

- There are two common cases where correlation can be ignored:
  1. Source resistance R_s is small compared to input resistance R_i → i.e., voltage source input
  2. Source resistance R_s is large compared to input resistance R_i → i.e., current source input

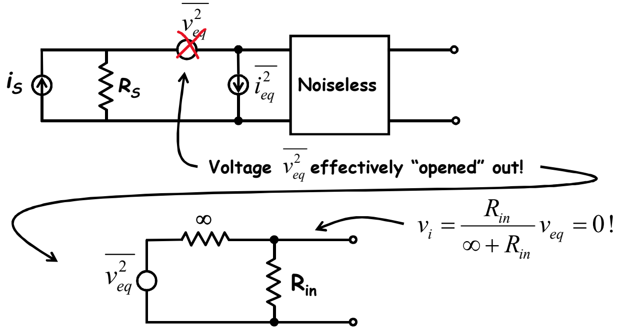
1) R_S = small (ideally = 0 for an ideal voltage source):

$\overline{i_{eq}^2}$ Current shorted out!

∴ For R_S= small, $\overline{i_{eq}^2}$ can be neglected → only $\overline{v_{eq}^2}$ is important!
(Thus, we need not deal with correlation)

Cases Where Correlation Is Not Important

2) R_S = large (Ideally = ∞ for an ideal current source)



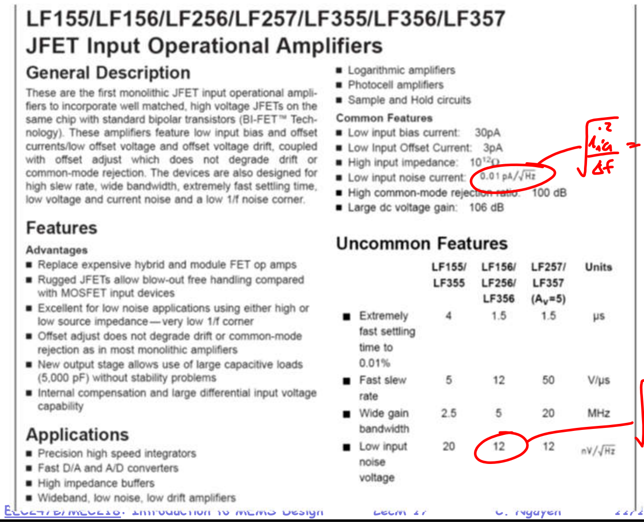
$$v_i = \frac{R_{in}}{\infty + R_{in}} v_{eq} = 0!$$

∴ For R_S= large, $\overline{v_{eq}^2}$ can be neglected!

→ only $\overline{i_{eq}^2}$ is important!

(… and again, we need not deal with correlation)

LF356 Op Amp Data Sheet



LF155/LF156/LF256/LF257/LF355/LF356/LF357

JFET Input Operational Amplifiers

General Description

These are the first monolithic JFET input operational amplifiers to incorporate well matched, high voltage JFETs on the same chip with standard bipolar transistors (BI-FET™ Technology). These amplifiers feature low input bias and offset currents/low offset voltage and offset voltage drift, coupled with offset adjust which does not degrade drift or common-mode rejection. The devices are also designed for high slew rate, wide bandwidth, extremely fast settling time, low voltage and current noise and a low 1/f noise corner.

Features

Advantages

- Replace expensive hybrid and module FET op amps
- Rugged JFETs allow blow-out free handling compared with MOSFET input devices
- Excellent for low noise applications using either high or low source impedance—very low 1/f corner
- Offset adjust does not degrade drift or common-mode rejection as in most monolithic amplifiers
- New output stage allows use of large capacitive loads (5,000 pF) without stability problems
- Internal compensation and large differential input voltage capability

Applications

- Precision high speed integrators
- Fast D/A and A/D converters
- High impedance buffers
- Wideband, low noise, low drift amplifiers
- Logarithmic amplifiers
- Photocell amplifiers
- Sample and Hold circuits

Common Features

- Low input bias current: 30pA
- Low Input Offset Current: 3pA
- High input impedance: $10^{12}\Omega$
- Low input noise current: 0.01 pA/√Hz
- High common-mode rejection ratio: 100 dB
- Large dc voltage gain: 106 dB

Uncommon Features

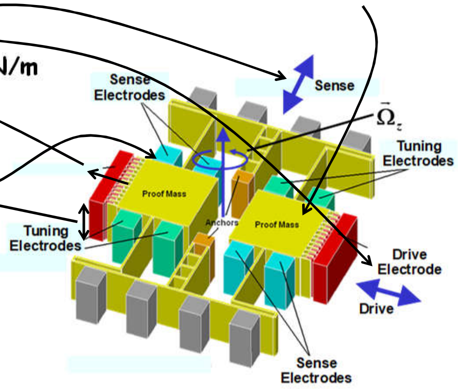
	LF155/ LF355	LF156/ LF256/ LF356	LF257/ LF357 (A_V=5)	Units
Extremely fast settling time to 0.01%	4	1.5	1.5	μs
Fast slew rate	5	12	50	V/μs
Wide gain bandwidth	2.5	5	20	MHz
Low input noise voltage	20	12	12	nV/√Hz

$$\sqrt{\frac{\overline{i_{ia}^2}}{\Delta f}} = 0.01\,pA/\sqrt{Hz}$$

$$\sqrt{\frac{\overline{v_{ia}^2}}{\Delta f}} = 12\,nV/\sqrt{Hz}$$

Example ARW Calculation

- **Example Design**:
  - Sensor Element:
    - m = (100μm)(100μm)(20μm)(2300kg/m³) = 4.6x10-10kg
    - ω_s = 2π(15kHz)
    - ω_d = 2π(10kHz)
    - $k_s = \omega_s^2 m$ = 4.09 N/m
    - x_d = 20 μm
    - Q_s = 50,000
    - V_P = 5V
    - h = 20 μm
    - d = 1 μm
  - Sensing Circuitry:
    - R_f = ~~100kΩ~~ → 1MΩ
    - i_{ia} = 0.01 pA/√Hz
    - v_{ia} = 12 nV/√Hz



Example ARW Calculation (cont)

Get rotation rate to output current scale factor:

$$A = 2\frac{\omega_d}{\omega_s} Q_s x_d \eta_e |\Theta(j\omega_d)| = 2\left(\frac{10k}{15k}\right)(50K)(20\mu)(5)(2000\epsilon_o)(0.000024) = \underline{2.83\times10^{-12}C}$$

$$\left[\Theta(j\omega_d) = \frac{(j\omega_d)(\omega_s/Q_s)}{-\omega_d^2 + \frac{j\omega_d\omega_s}{Q_s} + \omega_s^2} = \frac{j(10k)(15k)/(50k)}{(15k)^2 - (10k)^2 + \frac{j(10k)(15k)}{50K}} = \frac{j(3k)}{1.25\times10^8 + j(3k)}\right.$$

$$\left.\longrightarrow |\Theta(j\omega_d)| = \frac{3k}{\sqrt{(1.25\times10^8)^2 + (3k)^2}} = 0.000024\right]$$

8.854x10⁻⁸F/m

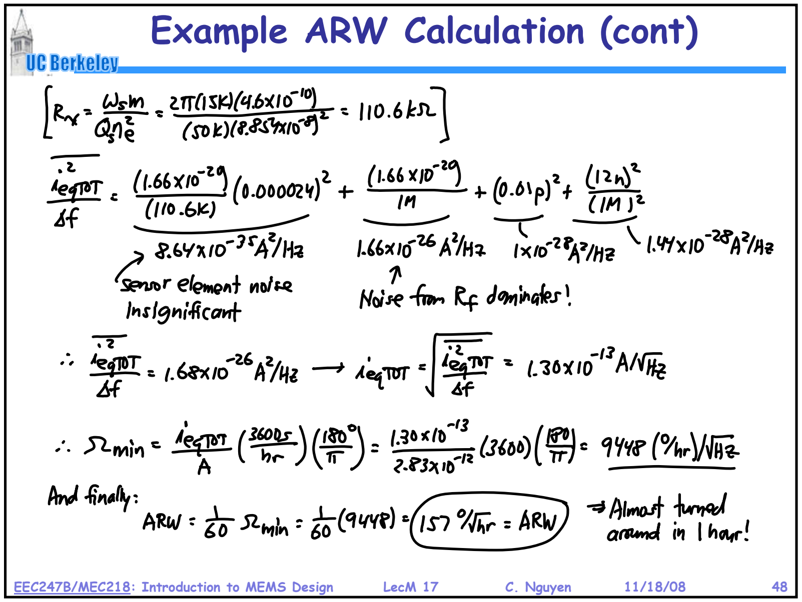
$$\left[\frac{\partial C}{\partial x} = \frac{C_o}{d} = \frac{\epsilon_o h W_p}{d} = \frac{\epsilon_o(20\mu)(100\mu)}{(1\mu)^2} = 2000\epsilon_o \rightarrow \eta_e = V_P\frac{\partial C}{\partial x} = 5(2000\epsilon_o)\right]$$

↑ Assume electrode covers the whole sidewall.

↗ 8.854x10⁻¹²F/m

Then, get noise:

$$\frac{\overline{i_{eqTOT}^2}}{\Delta f} = \frac{4kT}{R_x}|\Theta(j\omega_d)|^2 + \frac{4kT}{R_f} + \frac{\overline{i_{ia}^2}}{\Delta f} + \frac{\overline{v_{ia}^2}}{\Delta f}\left(\frac{1}{R_f^2}\right)$$



Example ARW Calculation (cont)

$$\left[R_x = \frac{\omega_s m}{Q_s \eta_e^2} = \frac{2\pi(15K)(4.6\times10^{-10})}{(50K)(8.854\times10^{-8})^2} = 110.6k\Omega\right]$$

$$\frac{\overline{i^2_{eqTOT}}}{\Delta f} = \underbrace{\frac{(1.66\times10^{-29})}{(110.6K)}(0.000024)^2}_{8.64\times10^{-35}A^2/Hz} + \underbrace{\frac{(1.66\times10^{-29})}{1M}}_{1.66\times10^{-26}A^2/Hz} + \underbrace{(0.01p)^2}_{1\times10^{-28}A^2/Hz} + \underbrace{\frac{(12n)^2}{(1M)^2}}_{1.44\times10^{-28}A^2/Hz}$$

sensor element noise Insignificant

Noise from R_f dominates!

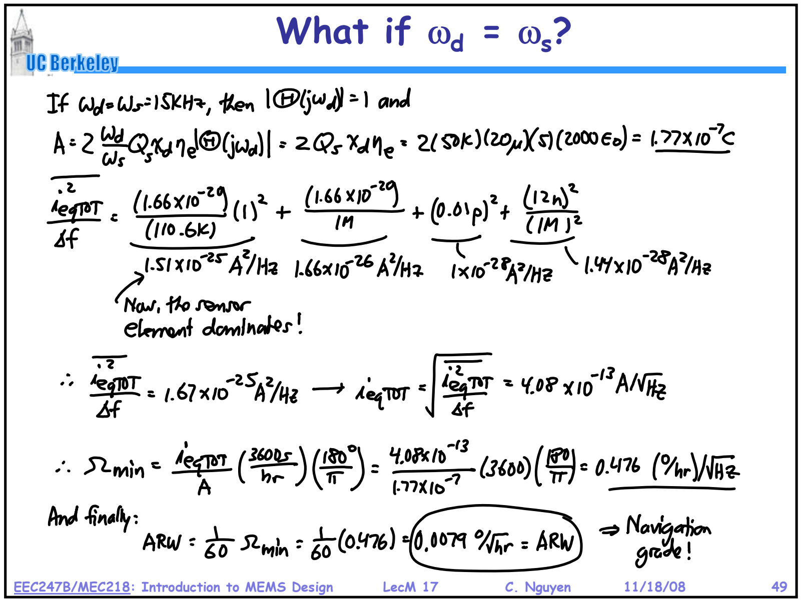
$$\therefore \frac{\overline{i^2_{eqTOT}}}{\Delta f} = 1.68\times10^{-26}A^2/Hz \longrightarrow i_{eqTOT} = \sqrt{\frac{\overline{i^2_{eqTOT}}}{\Delta f}} = 1.30\times10^{-13}A/\sqrt{Hz}$$

$$\therefore \Omega_{min} = \frac{i_{eqTOT}}{A}\left(\frac{3600s}{hr}\right)\left(\frac{180^\circ}{\pi}\right) = \frac{1.30\times10^{-13}}{2.83\times10^{-12}}(3600)\left(\frac{180}{\pi}\right) = \underline{9448\ (^\circ/hr)/\sqrt{Hz}}$$

And finally:

$$ARW = \frac{1}{60}\Omega_{min} = \frac{1}{60}(9448) = \boxed{157\ ^\circ/\sqrt{hr} = ARW}$$

⇒ Almost turned around in 1 hour!



What if $\omega_d = \omega_s$?

If $\omega_d = \omega_s = 15KHz$, then $|\Theta(j\omega_d)| = 1$ and

$$A = 2\frac{\omega_d}{\omega_s}Q_s x_d \eta_e |\Theta(j\omega_d)| = 2Q_s x_d \eta_e = 2(50K)(20\mu)(5)(2000\epsilon_0) = \underline{1.77\times10^{-7}C}$$

$$\frac{\overline{i^2_{eqTOT}}}{\Delta f} = \underbrace{\frac{(1.66\times10^{-29})}{(110.6K)}(1)^2}_{1.51\times10^{-25}A^2/Hz} + \underbrace{\frac{(1.66\times10^{-29})}{1M}}_{1.66\times10^{-26}A^2/Hz} + \underbrace{(0.01p)^2}_{1\times10^{-28}A^2/Hz} + \underbrace{\frac{(12n)^2}{(1M)^2}}_{1.44\times10^{-28}A^2/Hz}$$

Now, the sensor element dominates!

$$\therefore \frac{\overline{i^2_{eqTOT}}}{\Delta f} = 1.67\times10^{-25}A^2/Hz \longrightarrow i_{eqTOT} = \sqrt{\frac{\overline{i^2_{eqTOT}}}{\Delta f}} = 4.08\times10^{-13}A/\sqrt{Hz}$$

$$\therefore \Omega_{min} = \frac{i_{eqTOT}}{A}\left(\frac{3600s}{hr}\right)\left(\frac{180^\circ}{\pi}\right) = \frac{4.08\times10^{-13}}{1.77\times10^{-7}}(3600)\left(\frac{180}{\pi}\right) = \underline{0.476\ (^\circ/hr)/\sqrt{Hz}}$$

And finally:

$$ARW = \frac{1}{60}\Omega_{min} = \frac{1}{60}(0.476) = \boxed{0.0079\ ^\circ/\sqrt{hr} = ARW}$$

⇒ Navigation grade!