## Lecture 3-4: Benefits of Scaling II

- Announcements:
- As announced last time, I am on travel right now
- This is a pre-recorded video
- The notes from last time are online, as well as the video – both in the Lecture link table
- Modules 1 & 2 are online (also, in the Lecture link table)
- Get your computer accounts by following the instructions at the end of the Course Info Sheet
- HW#1 is online and due Thursday, Feb. 7, at 9 a.m. via Gradescope, which Kyle will set up for you
- ------------------------------------
- Today:
- Reading: Senturia, Chapter 1
- Lecture Topics:
  - Benefits of Miniaturization
  - Examples
    - —GHz micromechanical resonators
    - —Chip-scale atomic clock
    - —Micro gas chromatograph
- ------------------------------------
- Last Time:
- Going through Module 2, looking at how scaling vibrating RF MEMS provides both benefits and problems that one must circumvent
- Continue with this now

Scalings:

2x, ½x

① Scale all dimensions equally by a factor S

$f_o \sim \frac{S}{S^2} = \frac{1}{S}$

② If scale L only: $f_o = \frac{1}{S^2}$ → even faster rise in freq.!

(... but problems ...)

Example.

$L = 4\mu m \rightarrow f_o = (1.03)(8076)\frac{2\mu}{(4\mu)^2} = 1.04\ GHz$

ignore width effect → really need ~3μm

questionable thing to do

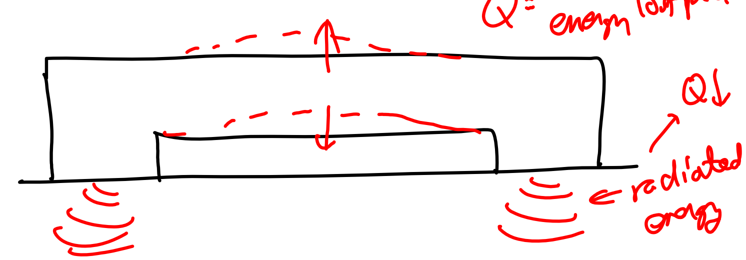
Remarks.

① Eq.(1) not accurate when L ≃ W ≃ h

② When L ≃ h (or when it isn't more than 10×h)

↳ get anchor loss problems

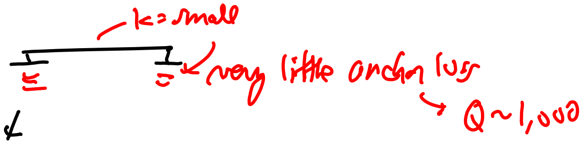
$Q = \frac{\text{energy per cycle}}{\text{energy lost per cycle}}$



↳ Soln:

③ Solution: use nanodimensions! ✓

↳ ex.: h=300nm, L~1μm

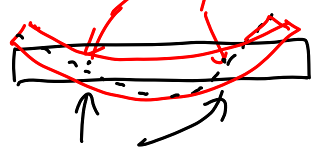


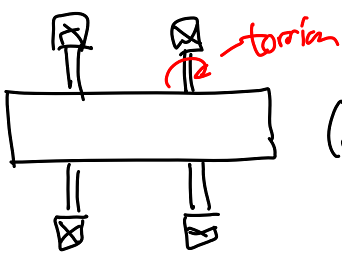

Problem: power handling ↓ when size ↓

↳ Soln: use massive numbers in arrays!

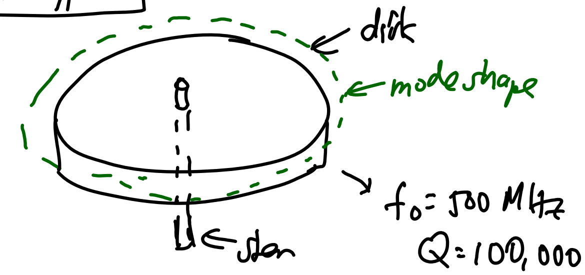
④ Better Soln: use other geometries

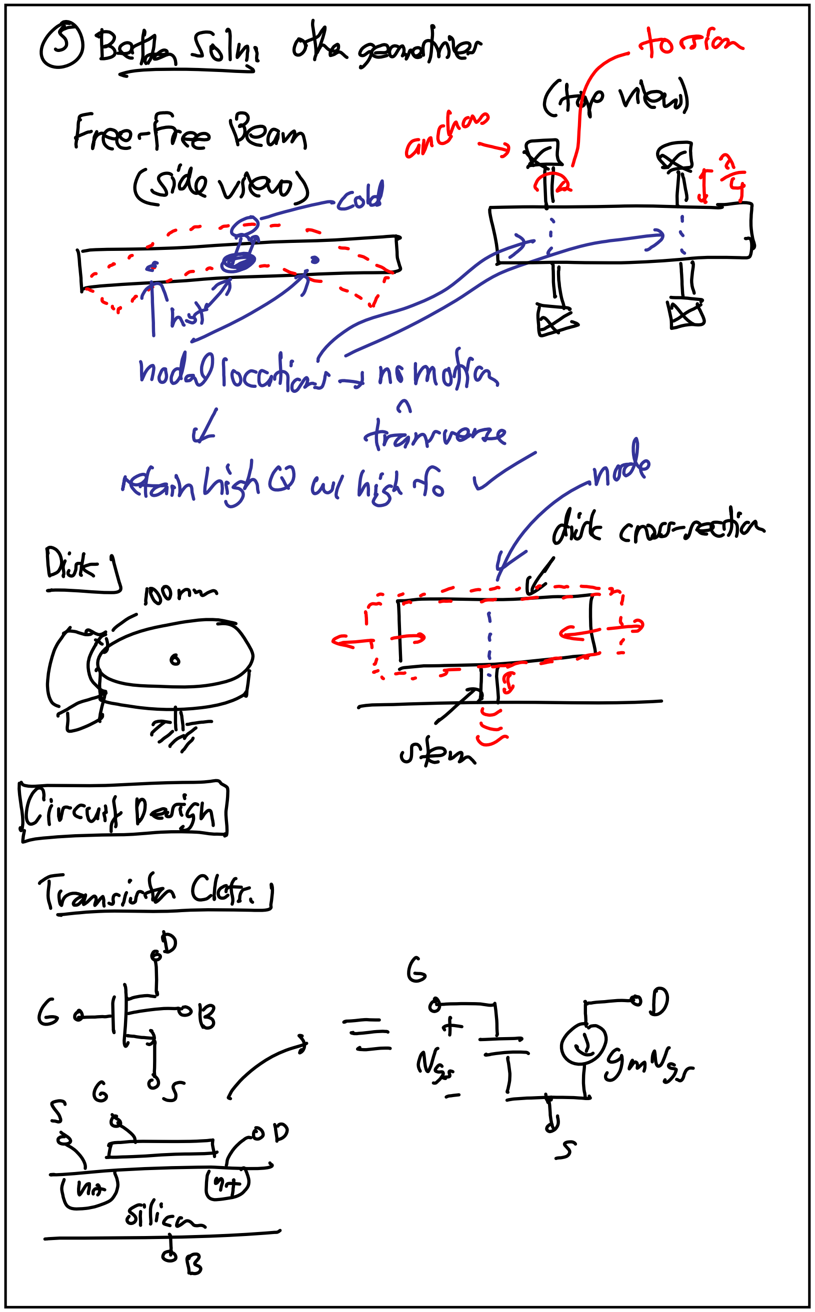
Free-Free Beam:

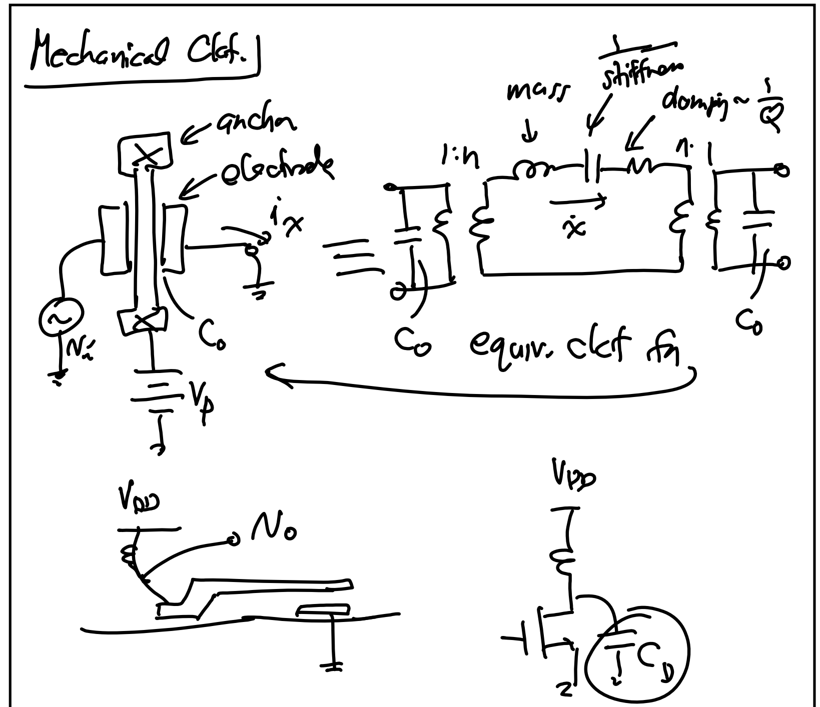



Even Better Approach



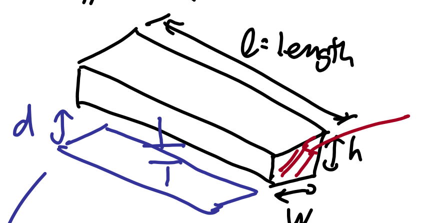



- Now go through Module 2, slides 9-30

## Review Electrical Resistance First

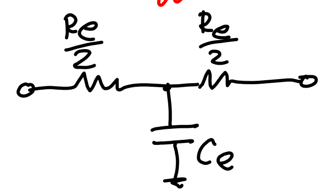
(then attack the thermal R analogy)



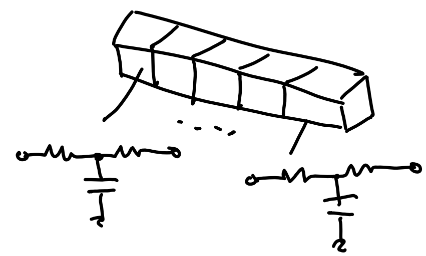

$R_e \triangleq$ electrical resistance $= \frac{l}{\sigma A}$ ← $\sigma$: electrical conductivity

$C_e$ = capacitance $= \frac{\epsilon_o W L}{d}$ ← $\epsilon_o$: permittivity

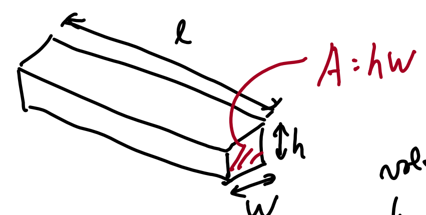
↳ Stored Energy (charge energy): $\frac{1}{2}CV^2 = E$ ← V: voltage across the capacitor




⇒ if want to be more exact:



## Thermal Ckt.




⇒ thermal capacitance: $C_{th} = \rho V C_p$

- $\rho$: density
- $V$: volume
- $C_p$: specific heat → stores thermal energy

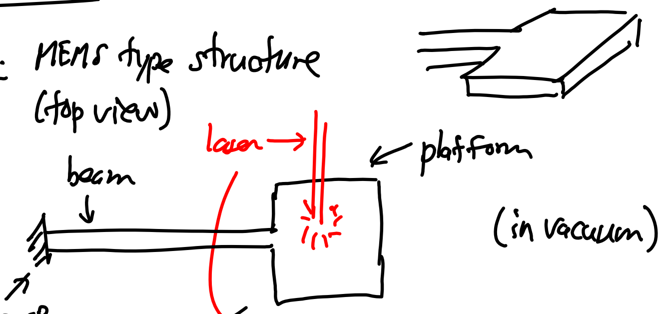
⇒ thermal resistance:

$R_{th} = \frac{l}{kA}$

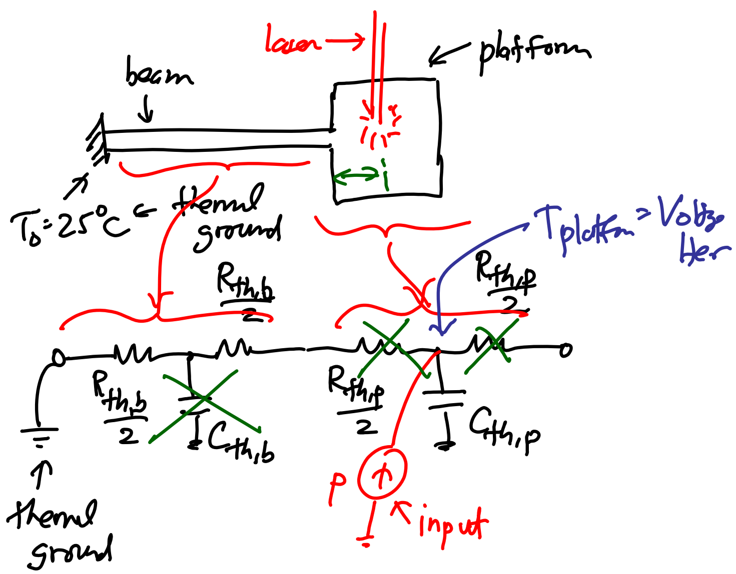
- $l$ ← length
- $A$ ← cross-sectional area
- $k$: thermal conductivity

## Examples

Ex. MEMS type structure (top view)




What is the temperature on the platform?




## Analogies:

(temperature) $T \rightarrow V$ (voltage)

(power) $P \rightarrow I$ (current)

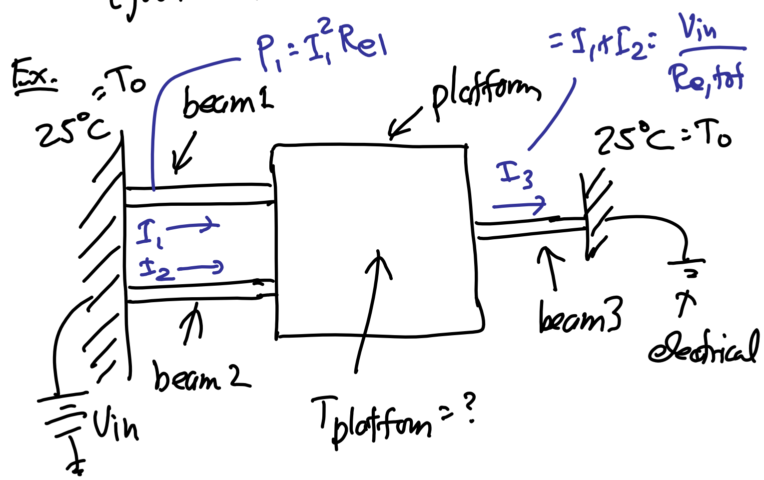
thermal ← → electrical

### To Analyze:

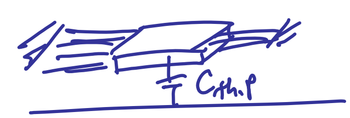
1. Remove small elements to simplify the ckt.
2. $T_{platform} = P R_{th,b}$ (in steady-state)

   (just like $V = IR$ in an electrical system)

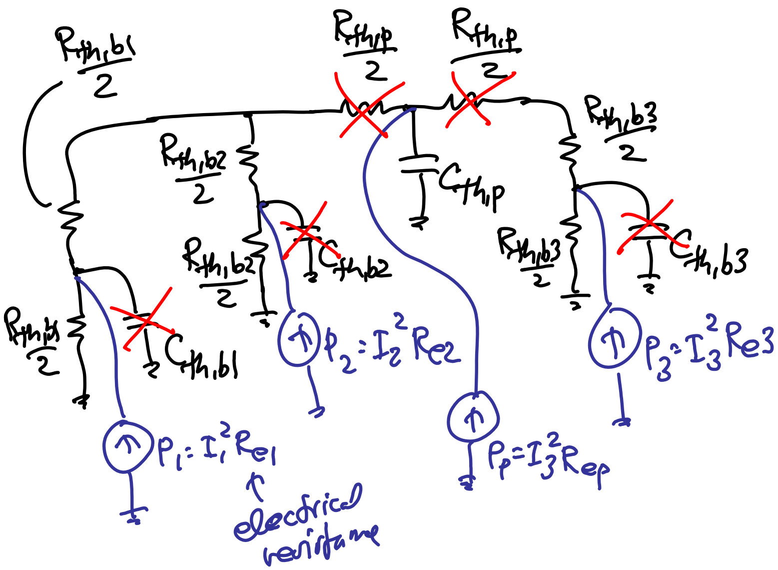
Ex.




⇒ coupled thermal, electrical system



⇒ Draw the thermal ckt:




To Analyze:

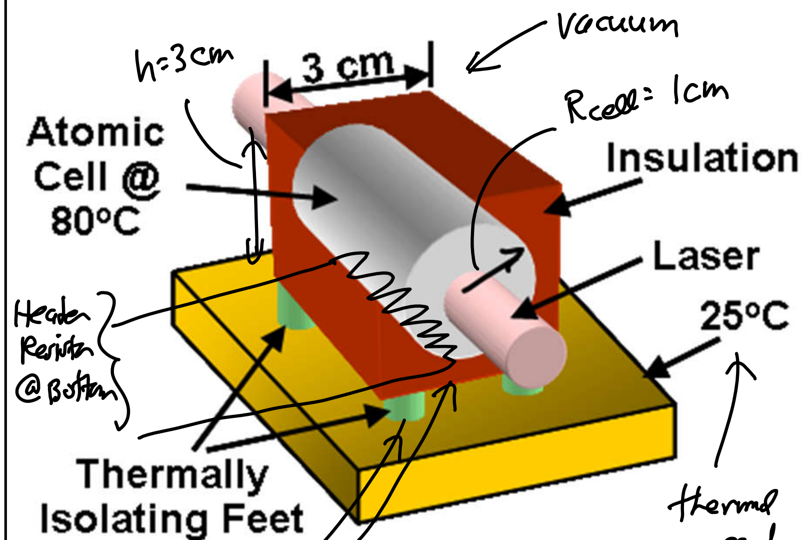
① $I_3 = I_1 + I_2 = \frac{V_{in}}{R_{e,tot}}$ (electrical analysis)

② Get $P_i$'s. (powers)

③ Use superposition to solve the thermal ckt. handle one power source at a time & sum the temperatures (i.e., thermal voltages) to get the total temperature at any node

Example: Thermal Ckt.

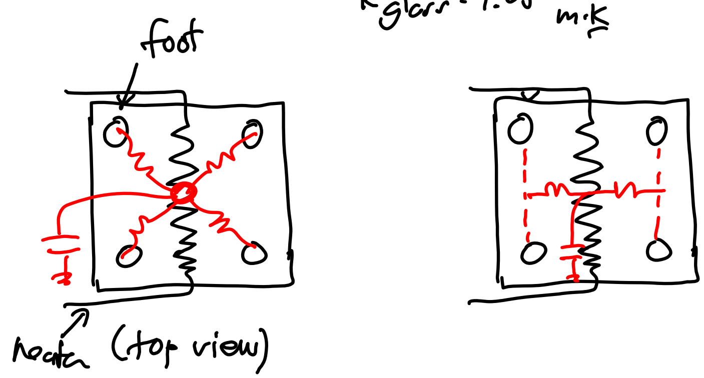
⇒ determine the power needed to get this atomic cell to 80°C (from RT) & how fast

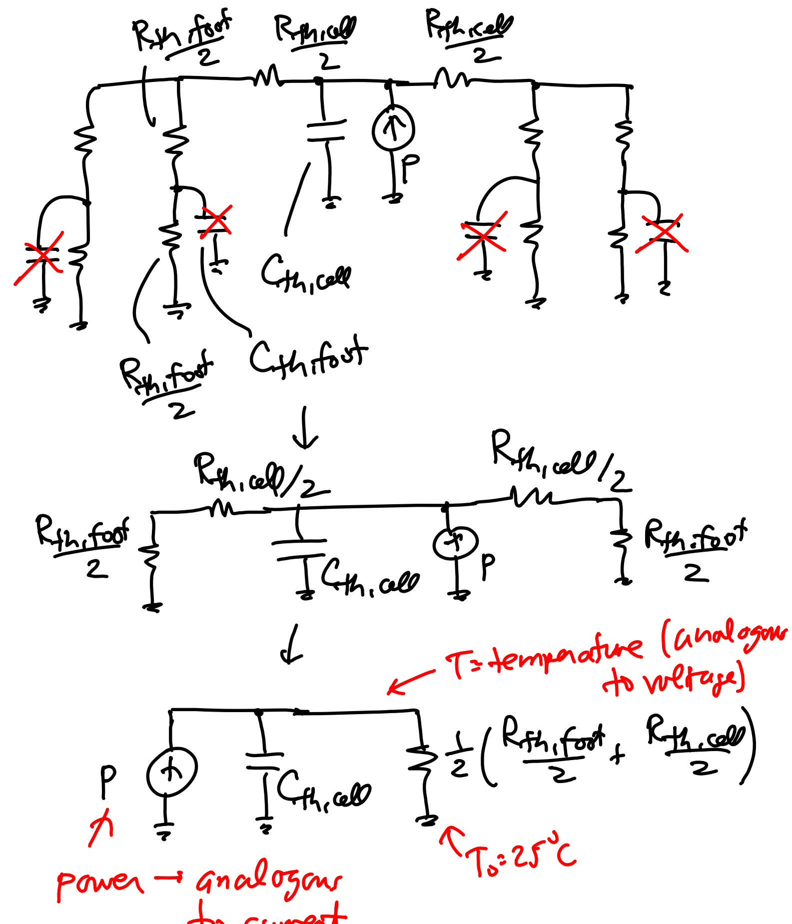



all glass: $\rho_{glass} = 2500\ kg/m^3$

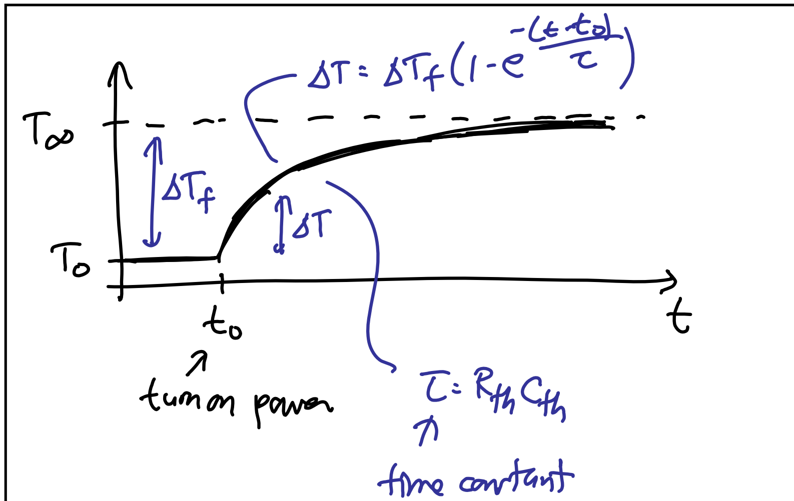
$C_{p,glass} = 0.5\ J/(g\cdot K)$

$K_{glass} = 1.05\ \frac{W}{m\cdot K}$



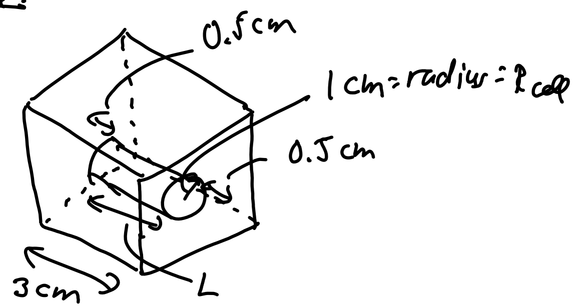



$\Delta T = T - T_o = P R_{th}$ (in steady-state)




Find $C_{th,cell}$:

First, find the cell volume:

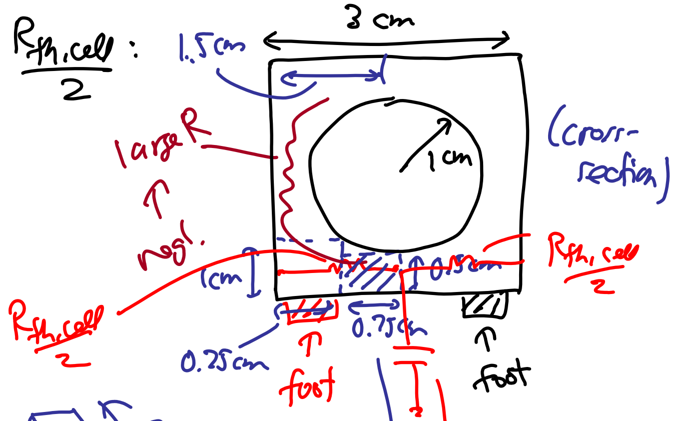


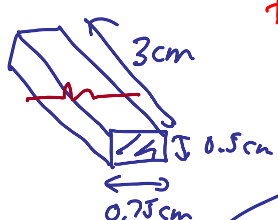

$$V_{cell} = hWL - \pi R_{cell}^2 \cdot L$$
$$= (3cm)(3cm)(3cm) - \pi(1cm)^2(2cm) = 20.7\ cm^3$$

$$C_{th,cell} = \rho_{glass} V_{cell} C_{p,glass}$$
$$= \left(2500 \frac{kg}{m^3}\right)\left(1000 \frac{g}{kg}\right)\left(\frac{1}{100^3} \frac{m^3}{cm^3}\right) \times (20.7\ cm^3)\left(0.5 \frac{J}{g \cdot K}\right)$$

$$\Rightarrow C_{th,cell} = 25.9\ J/K$$

Find $\frac{R_{th,cell}}{2}$:

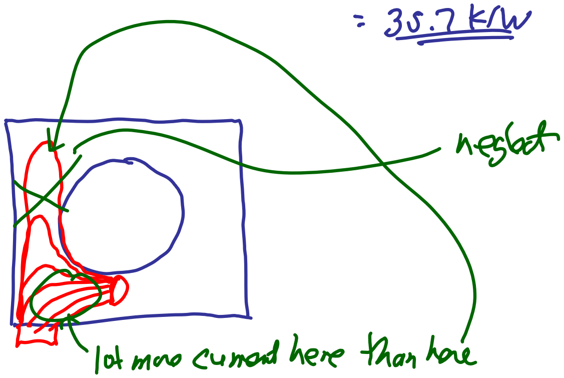


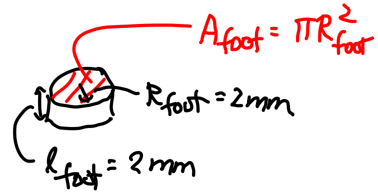

$$\frac{R_{th,cell}}{2} = \frac{\frac{3}{4}}{k(3)(\frac{1}{2})} + \frac{\frac{3}{4}}{k(3)(1)} = \frac{1}{k}\left(\frac{1}{8}+\frac{1}{4}\right) = \frac{3}{8}\frac{1}{k}$$

$\left[R_{th} = \frac{l}{kA}\right]$

$$\therefore \frac{R_{th,cell}}{2} = \frac{3}{8}\frac{1}{1.05} \times \left(100\,\frac{cm}{m}\right) = 35.7\ K/W$$




Find $R_{th,foot}$:




$$\therefore R_{th,foot} = \frac{l_{foot}}{kA_{foot}} = \frac{2\,mm}{(1.05\frac{W}{m \cdot K})\pi(2mm)^2} = 151.6\ \frac{K}{W}$$

Then:

$$R_{th} = \frac{1}{2}\left(\frac{R_{th,foot}}{2} + \frac{R_{th,cell}}{2}\right) = \frac{1}{2}\left(\frac{151.6}{2} + 35.7\right)$$

$$\Rightarrow R_{th} = 55.8\ K/W$$

⇒ Find the power req'd to maintain $T_\infty = 80°C$ in steady-state:

$$P = \frac{T_\infty - T_o}{R_{th}} = \frac{(80-25)}{55.8} = 0.99W \sim 1W$$

⇒ Find the time constant:

$$\tau = R_{th} C_{th,cell} = \underbrace{(55.8)}_{K/W}\underbrace{(25.9)}_{J/K} = 24\ min.$$

It takes ~3τ to reach steady-state

∴ must wait 72 min. before using this atomic cell

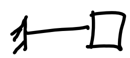
## How about using MEMS?

⇒ same question as: "How about scaling this?"

⇒ much smaller cell volume → weight↓
↘ Volume↓ → $C_{th}$↓

⇒ can get away from feet support.
↳ move to tethers

Macro:

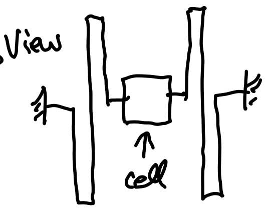


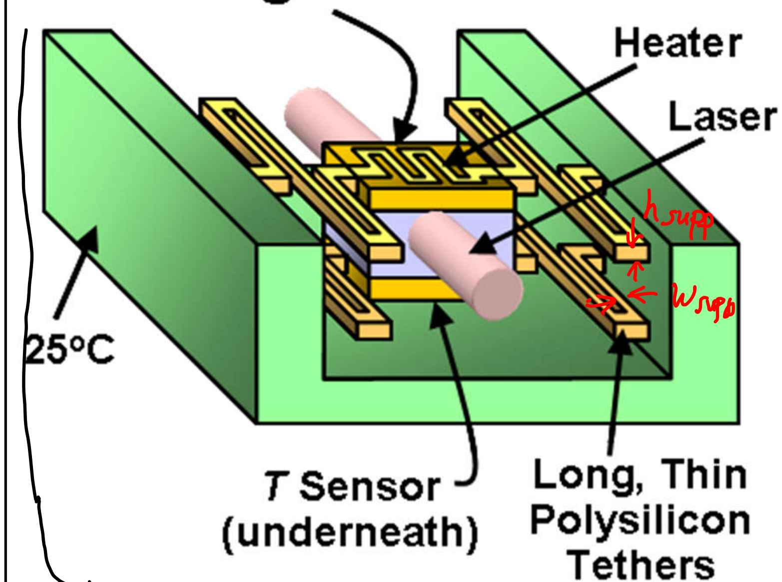
↓ shrink dimensions

Micro:

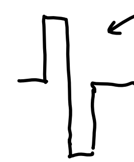
much longer, thinner support to suspend the cell (w/o droop)


## MEMS Atomic Cell




Cell is hollow w/ 10μm-thick walls.



folded to save space & relieve stress
Unfolded length = 500μm-long
10μm-thick, 20μm-wide

$V_{cell} = (300\mu)(300\mu)(300\mu) - (280\mu)(280\mu)(280\mu)$
$= 5.048 \times 10^{-12}\ m^3$
(much smaller than macro cell!)

$$C_{th,cell} = \rho_{glass} V_{cell} C_{p,glass}$$
$$= \left(2500 \frac{kg}{m^3}\right)(5.048\times10^{-12} m^3)\left(500 \frac{J}{kg\cdot K}\right)$$
$$= 6.31\times10^{-6}\ J/K$$ ← ~4 million X smaller than macro!

$$R_{th,supp} = \frac{l_{supp}}{k_{polysi} W_{supp} \cdot h_{supp}} = \frac{500\mu}{\left(30 \frac{W}{mK}\right)(20\mu)(10\mu)}$$

$$\rightarrow R_{th,supp} = 83{,}333\ K/W$$ → 1493X larger than macro!

and...

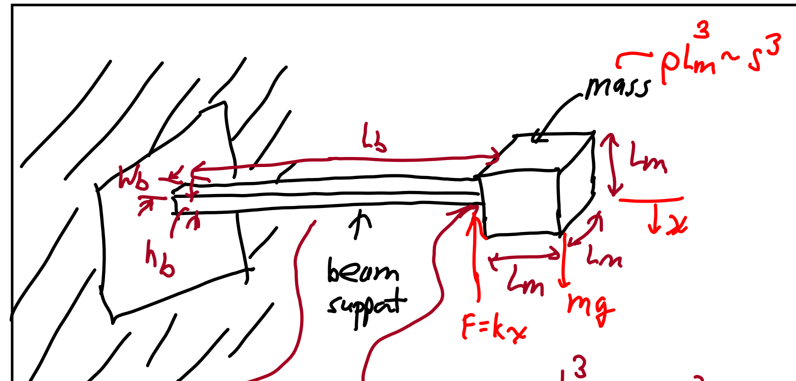
$$P = \frac{(80-25)}{83{,}333} = 0.66\ mW$$ ← 1493X smaller than macro!

$$t = 0.53s$$ ← 2727X faster than macro!

All due to scaling!

Remarks. (What makes this possible?)

1. Scaling reduces $C_{th} \sim l^3 \rightarrow s^3$ ↘ $s\downarrow \rightarrow C_{th}\downarrow\downarrow\downarrow$
2. Scaling allows the use of long, thin tethers → $R_{th}\uparrow\uparrow\uparrow$



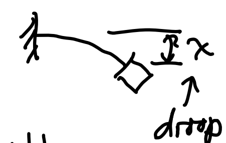

$$k \triangleq \text{stiffness @ this pt.} = \frac{1}{4} E W_b \frac{h_b^3}{L_b^3} \sim s \frac{s^3}{s^3} \sim s$$

@ static equilibrium:

Force due to Gravity = Spring Force

$$mg = kx$$ (g: acceleration due to gravity; x: displacement) *

$$x = \frac{m}{k} g \sim \frac{s^3}{s} \sim s^2$$ → as $s\downarrow \rightarrow x\downarrow\downarrow$




$$R_{th} = \frac{L_b}{k_{th} W_b h_b}$$ → want to raise this (for a lower power consuming atomic cell) ↘ but maintain the same drop $x$

$$* \leftarrow \rho L_m^3 g = \frac{1}{4} E W_b \frac{h_b^3}{L_b^3} x$$

$$\frac{L_b}{W_b h_b} = \frac{1}{4} E \frac{h_b^2}{L_b^3} x \frac{1}{\rho L_m^3 g} \sim \frac{s^2}{s^2} \frac{1}{s^3} \sim \frac{1}{s^3}$$

$\sim R_{th}$

as $s \downarrow \rightarrow \frac{L_b}{W_b h_b} \sim R_{th} \uparrow\uparrow\uparrow$

- Go through slides 30-31 and 37-48 in Module 2 to finish up Thermal Circuits and cover Micro Gas Analyzers