Lecture 2: Benefits of Scaling I

- Announcements:
- The notes from last time are online in the Lecture link table; video will go up in a day or so
- Modules 1 & 2 are also online (also, in the Lecture link table)
- HW#1 online and due Feb. 7 at 9 a.m.
- As announced last time, I will be traveling next week (at the IEEE MEMS Conference)
  - Next week's lectures will be by recorded video
  - The videos will be online in the Lecture link table in the far right column
  - Please watch the videos before the week after next to avoid falling behind
  - You'll need to watch them, anyway, in order to do the homework
- Get your computer accounts by following the instructions at the end of the Course Info Sheet
- You all have received invites to join the class Piazza group
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- Today:
- Reading: Senturia, Chapter 1
- Lecture Topics:
  - Benefits of Miniaturization
  - Examples
    - GHz micromechanical resonators
    - Chip-scale atomic clock
    - Micro gas chromatograph
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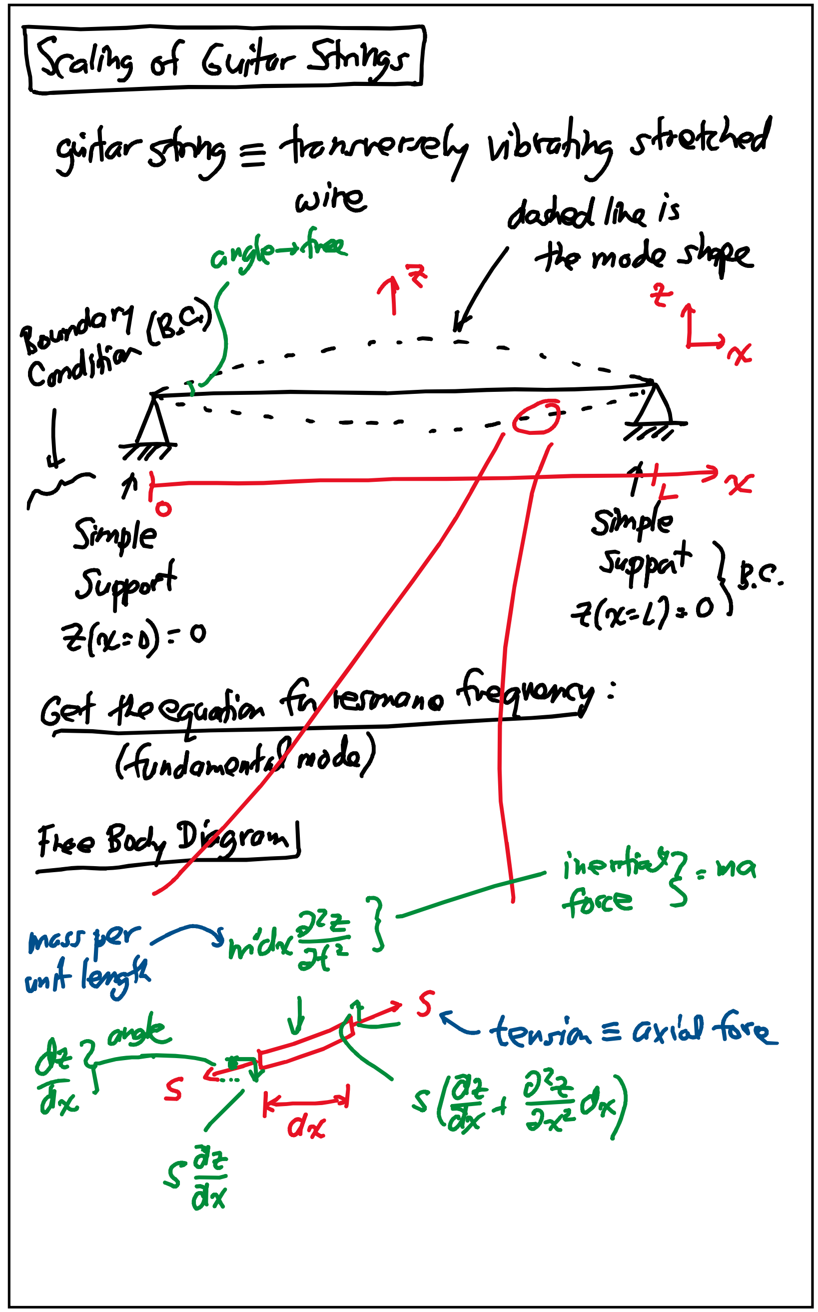
- Last Time: Going through Module 1
- Finish Module 1, then start going through Module 2



⇒ Condition for dynamic equilibrium:

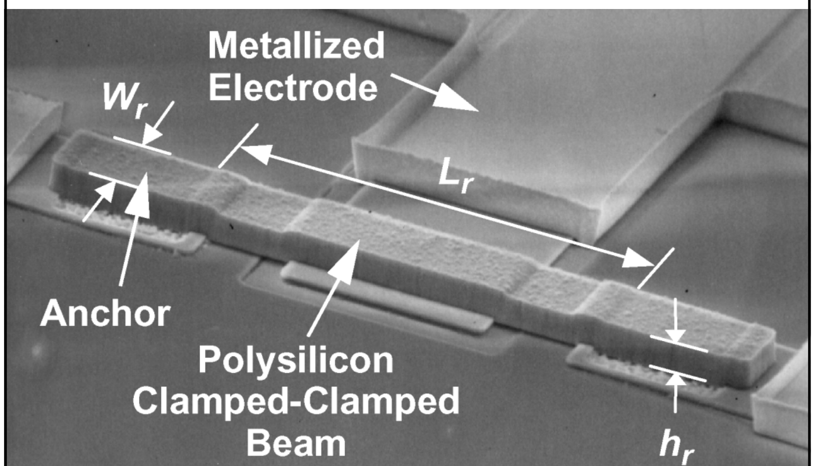
$$S\left(\frac{\partial z}{\partial x}+\frac{\partial^2 z}{\partial x^2}dx\right) - S\frac{\partial z}{\partial x} - m' dx\frac{\partial^2 z}{\partial t^2} = 0$$

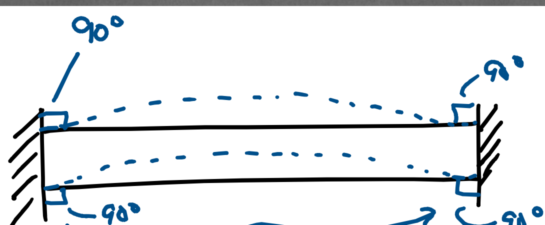
Solve

$$\boxed{f_i = \frac{i}{2L}\sqrt{\frac{S}{m'}}}$$ ← frequency

i: mode: 1, 2, 3, ...

$\boxed{\text{Clamped-Clamped Beam}}$





Clamped (a fixed) Boundary Condition

Eq. for Resonance: $$f_o = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 1.03\sqrt{\frac{E}{\rho}}\frac{h}{L^2} \qquad (1)$$

where $E \triangleq$ Young's modulus [GPa]
$\rho \triangleq$ density [kg/m³]
$h \triangleq$ thickness [m]
$L \triangleq$ length [m]

Example. $L = 40\mu m$, $h = 2\mu m$

polySi → $E = 150\,GPa$, $\rho = 2300\ kg/m^3$

$$\therefore f_o = (1.03)\underbrace{\sqrt{\frac{150G}{2300}}}\frac{2\mu}{(40\mu)^2} \Rightarrow \boxed{f_o = 10.4\,MHz}$$

$\sqrt{\frac{E}{\rho}}$ ≡ acoustic velocity

Scaling:

2x, ½x ↓

① Scale all dimensions equally by a factor S

$$f_o = \frac{S}{S^2} = \frac{1}{S}$$

② If scale L only: $f_o \sim \frac{1}{S^2}$ → even faster rise in f_o!
(but... problems)

Example.

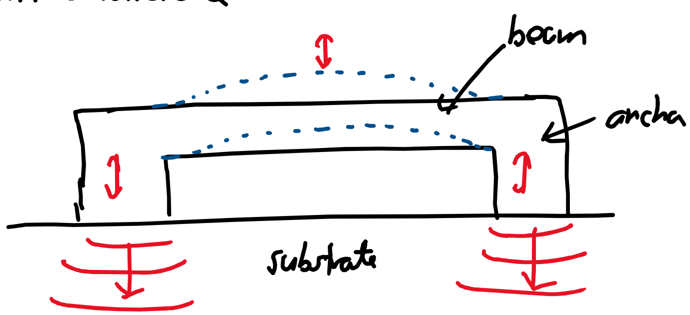
$L = 4\mu m \longrightarrow f_o = (1.03)(8076)\frac{3\mu}{(4\mu)^2}$ (m/s)

$f_o = 1.04\,GHz$

ignore width effects → actually frequency will be different

~ 300 MHz ⇒ but this an okay approximation ...

- Remarks:
- Eq. (1) not accurate when L ≈ h
- Anchor loss when L ≈ h ! → beam becomes too stiff → lowers Q



energy radiates into the substrate! } anchor loss!

$$Q = \frac{\text{Energy per Cycle}}{\text{Energy Lost per Cycle}}$$ ↓ ↗

Example of a con of scaling this way!

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Solution: nanodimensions.

↳ this means: h ~ 300 nm, L ~ 1μm

↳ Problem: size ↓ → power handling ↓

↳ Solution: array!

⇒ use many & add up responses!

④ Better Solution: other geometries

Free-Free Beam: (as opposed to clamped-clamped)