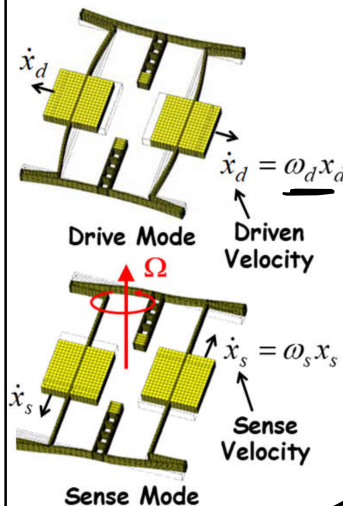
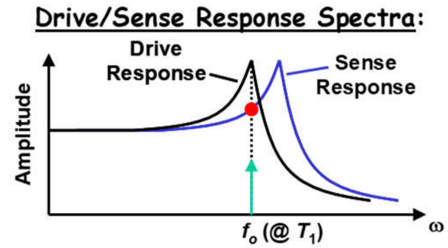
Lecture 27: Gyro Minimum Detectible Signal

- Announcements:
- HW#7 online and due Friday, 5/10, at 9 a.m.
- Module 17 is online (on Noise and MDS)
- Project outbrief scheduling done
  - Everyone has received a calendar invite
- Final Exam Info Sheet online
  - Will go through this in class
  - Also pass out some old Final Exam solutions
- -----------------------------------
- Reading: Senturia Chpt. 16
- Lecture Topics:
  - Minimum Detectable Signal
  - Noise
    - Circuit Noise Calculations
    - Noise Sources
    - Equivalent Input-Referred Noise
  - Gyro MDS
    - Equivalent Noise Circuit
    - Example ARW Determination
- -----------------------------------
- Last Time:
- Did an example noise calculation and started into gyro MDS … now continue with this

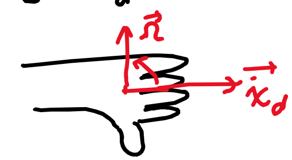
Gyroscope Drive-to-Sense Xfer Fcn





Rotation-Induced Coriolis Force:

$\vec{a}_c = 2\vec{\dot{x}}_d \times \vec{\Omega}$



$\vec{a}_c = 2\omega_d \vec{x}_d \times \vec{\Omega}$ → acts in the sense direction

$a_s = a_c = 2\omega_d x_d \Omega \sin 90°$

$\boxed{a_s = 2\omega_d x_d \Omega}$

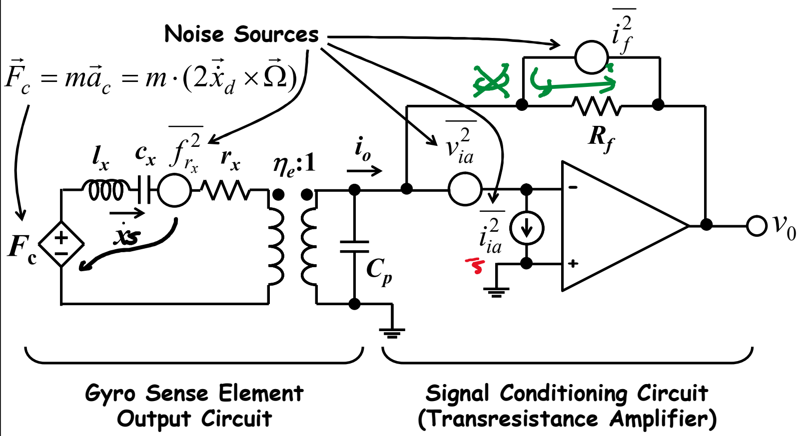
rotation rate

drive motion frequency

drive displacement amplitude

$F_s = M_s a_s = m_s a_c$

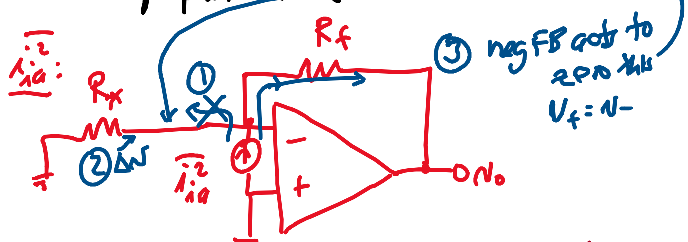
Determine MDS



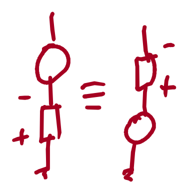
① Determine $\overline{v_o^2}$ (total output noise)

⇒ use superposition ~~one source at a time~~

$\overline{i_{ia}^2}$:

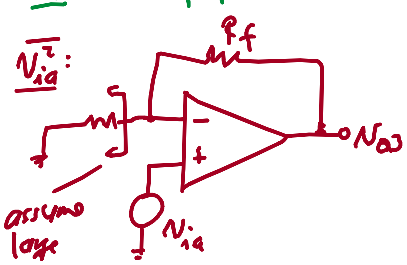


$v_{o1} = i_{ia} R_F \longrightarrow \overline{v_{o1}^2} = \overline{i_{ia}^2} R_f^2$



$\overline{i_f^2}$: $v_{o2} = i_f R_f \longrightarrow \overline{v_{o2}^2} = \overline{i_f^2} R_f^2$

$\overline{v_{ia}^2}$:



$v_{o3} = v_{ia}$
⇓
$\overline{v_{o3}^2} = \overline{v_{ia}^2}$

$\overline{f_{r_x}^2}$: $v_{o4} = f_{r_x}\underbrace{\left(\frac{1}{r_x}\Theta_s(j\omega_d)\right)}_{\dot{x}_s}\eta_e R_f$

$$\overline{v_{o4}^2} = \frac{\overline{f_{r_x}^2}}{r_x^2}\left|\Theta(j\omega_d)\right|^2\eta_e^2 R_f^2$$

$\left[\overline{f_{r_x}^2} = 4kTr_x\Delta f\right] \Rightarrow \; = 4kT\left|\Theta(j\omega_d)\right|^2\eta_e^2\frac{R_f^2}{r_x}$

Bandwidth

Δf

$$\therefore \; \overline{v_o^2} = \overline{i_{ia}^2}R_f^2 + \overline{v_{ia}^2} + 4kTR_f\left(1+\eta_e^2\left|\Theta(j\omega_d)\right|^2\frac{R_f}{r_x}\right)\Delta f$$

get from Data Sheet

Total output mean-square voltage value

② Find v_o in terms of rotation rate Ω:

⇒ find the rotation-to-v_o transfer fcn:

$$\dot{x}_s = F_c\left(\frac{1}{r_x}\Theta_s(j\omega_d)\right) = F_c\left(\frac{\omega_s Q_s}{k_s}\Theta_s(j\omega_d)\right)$$

$\left[\frac{1}{r_x} = \frac{\omega_s Q_s}{k_s}\right]$

$$= \frac{\omega_s Q_s}{k_s}\cdot 2\omega_d x_d \Omega m \cdot \Theta_s(j\omega_d)$$

$\frac{1}{\omega_s^2}$

$\left[F_c = ma_c = 2\omega_d x_d \Omega m\right]$

$$\dot{x}_s = 2\frac{\omega_d}{\omega_s}Q_s x_d \Theta_s(j\omega_d)\cdot\Omega$$

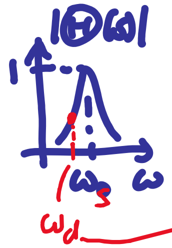
*

$$\Theta(s) = \frac{s(\omega_0/Q)}{s^2 + s(\omega_0/Q) + \omega_0^2}$$

$s=0$: $\Theta(0) = 0$

$s=j\omega_0$: $\Theta(j\omega_0) = 1$

$s=\infty$: $\Theta(\infty) = 0$



*

$i_o = \eta_e \dot{x}_s$

$$v_o = \eta_e \dot{x}_s R_f = \underbrace{2R_f \frac{\omega_d}{\omega_s} Q_s x_d \eta_e \Theta_s(j\omega_d)}_{A = \text{scale factor}} \cdot \underbrace{\Omega}_{\text{rotation rate}}$$

③ $\Omega = \Omega_{min}$ when $|v_o| = \sqrt{\overline{v_o^2}}$ rms noise voltage

minimum detectable rotation rate (MDS)

signal-to-noise ratio = 1 (SNR)

$$v_o = \sqrt{\overline{v_o^2}}$$

$$A \cdot \Omega_{min} = \sqrt{\overline{v_o^2}} \rightarrow \Omega_{min} = \frac{\sqrt{\overline{v_o^2}}}{A}$$

$$\boxed{\Omega_{min} = \frac{\sqrt{\left(\overline{i_{ia}^2} R_f^2 + \overline{v_{ia}^2} + 4kTR_f\left(1 + \eta_e^2 |\Theta_s(j\omega_d)|^2 \frac{R_f}{\nu_x}\right)\right)\Delta f}}{2R_f \frac{\omega_d}{\omega_s} Q_s x_d \eta_e \Theta_s(j\omega_d)}}$$

Often, most interested in spectral density:

$$\boxed{\frac{\Omega_{min}}{\sqrt{\Delta f}} = \Omega_{min} \times \left(\frac{1}{\sqrt{\Delta f}}\right) \times \left(\frac{3600\,s}{hr}\right)\left(\frac{180^\circ}{\pi}\right) \rightarrow \left[(^\circ/hr)/\sqrt{Hz}\right]}$$

$$\boxed{\text{Angle Random Walk} = \text{ARW} = \frac{1}{60}\frac{\Omega_{min}}{\sqrt{\Delta f}} \quad \left[^\circ/\sqrt{hr}\right]}$$

Easier to determine directional error as a function of elapsed time