Lecture Outline

• Reading: Senturia, Chpt. 14
• Lecture Topics:
  ◦ Detection Circuits
    • Velocity Sensing
    • Position Sensing
Velocity-to-Voltage Conversion

* To convert velocity to a voltage, use a resistive load

Consider the mechanical device by itself first, with output shorted.

\[ \frac{V_o}{V_i} = \frac{n \cdot \text{Velocity}}{R_L} \]

Voltage representing Velocity

\[ \omega = \frac{\omega_0}{k} \]

Solve the problem at resonance first, then multiply by \( \Omega(c) \)
**Velocity-to-Voltage Conversion**

* To convert velocity to a voltage, use a resistive load

\[ V_O = \frac{R_b}{R_x + R_b} V_P \]

Since this structure has completely symmetrical I/O ports:

\[ Q^+ : \frac{R_b}{R_x + R_b} \rightarrow Q^+ \frac{R_b}{R_x} \frac{R_b}{R_b + R_0} \]

\[ Q^- : \frac{1}{C_x} \frac{V_O}{V_O} \]

\[ N_x(x) = \frac{R_b}{R_x + R_b} \frac{1}{s^2 + s^2 + s(\omega_0^2 + \frac{1}{C_x} + \frac{1}{C_0})} \]

**Position-to-Voltage Conversion**

* To sense position (i.e., displacement), use a capacitive load

\[ V_O = \frac{1}{s^2 + s^2} C_x \frac{C_0}{s^2 + s^2 + s(\omega_0^2 + \frac{1}{C_x} + \frac{1}{C_0})} \]

\[ N_x(x) = \frac{1}{s^2 + s^2 + s(\omega_0^2 + \frac{1}{C_x} + \frac{1}{C_0})} \]

Again, have port-to-port I/O symmetry:

\[ N_x(x) = \frac{1}{s^2 + s^2 + s(\omega_0^2 + \frac{1}{C_x} + \frac{1}{C_0})} \]
Position-to-Voltage Conversion

* To sense position (i.e., displacement), use a capacitive load

\[
\frac{V_o(s)}{V_i(s)} = \frac{C_D/C_0}{1 + \frac{C_D}{C_0} \left( \frac{\omega_0}{s} \right)^2}
\]

\[
\omega_0^2 = \frac{2}{C_D/C_0}
\]

DC Gain Term

Low-Pass Biquad

To maximize gain \( \rightarrow \infty \), need \( C_D \approx C_0 \).

Note: Can use similar short-cut to the R case.

1. Get DC response \( C_r \) dominant.
2. Then:

\[
V_o(s) = (DC Gain) \cdot \frac{1}{s} \cdot \omega_0^2 \cdot \Theta(s, \omega_0, \omega_c, \cdot \cdot \cdot)
\]

Velocity Sensing Circuits
**Velocity-to-Voltage Conversion**

* To convert velocity to a voltage, use a resistive load.

Since this structure has completely symmetrical 30° pairs:

\[
\begin{align*}
V_o &= \frac{R_o}{R_o + R_D} \left( \frac{\frac{\pi}{2}}{\alpha} \right) \left( \frac{R_o}{R_o + R_D} \right) \\
\end{align*}
\]

Work @ resonance: (to simplify the analysis)

\[
\frac{V_o}{V_i} = \frac{R_o}{R_o + R_D}
\]

Then, generate to offset resonance:

\[
\frac{V_o}{V_i} = \frac{R_o}{R_o + R_D} \left( \frac{\pi}{2} \right)
\]

Problems With Purely Resistive Sensing

Now, we get (approximately)

\[
\frac{V_o}{V_i} \sim \frac{R_o}{R_o + R_D} \left( \frac{\pi}{2} \right)
\]

\[
\frac{1}{\omega_p} = \frac{1}{(R_o + R_D) C_p}
\]

Depending on both \(R_o + R_D\) and \(C_p\).

Impact depends on \(\omega\) relative to \(\omega_p\).

Includes \(C_o\), line \(C\), bond pad \(C\), and next stage \(C\).

\[
\begin{align*}
|V_o|/|V_i| &< \text{Not Good} \\
\omega_D &< \text{Okay}
\end{align*}
\]
Problems With Purely Resistive Sensing

In general, the sensor output must be connected to the inputs of further signal conditioning circuits \( \rightarrow \) input \( R_i \) of these circuits can load \( R_D \nabla \)

These change w/ hook-up \( \rightarrow \) not good.

Problem: need a sensing circuit that is immune to parasitics or loading.

Soln: use op amps.

The TransR Amplifier Advantage

* The virtual ground provided by the ideal op amp eliminates the parasitic capacitance \( C_p \) and \( R_i \nabla \)

* The zero output resistance of the (ideal) op amp can drive virtually anything

\( R_o = 0 \Omega \nabla \)

Virtual Ground \( \Rightarrow \) no voltage across \( C_p \nabla \)

\( C_p \) effectively infinite!
Position Sensing Circuits

Problems With Pure-C Position Sensing

- To sense position (i.e., displacement), use a capacitive load.

\[ v_o = \frac{C_v}{1+\frac{C_v C_d}{C_b}} \cdot \frac{1}{\omega} \cdot \frac{1}{2} \cdot \theta \left( \frac{\omega}{\omega_b} \right) \cdot \omega \cdot \theta \]

Integration yields displacement.

To maximize gain, minimize \( C_b \).

\[ C_v \rightarrow C_b + C_{ps} + C_p \]

\[ \text{DC Gain: } \frac{C_v}{C_v + C_b + C_{ps} + C_p} \]

Output will get smaller.

Remedy: Suppress \( C_p \) via use of op amps.
The Op Amp Integrator Advantage

- The virtual ground provided by the ideal op amp eliminates the parasitic capacitance $C_p$

Differential Position Sensing
**Differential Position Sensing**

- **Example:** ADXL-50

  - Tethers with fixed ends
  - Fixed Electrodes
    - $C_1$, $C_2$
  - Proof Mass
  - Sense Finger

  - Suspension Beam in Tension

  
  \[
  V_0 = -V_p + \left( \frac{2V_p}{C_1+C_2} \right) \frac{V_p}{C_1+C_2} \\
  V_p = \frac{(C_1+C_2)}{C_1+C_2} V_p \\
  V_0 = V_p \frac{C_1+C_2}{C_1+C_2+C_p} V_p
  \]

  - Issue: Parasitic Capacitance

  - **As before, $C_p$ reduces gain → Solution: Use op amp!**

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**Buffer-Bootstrapped Position Sensing**

- Includes capacitance from interconnects, bond pads, and $C_{gs}$ of the op amp

- *Bootstrap the ground lines around the interconnect and bond pads
  - No voltage across $C_p$
  - It's effectively not there!

- **Unity Gain Buffer**

  \[
  C_{gd} = \text{gate-to-drain capacitance of the input MOS transistor}
  \]
**Effect of Finite Op Amp Gain**

Total ADXL-50 Sense C ~ 100fF

\[ V_p \]

\[ -V_p \]

\[ N_v = \frac{A_0(N_i - N_f) \cdot A_{cp}(N_i - N_c)}{N_v} \cdot A_{cp} \cdot \frac{N_v}{1 + A_0} \]

\[ C_{ct} \]

\[ C_{gd} \]

\[ V_{0} \]

Unity Gain Buffer

\[ \frac{N_c}{N_i} \]

\[ \frac{C_p}{1 + A_0} \]

\[ C_{eff} = \frac{C_p}{1 + A_0} \]

No longer poor!

Ex: \[ A_0 = 100, C_p = 2pF \]

\[ \Rightarrow C_{eff} = \frac{2pF}{100} = 20fF \]

Not negligibly compared w/ ADXL-50 C_{eff} ~ 100fF!

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**Integrator-Based Diff. Position Sensing**

\[ V_p \]

\[ \frac{\dot{i}_1}{C_1} \]

\[ \frac{\dot{i}_2}{C_2} \]

\[ i_0 = i_1 + i_2 = \frac{N_v}{C_1} \cdot \frac{N_v}{C_2} \]

\[ = \frac{V_p}{s(C_1 + C_2)} \]

\[ C_p \]

\[ R_2 \]

\[ R_2 >> \frac{1}{sC_2} \]

(for biasing)

\[ R_o \]

\[ \Delta V \]

\[ \frac{V_o}{V_p} = \frac{C_1 \cdot C_2}{C_F} \]

A seemingly perfect differential sensor/amplifier output...but only when the op amp is ideal...