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EE C247B – ME C218
Introduction to MEMS Design
Spring 2019

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Lecture Module 14: Sensing Circuits

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Lecture Outline

- Reading: Senturia, Chpt. 14
- Lecture Topics:
  - Detection Circuits
    - Velocity Sensing
    - Position Sensing

Velocity-to-Voltage Conversion

- To convert velocity to a voltage, use a resistive load

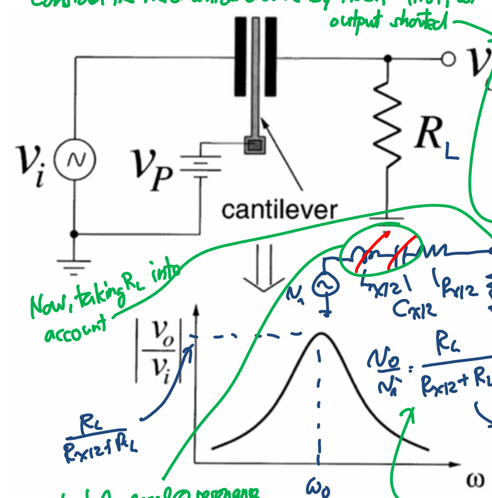
Consider the mechanical device by itself first, w/ output shorted

$$\frac{\dot{x}}{F_{d1}}(s) = \frac{\omega_o Q}{k}\Theta(s)$$

$$\left[F_{d1} = \eta_{e1} v_i\right] \Rightarrow \frac{\dot{x}}{v_i}(s) = \eta_{e1}\frac{\omega_o Q}{k}\Theta(s)$$

$$\left[i_o = \eta_{e2}\dot{x}\right] \Rightarrow \frac{i_o}{v_i}(s) = \eta_{e1}\eta_{e2}\frac{\omega_o Q}{k}\Theta(s) = \frac{\eta_{e1}\eta_{e2}Q}{m\omega_o}\Theta(s)$$

$$\frac{1}{R_{x12}}$$



Now, taking R_L into account

v_i, L_{x12}, C_{x12}, R_{x12}, R_L, v_o

$$\frac{v_o}{v_i} = \frac{R_L}{R_{x12}+R_L} \qquad \frac{i_o}{v_i}(s) = \frac{1}{R_{x12}}\Theta(s)$$

$$\boxed{\frac{v_o}{v_i}(s) = \frac{R_L}{R_{x12}+R_L}\Theta(s)} \rightarrow Q' = Q\left(\frac{R_{x12}}{R_{x12}+R_L}\right)$$

w/ altered Q

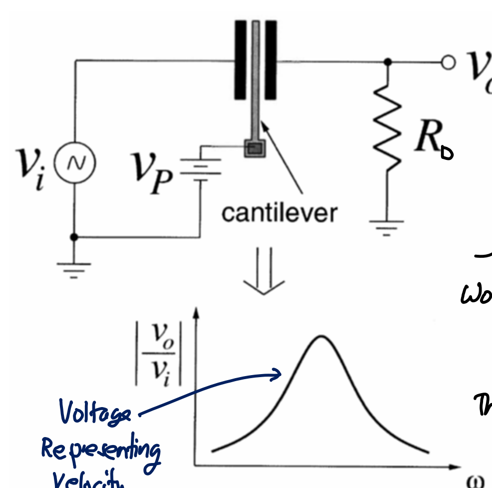
$\frac{R_L}{R_{x12}+R_L}$

L_x & C_x cancel @ resonance $\qquad \omega_0$

Solve the problem @ resonance first, then multiply by $\Theta(s)$

Velocity-to-Voltage Conversion

- To convert velocity to a voltage, use a resistive load



Voltage Representing Velocity

Since this structure has completely symmetrical I/O ports:

v_i, v_o, R_D

$$R_x = \frac{b}{\eta_e^2} \qquad C_x = \frac{\eta_e^2}{k} \qquad L_x = \frac{m}{\eta_e^2}$$

C_x & L_x cancel

Work @ resonance: (to simplify the analysis)

$$\frac{v_o}{v_i} = \frac{R_D}{R_x+R_D} \quad \text{(@ resonance)}$$

Then, generalize to off resonance:

$$\frac{v_o}{v_i} = \frac{R_D}{R_x+R_D}\Theta(s,Q'), \text{ where } Q' = Q\left(\frac{R_x}{R_x+R_D}\right)$$

Velocity-to-Voltage Conversion

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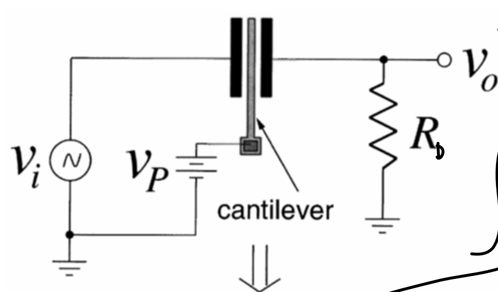
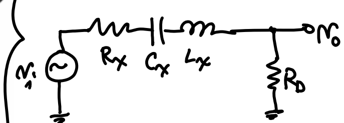
- To convert velocity to a voltage, use a resistive load



Since this structure has completely symmetrical I/O ports:



$$\left[Q = \frac{\omega_0 L_x}{R_x} \rightarrow Q' = \frac{\omega_0 L_x}{R_x + R_D} \rightarrow \frac{R_x + R_D}{L_x} = \frac{\omega_0}{Q'}\right]$$

$$\left[\frac{1}{L_x C_x} = \omega_0^2\right]$$

Brute force approach:

$$\frac{v_o}{v_i}(s) = \frac{R_D}{R_x + \frac{1}{sC_x} + sL_x + R_D} = \frac{sR_D C_x}{sR_x C_x + 1 + s^2 L_x C_x + sR_D C_x} = \frac{s\frac{R_D}{L_x}}{s^2 + s\frac{R_x + R_D}{L_x} + \frac{1}{L_x C_x}}$$

$$= \frac{R_D}{R_x + R_D} \frac{s\left(\frac{R_x + R_D}{L_x}\right)}{s^2 + s\left(\frac{R_x + R_D}{L_x}\right) + \frac{1}{L_x C_x}} \Rightarrow \frac{v_o}{v_i}(s) = \frac{R_D}{R_x + R_D} \frac{s(\omega_0/Q')}{s^2 + s(\omega_0/Q') + \omega_0^2} = \frac{R_D}{R_x + R_D} \Theta(s, Q')$$

$$Q' = Q\left(\frac{R_x}{R_x + R_D}\right)$$

Position-to-Voltage Conversion

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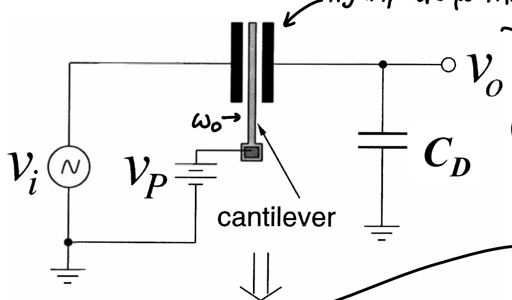
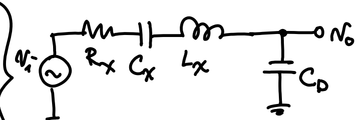
- To sense position (i.e., displacement), use a capacitive load

Again, have port-to-port I/O symmetry:





Brute force approach:

$$\frac{v_o}{v_i}(s) = \frac{\frac{1}{sC_D}}{R_x + \frac{1}{sC_x} + sL_x + \frac{1}{sC_D}}$$

$$\frac{v_o}{v_i}(s) = \frac{\frac{\cancel{s}C_x}{\cancel{s}C_D}}{sR_x C_x + 1 + s^2 L_x C_x + \frac{\cancel{s}C_x}{\cancel{s}C_D}} = \frac{C_x/C_D}{1 + C_x/C_D} \frac{1}{1 + \frac{sR_x C_x}{1 + C_x/C_D} + s^2 \frac{L_x C_x}{1 + C_x/C_D}}$$

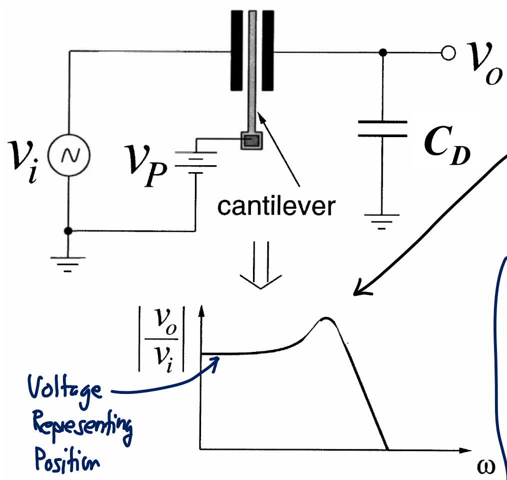
$$= \frac{C_x/C_D}{1 + C_x/C_D} \frac{\frac{1 + C_x/C_D}{L_x C_x}}{s^2 + s\left(\frac{R_x}{L_x}\right) + \frac{(1 + C_x/C_D)}{L_x C_x}}$$

$$\left[\omega_0^2 = \frac{1}{L_x C_x} \rightarrow (\omega_0')^2 = \omega_0^2(1 + C_x/C_D)\right]$$

$$\left[Q' = \frac{\omega_0' L_x}{R_x} \rightarrow \frac{R_x}{L_x} = \frac{\omega_0'}{Q'},\ Q' = Q\sqrt{1 + C_x/C_D}\right]$$ over

Position-to-Voltage Conversion

- To sense position (i.e., displacement), use a capacitive load



$$\frac{v_o}{v_i}(s) = \underbrace{\frac{C_x/C_D}{1+C_x/C_D}}_{\text{DC Gain Term}} \; \underbrace{\frac{(\omega_o')^2}{s^2 + \left(\frac{\omega_o'}{Q'}\right)s + (\omega_o')^2}}_{\text{Low-Pass Biquad}}$$

To maximize gain → 1, need $C_D \ll C_x$. (must minimize C_D)

Note: Can use similar short-cut to the R case.
1. Get DC response → C's dominate.
2. Then:

$$\frac{v_o}{v_i}(s) = (\text{DC Gain}) \cdot \frac{1}{s} \cdot H(s, \omega_o', Q') \cdot \omega_o' Q'$$

Velocity Sensing Circuits

Velocity-to-Voltage Conversion

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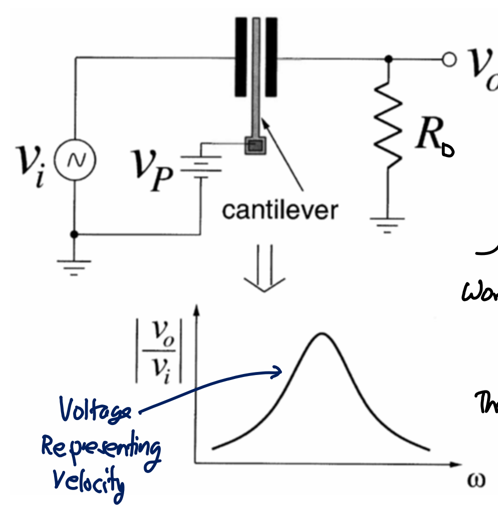
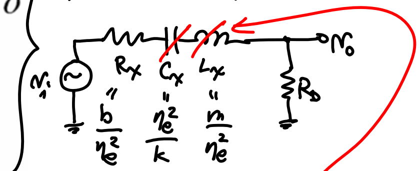
- To convert velocity to a voltage, use a resistive load



Since this structure has completely symmetrical I/O ports:



C_x & L_x cancel

Work @ resonance: (to simplify the analysis)

$$\frac{v_o}{v_i} = \frac{R_D}{R_x+R_D} \quad \text{(@ resonance)}$$

Then, generalize to off resonance:

$$\frac{v_o}{v_i} = \frac{R_D}{R_x+R_D} H(s,Q'), \text{ where } Q' = Q\left(\frac{R_x}{R_x+R_D}\right)$$

Problems With Purely Resistive Sensing

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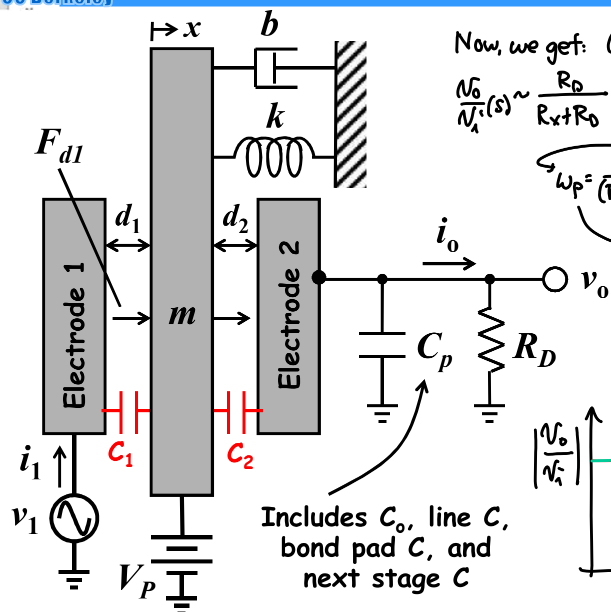


Includes C_o, line C, bond pad C, and next stage C

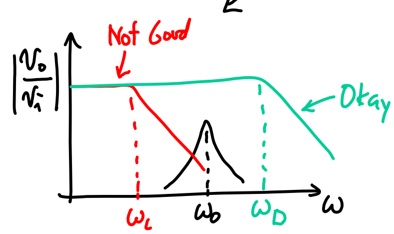
Now, we get: (approximately)

$$\frac{v_o}{v_i}(s) \sim \frac{R_D}{R_x+R_D} \cdot \frac{1}{1+\frac{s}{\omega_p}} \cdot H(s,\omega_o',Q')$$

$$\omega_p = \frac{1}{(R_x \| R_D)C_p}$$

Depend on both R_D & C_p.

Impact depends on where ω_p is relative to ω_o



Problems With Purely Resistive Sensing

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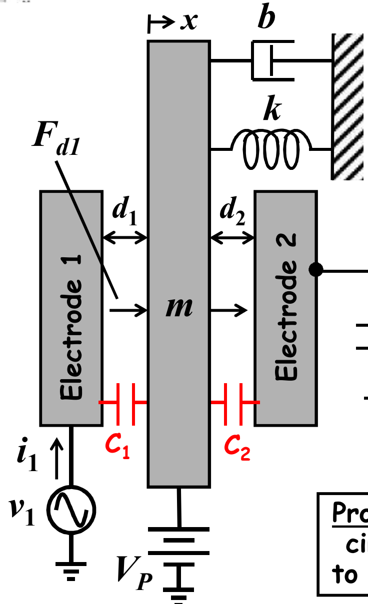
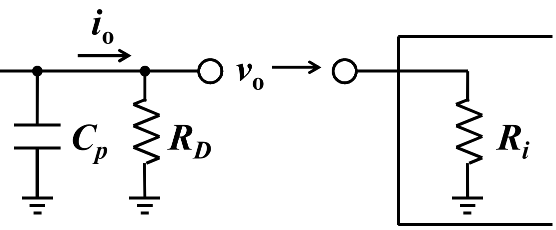


- In general, the sensor output must be connected to the inputs of further signal conditioning circuits → input R_i of these circuits can load R_D



These change w/ hook-up → not good.

Problem: need a sensing circuit that is immune to parasitics or loading.

Soln: use op amps.

The TransR Amplifier Advantage

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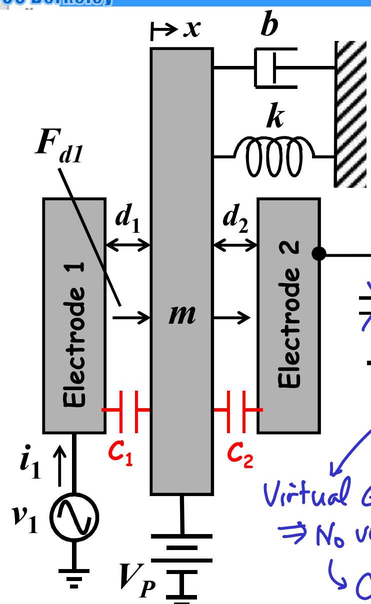
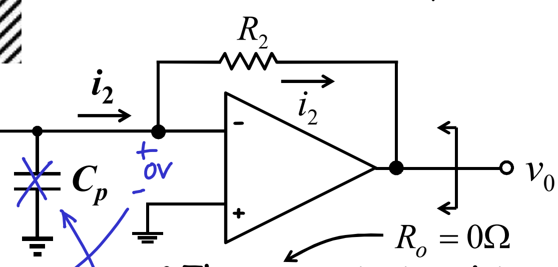


- The virtual ground provided by the ideal op amp eliminates the parasitic capacitance C_p and R_i



- The zero output resistance of the (ideal) op amp can drive virtually anything

Virtual Ground
⇒ No voltage across C_p
↳ C_p effectively isn't there!

$$\boxed{v_0 = -\frac{R_2}{R_x}\,\Theta(s)\,v_i}$$

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Position Sensing Circuits

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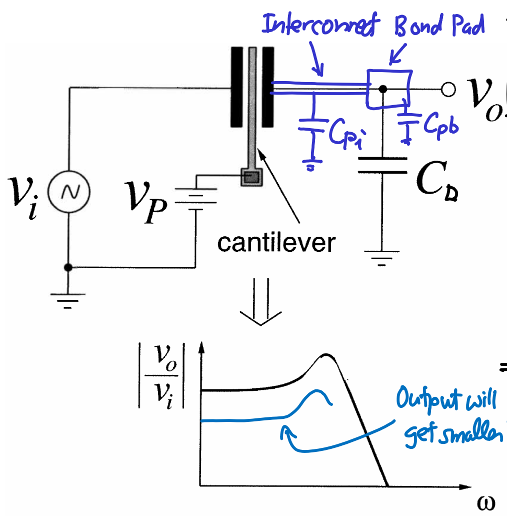
Problems With Pure-C Position Sensing

- To sense position (i.e., displacement), use a capacitive load



$$\boxed{\frac{v_o}{v_i}(s) = \frac{C_x/C_D}{1+C_x/C_D}\cdot\frac{1}{s}\cdot H(s,\omega_o',Q')\cdot\omega_o'Q'}$$

Integration yields displacement.

To maximize gain, minimize C_D.

$\Rightarrow$ <u>Problem</u>: parasitic capacitance

$C_D \rightarrow C_D + C_{Pi} + C_{pb}$

$\Rightarrow$ DC Gain: $\dfrac{C_x/(C_D+C_{Pi}+C_{pb})}{1+C_x/(C_D+C_{Pi}+C_{pb})}$

<u>Remedy</u>: Suppress C_p via use of op amps.

The Op Amp Integrator Advantage

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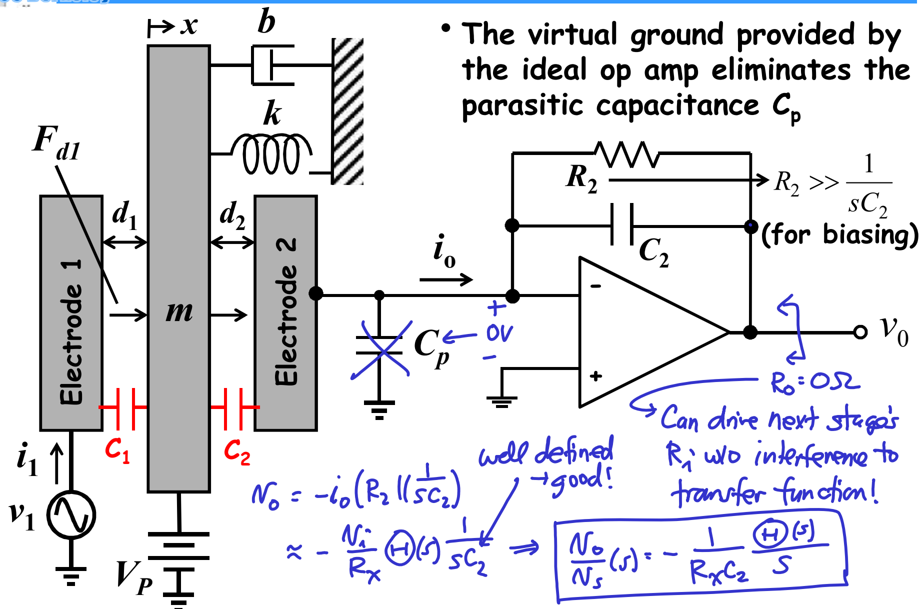
- The virtual ground provided by the ideal op amp eliminates the parasitic capacitance C_p



$$v_o = -i_o\left(R_2 \,\|\, \frac{1}{sC_2}\right)$$

$$\approx -\frac{v_i}{R_x} H(s) \frac{1}{sC_2} \Rightarrow \boxed{\frac{v_o}{v_s}(s) = -\frac{1}{R_x C_2}\frac{H(s)}{s}}$$

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Differential Position Sensing

Differential Position Sensing

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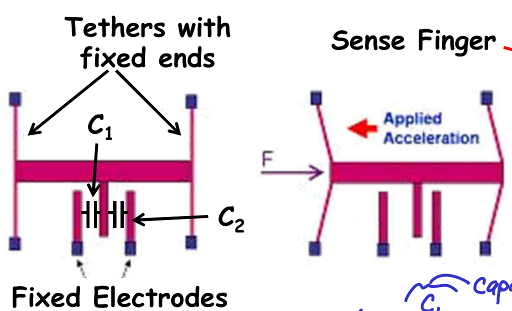
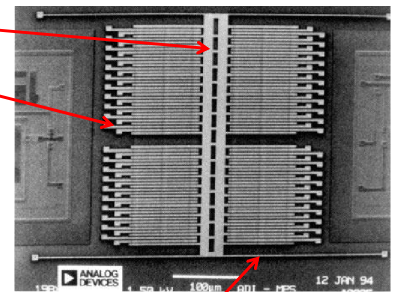
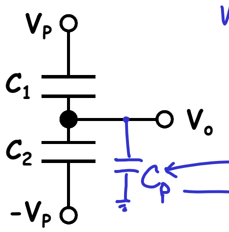
- Example: ADXL-50







$V_o = -V_p + (2V_p)\underbrace{\frac{C_1}{C_1+C_2}}_{\text{capacitive divider}}$

$= \frac{-V_pC_1 - V_pC_2 + 2V_pC_1}{C_1+C_2} = \boxed{V_p\left(\frac{C_1-C_2}{C_1+C_2}\right) = V_o}$

Issue: Parasitic Capacitance

$\boxed{V_o = \frac{C_1-C_2}{C_1+C_2+C_p}V_p}$

As before, C_p reduces gain → Soln: use op amp!

Buffer-Bootstrapped Position Sensing

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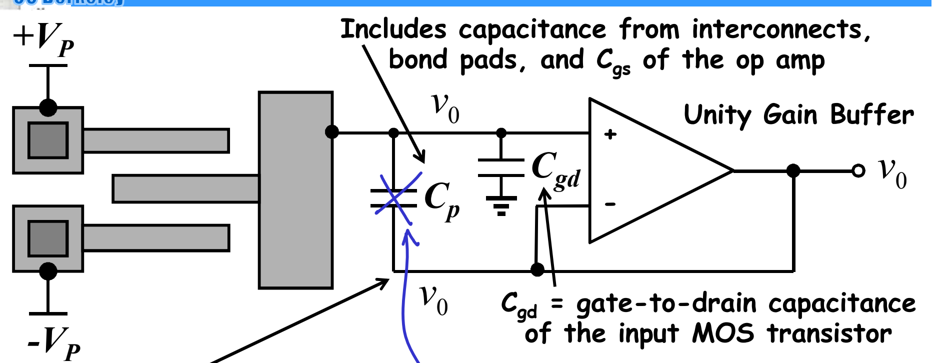
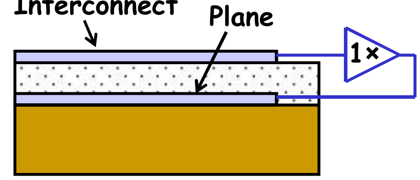


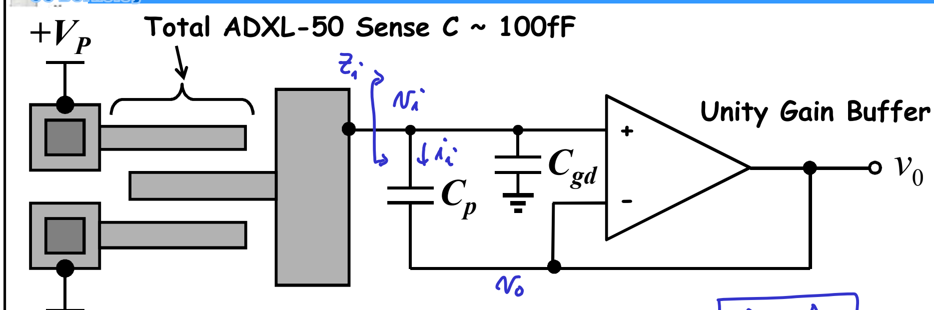
Includes capacitance from interconnects, bond pads, and C_{gs} of the op amp

C_{gd} = gate-to-drain capacitance of the input MOS transistor

- Bootstrap the ground lines around the interconnect and bond pads
  - No voltage across C_p
  - It's effectively not there!



Effect of Finite Op Amp Gain



$v_o = A_o(v_+ - v_-) = A_o(v_i - v_o) \rightarrow v_o(1+A_o) = A_o v_i \rightarrow \boxed{\frac{v_o}{v_i} = \frac{A_o}{1+A_o}}$

Get $Z_i = \frac{v_i}{i_i}$: $i_i = (v_i - v_o)sC_p = v_i\left(1 - \frac{A_o}{1+A_o}\right)sC_p = v_i \frac{1}{1+A_o} sC_p$

$\therefore \frac{v_i}{i_i} = Z_i = \frac{1}{s\left[\frac{C_p}{1+A_o}\right]}$ $\rightarrow$ $\boxed{C_{eff} = \frac{C_p}{1+A_o}}$

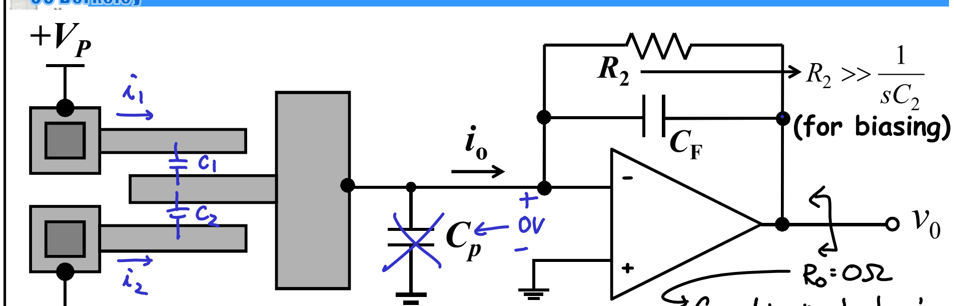
No longer zero!

Ex: $A_o = 100$, $C_p = 2pF$

$\Rightarrow C_{eff} = \frac{2p}{101} = 20fF$

Not negligible compared w/ ADXL-50 $C_{tot} \sim 100fF$!

Integrator-Based Diff. Position Sensing



Can drive next stage's R_i w/o interference to transfer function!

$i_o = i_1 + i_2 = v_P(sC_1) - v_P(sC_2)$

$= v_P s(C_1 - C_2)$

$\therefore v_o = -i_o\left(\frac{1}{sC_F}\right) = -v_P\left(\frac{C_1 - C_2}{C_F}\right)$

$\boxed{\frac{v_o}{v_P} = -\frac{C_1 - C_2}{C_F}}$ $\Rightarrow$ A seemingly perfect differential sensor/amplifier output! ... but only when the op amp is ideal...