EE C247B – ME C218
Introduction to MEMS Design
Spring 2019

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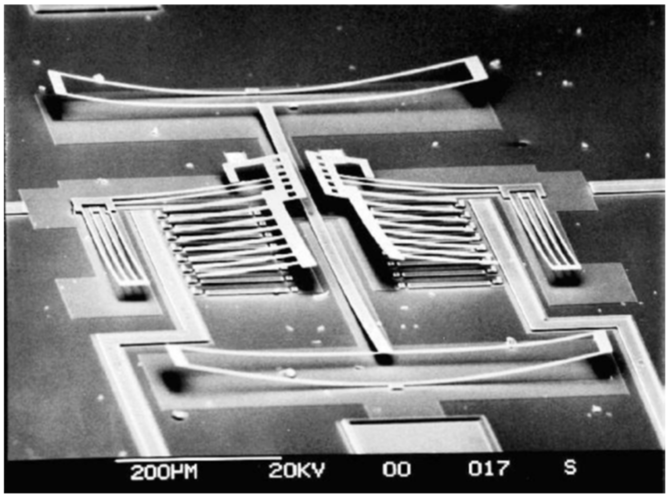
Lecture Module 7: Mechanics of Materials

Outline

- Reading: Senturia, Chpt. 8
- Lecture Topics:
  - Stress, strain, etc., for isotropic materials
  - Thin films: thermal stress, residual stress, and stress gradients
  - Internal dissipation
  - MEMS material properties and performance metrics

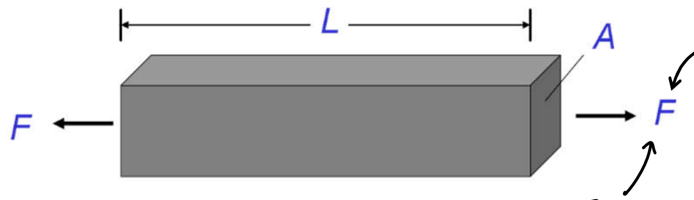
Vertical Stress Gradients

- Variation of residual stress in the direction of film growth
- Can warp released structures in z-direction



Elasticity

Normal Stress (1D)

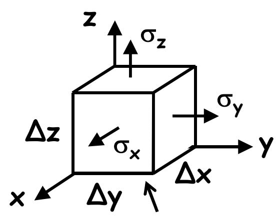


If the force acts normal to a surface, then the stress is called a **normal stress**

Force assumed uniform over the whole area A

$$\text{Stress} = \left\{ \begin{matrix}\text{Force per} \\ \text{unit area}\end{matrix} \right\} = \sigma = \frac{F}{A} \quad [N/m^2 = Pa]$$

↖ standard mks unit

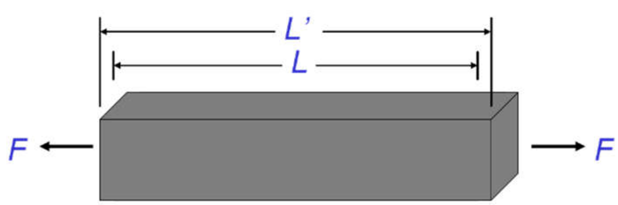


Differential volume element

⇒ Microscopic Definition: force per unit area acting on the surface of a differential volume element of a solid body

⇒ Note: assume stress acts uniformly across the entire surface of the element, not at just a point

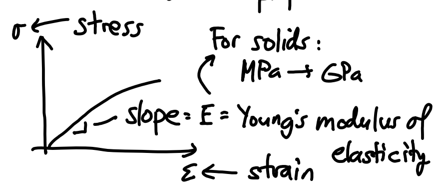
Strain (1D)



Sometimes a unit called the "microstrain" is used, where
$1\mu\varepsilon = \frac{\Delta L}{L}$ of 1 part in 10^6

$$\text{Strain} = \left\{ \begin{matrix}\text{Fractional Change} \\ \text{in Length}\end{matrix} \right\} = \varepsilon = \frac{L'-L}{L} = \frac{\Delta L}{L} \quad [\text{unitless}]$$

In the elastic regime (i.e., for "small" stresses at "low" temperatures), strain is found to be proportional to stress



For solids: MPa → GPa

$$\sigma = \varepsilon E \rightarrow \boxed{\varepsilon = \frac{\sigma}{E} \quad [\text{unitless}]}$$

Thus, the units of E are the same as σ → Pa

The Poisson Ratio

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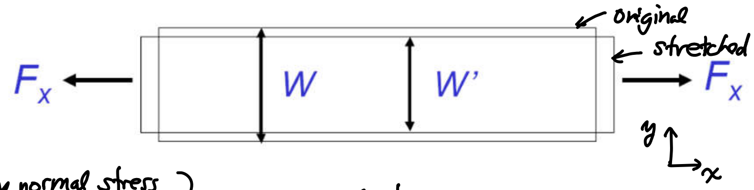

Apply normal stress to a free-standing object → uniaxial strain
→ but also get contraction in directions transverse to the uniaxial strain

⇒ contraction creates a (−) strain:

$$\varepsilon_y = \frac{W'-W}{W} = \frac{\Delta W}{W} = -\nu \varepsilon_x$$

ν = Poisson ratio [unitless]

Typical values: 0 → 0.5

⇒ inorganic solids: 0.2 → 0.3

⇒ elastomers (e.g., rubber): ~0.5

Shear Stress & Strain (1D)

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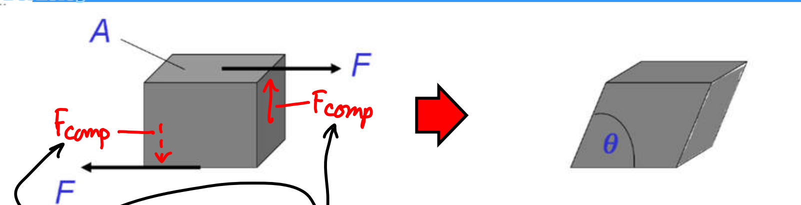


Note: Assume compensating forces are applied to the vertical faces to avoid a net torque. (This by convention.)

$$\text{Shear Stress} = \left\{\begin{array}{l}\text{Force Per Unit Area} \\ \text{Parallel to the Surfaces}\end{array}\right\} = \tau = \frac{F}{A} \quad [\text{Pa}]$$

Generates a shear strain:

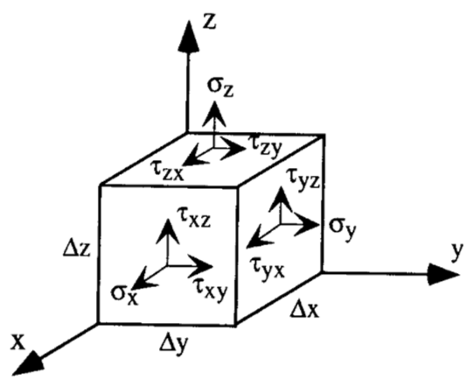
$$\text{Shear Strain} = \theta = \frac{\tau}{G}$$

$G \triangleq$ shear modulus

$$G = \frac{E}{2(1+\nu)}$$

2D and 3D Considerations

- Important assumption: the differential volume element is in static equilibrium → no net forces or torques (i.e., rotational movements)
  - Every σ must have an equal σ in the opposite direction on the other side of the element
  - For no net torque, the shear forces on different faces must also be matched as follows:

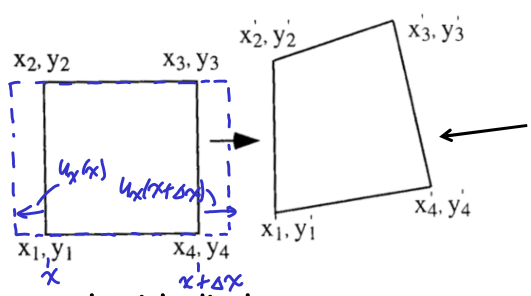


Stresses acting on a differential volume element

$$\tau_{xy} = \tau_{yx} \qquad \tau_{xz} = \tau_{zx} \qquad \tau_{yz} = \tau_{zy}$$

2D Strain

- In general, motion consists of
  - rigid-body displacement (motion of the center of mass)
  - rigid-body rotation (rotation about the center of mass)
  - Deformation relative to displacement and rotation



Area element experiences both displacement and deformation

- Must work with displacement vectors
- Differential definition of axial strain: ⟶ $\varepsilon_x = \dfrac{u_x(x+\Delta x) - u_x(x)}{\Delta x} = \dfrac{\partial u_x}{\partial x}$

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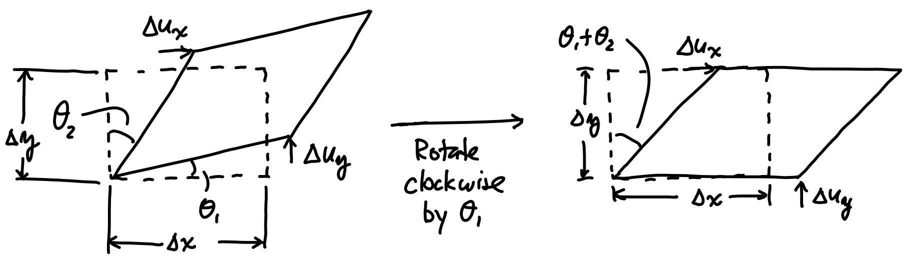
2D Shear Strain



⇒ For shear strains, must remove any rigid body rotation that accompanies the deformation

↳ use a symmetric definition of shear strain:

$$\gamma_{xy} = \theta_2 + \theta_1 \approx \left(\frac{\Delta u_x}{\Delta y} + \frac{\Delta u_y}{\Delta x}\right) = \left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right)$$

↑ For small amplitude deformations.

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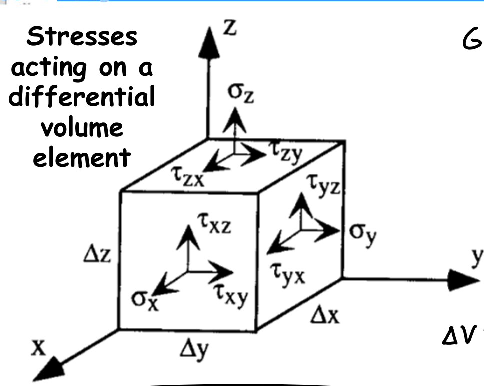
Volume Change for a Uniaxial Stress

Stresses acting on a differential volume element



Given an x-directed uniaxial stress, σ_x:

$\Delta x \rightarrow \Delta x(1+\epsilon_x)$

$\Delta y \rightarrow \Delta y(1-\nu\epsilon_x)$

$\Delta z \rightarrow \Delta z(1-\nu\epsilon_x)$

↓ The resulting change in volume ΔV

$$\Delta V = \Delta x \Delta y \Delta z (1+\epsilon_x)(1-\nu\epsilon_x)^2 - \Delta x \Delta y \Delta z$$
$$= \Delta x \Delta y \Delta z \left[(1+\epsilon_x)(1-\nu\epsilon_x)^2 - 1\right]$$

{Assume small strains} ⇒ $\Delta V = \Delta x \Delta y \Delta z \left[(1+\epsilon_x)(1-2\nu\epsilon_x) - 1\right]$

$[(1+mx)^n \approx 1+nmx]$ ⇒ $\approx \Delta x \Delta y \Delta z \left[1 + \epsilon_x - 2\nu\epsilon_x - 2\nu\epsilon_x^2 - 1\right]$ (negl.)

$$\boxed{\Delta V \approx \Delta x \Delta y \Delta z (1-2\nu)\epsilon_x}$$

For $\nu = 0.5$ (rubber) → no ΔV!

$\nu < 0.5$ → finite ΔV

Isotropic Elasticity in 3D

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- Isotropic = same in all directions
- The complete stress-strain relations for an isotropic elastic solid in 3D: (i.e., a generalized Hooke's Law)

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right] \qquad \gamma_{xy} = \frac{1}{G}\tau_{xy}$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu\left(\sigma_z + \sigma_x\right)\right] \qquad \gamma_{yz} = \frac{1}{G}\tau_{yz}$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right] \qquad \gamma_{zx} = \frac{1}{G}\tau_{zx}$$

Basically, add in off-axis strains from normal stresses in other directions

Important Case: Plane Stress

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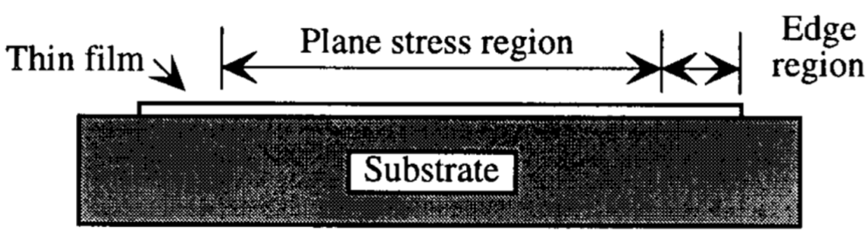
- Common case: very thin film coating a thin, relatively rigid substrate (e.g., a silicon wafer)



- At regions more than 3 thicknesses from edges, the top surface is stress-free → $\sigma_z = 0$
- Get two components of in-plane stress:

$$\varepsilon_x = (1/E)[\sigma_x - \nu(\sigma_y + 0)]$$

$$\varepsilon_y = (1/E)[\sigma_y - \nu(\sigma_x + 0)]$$

Important Case: Plane Stress (cont.)

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- Symmetry in the xy-plane → $\sigma_x = \sigma_y = \sigma$
- Thus, the in-plane strain components are: $\varepsilon_x = \varepsilon_y = \varepsilon$ where

$$\varepsilon_x = (1/E)[\sigma - \nu\sigma] = \frac{\sigma}{[E/(1-\nu)]} = \frac{\sigma}{E'}$$

and where

$$\text{Biaxial Modulus} \triangleq E' = \frac{E}{1-\nu}$$

Edge Region of a Tensile (σ>0) Film

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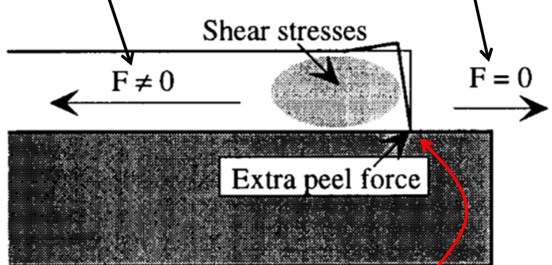
Net non-zero in-plane force (that we just analyzed)

At free edge, in-plane force must be zero

Film must be bent back, here

There's no Poisson contraction, so the film is slightly thicker, here



Discontinuity of stress at the attached corner → stress concentration

Peel forces that can peel the film off the surface

Linear Thermal Expansion

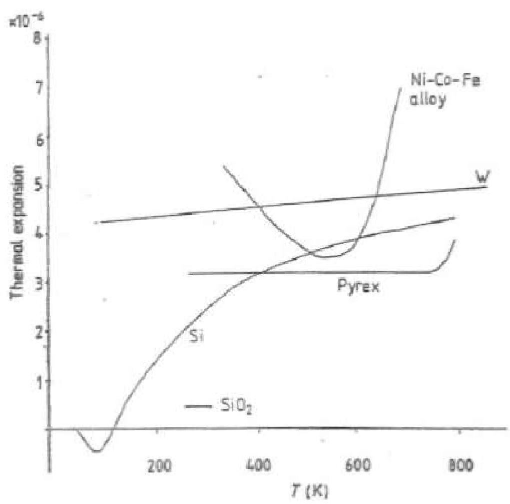
- As temperature increases, most solids expand in volume
- Definition: linear thermal expansion coefficient

Linear thermal expansion coefficient $\triangleq \alpha_T = \dfrac{d\varepsilon_x}{dT}$ [Kelvin^{-1}]

Remarks:

- α_T values tend to be in the 10^{-6} to 10^{-7} range
- Can capture the 10^{-6} by using dimensions of μstrain/K, where 10^{-6} K^{-1} = 1 μstrain/K
- In 3D, get volume thermal expansion coefficient → $\dfrac{\Delta V}{V} = 3\alpha_T \Delta T$
- For moderate temperature excursions, α_T can be treated as a constant of the material, but in actuality, it is a function of temperature

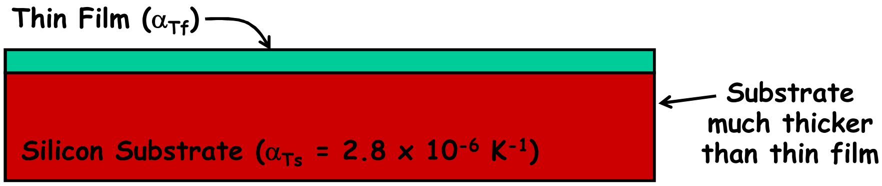
α_T As a Function of Temperature



[Madou, Fundamentals of Microfabrication, CRC Press, 1998]

- Cubic symmetry implies that α is independent of direction

Thin-Film Thermal Stress



- Assume film is deposited stress-free at a temperature T_r, then the whole thing is cooled to room temperature T_r
- Substrate much thicker than thin film → substrate dictates the amount of contraction for both it and the thin film

Thermal strain of the substrate: (in one in-plane dimension)

$$\epsilon_S = -\alpha_{Ts}\Delta T \quad , \text{ where } \Delta T = T_d - T_r$$

If the film were not attached to the substrate: $\epsilon_{f,free} = -\alpha_{Tf}\Delta T$ ↻ over

Linear Thermal Expansion

But the film is attached to the substrate, so the actual strain in the film is the same as that in the substrate.

$$\epsilon_{f,attached} = -\alpha_{Ts}\Delta T$$

Thus:

$$\boxed{\text{Thermal Mismatch Strain} = \epsilon_{f,mismatch} = (\alpha_{Tf} - \alpha_{Ts})\Delta T}$$

↳ Note that this is biaxial strain
↳ it can only be developed by an in-plane biaxial stress:

$$\sigma_{f,mismatch} = \left(\frac{E}{1-\nu}\right)\epsilon_{f,mismatch}$$

Ex. Thin-film is polyimide → $\alpha_{Tf} = 70\times10^{-6}\,K^{-1}$, $E' = 4GPa$ (e.g., SiO_2)
deposited @ 250°C, then cooled to RT = 25°C → $\Delta T = 225K$

$$\epsilon_{f,mismatch} = (70-2.8)\mu(225) = 1.5\times10^{-2}$$

↳ $\sigma_{f,mismatch} = (4G)(1.5\times10^{-2}) = \underline{60.5MPa}$ ← stress is (+), ∴ tensile [(-) would be compressive]

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MEMS Material Properties

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Material Properties for MEMS

Material	Density, ρ, Kg/m³	Modulus, E, GPa	E/ ρ GN/kg-m
Silicon	2330	165	72
Silicon Oxide	2200	73	36
Silicon Nitride	3300	304	92
Nickel	8900	207	23
Aluminum	2710	69	25
Aluminum Oxide	3970	393	99
Silicon Carbide	3300	430	130
Diamond	3510	1035	295

Units: $(m/s)^2$

$\sqrt{(E/\rho)}$ is acoustic velocity

[Mark Spearing, MIT]

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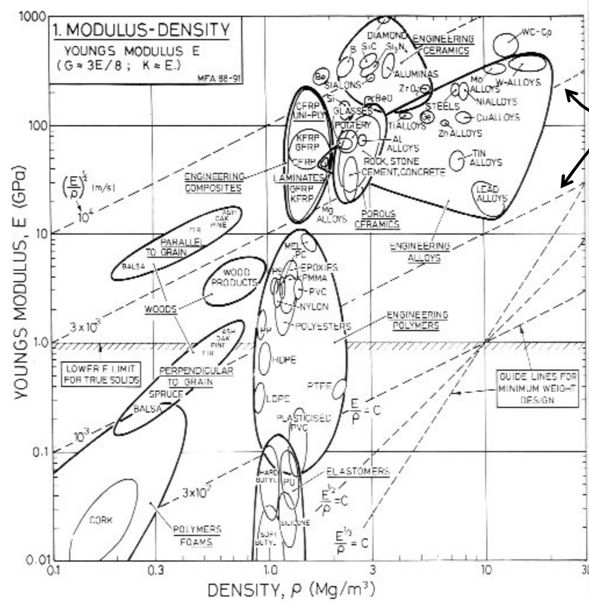
Young's Modulus Versus Density



Lines of constant acoustic velocity

[Ashby, Mechanics of Materials, Pergamon, 1992]

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Yield Strength

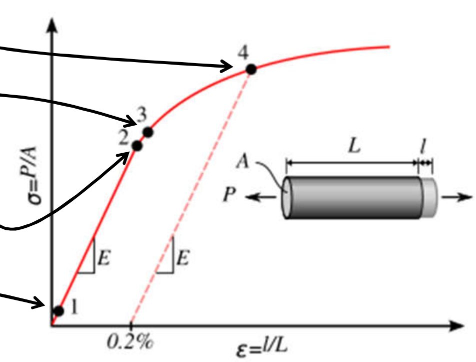
- Definition: the stress at which a material experiences significant plastic deformation (defined at 0.2% offset pt.)
- Below the yield point: material deforms elastically → returns to its original shape when the applied stress is removed
- Beyond the yield point: some fraction of the deformation is permanent and non-reversible

Yield Strength: defined at 0.2% offset pt.

Elastic Limit: stress at which permanent deformation begins

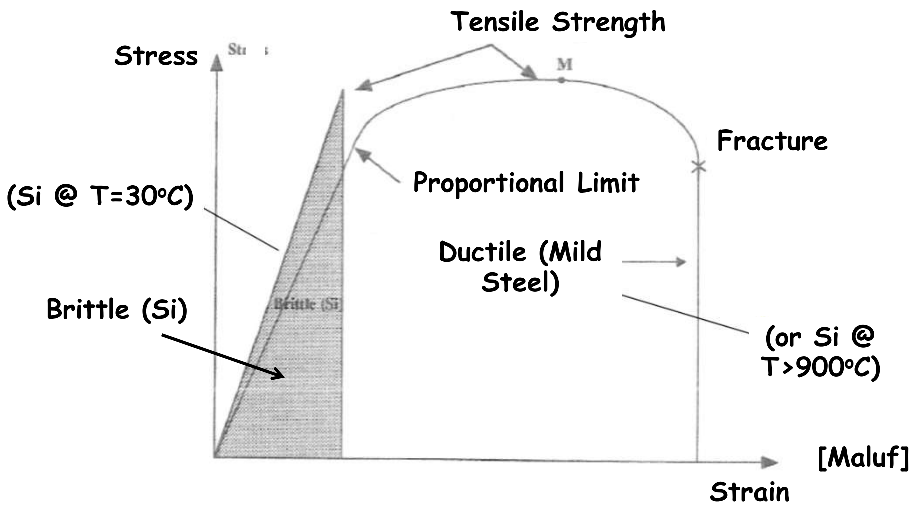
Proportionality Limit: point at which curve goes nonlinear

True Elastic Limit: lowest stress at which dislocations move



Yield Strength (cont.)

- Below: typical stress vs. strain curves for brittle (e.g., Si) and ductile (e.g. steel) materials



Young's Modulus and Useful Strength

Stored mechanical energy

Material	Modulus, E, GPa	Useful Strength*, σ_f, MPa	$\frac{\sigma_f}{E}$ (-) x 10^{-3}	$\frac{\sigma_f^2}{E}$ MJ/m³
Silicon	165	4000	24	97
Silicon Oxide	73	1000	13	14
Silicon Nitride	304	1000	3	4
Nickel	207	500	2	1.2
Aluminum	69	300	4	1.3
Aluminum Oxide	393	2000	5	10
Silicon Carbide	430	2000	4	9.3
Diamond	1035	1000	1	0.9

From Mark Spearing, MIT, *Future of MEMS Workshop*, Cambridge, England, May 2003

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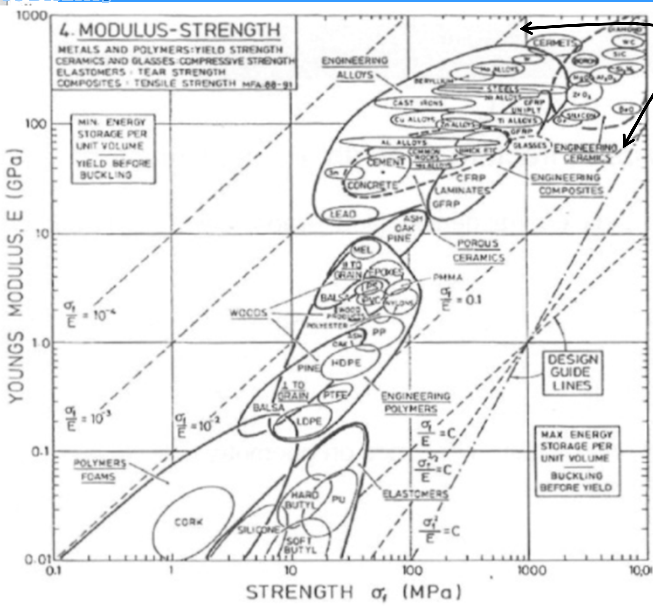
Young's Modulus Versus Strength



Lines of constant maximum strain

[Ashby, Mechanics of Materials, Pergamon, 1992]

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Quality Factor (or Q)

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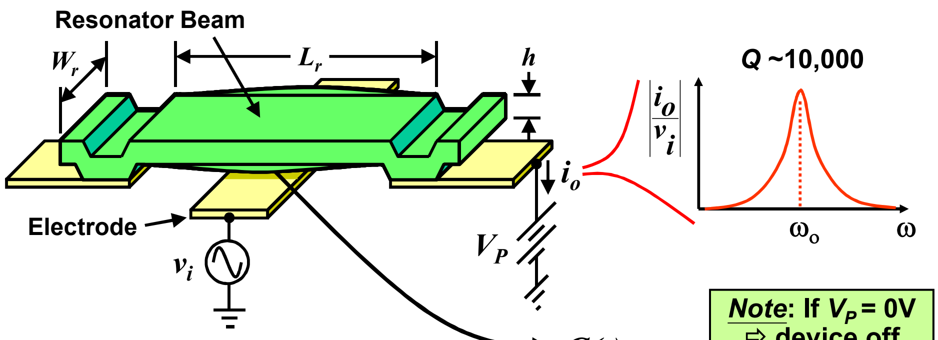
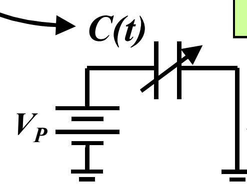
Clamped-Clamped Beam μResonator





$$i_o = V_P \frac{dC}{dt}$$

Note: If V_P = 0V ⇨ device off

Frequency:

$$f_o = \frac{1}{2\pi}\sqrt{\frac{k_r}{m_r}} = 1.03\sqrt{\frac{E}{\rho}}\frac{h}{L_r^2}$$

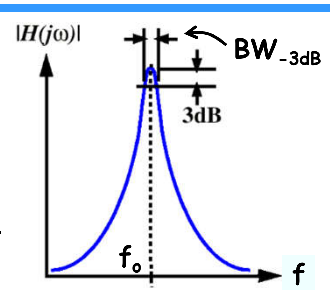
Stiffness (k_r), Mass (m_r), Young's Modulus (E), Density (ρ)

Mass ➡ (e.g., m_r = 10^{-13} kg) ➡ Smaller mass ⇨ higher freq. range and lower series R_x

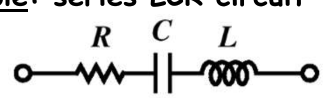
Quality Factor (or Q)

- Measure of the frequency selectivity of a tuned circuit
- Definition:

$$Q = \frac{\text{Total Energy Per Cycle}}{\text{Energy Lost Per Cycle}} = \frac{f_o}{BW_{3dB}}$$

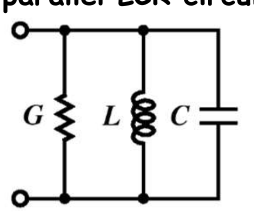


- Example: series LCR circuit

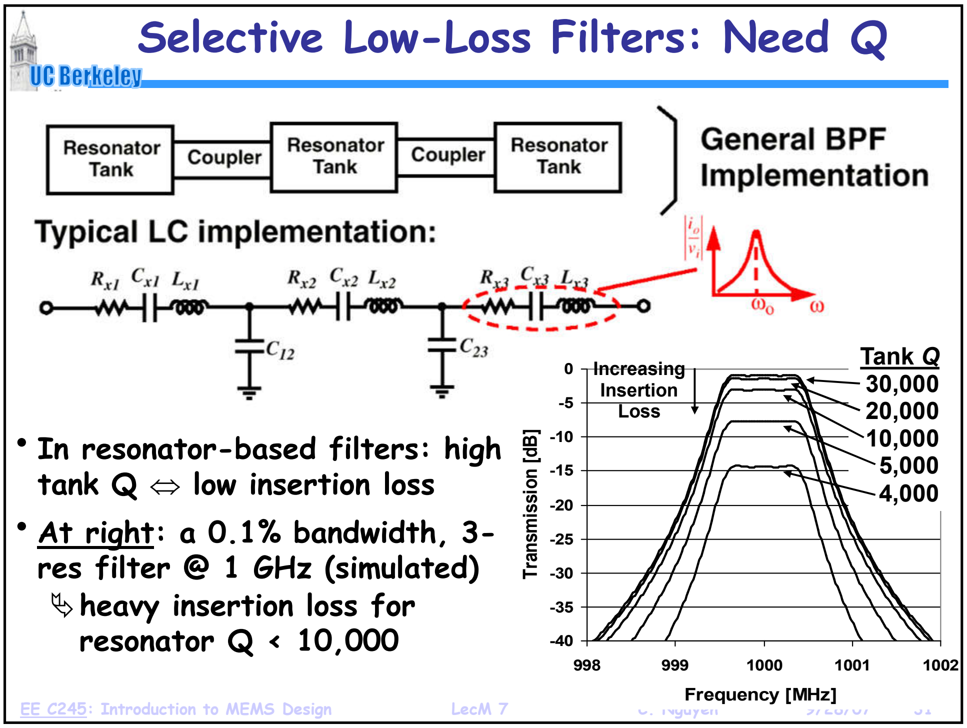


$$\Rightarrow \quad Q = \frac{\mathrm{Im}(Z)}{\mathrm{Re}(Z)} = \frac{\omega_o L}{R} = \frac{1}{\omega_o CR}$$

- Example: parallel LCR circuit



$$\Rightarrow \quad Q = \frac{\mathrm{Im}(Y)}{\mathrm{Re}(Y)} = \frac{\omega_o C}{G} = \frac{1}{\omega_o LG}$$


Selective Low-Loss Filters: Need Q
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Resonator Tank
Coupler
Resonator Tank
Coupler
Resonator Tank
General BPF Implementation
Typical LC implementation:
R_{x1} C_{x1} L_{x1}
R_{x2} C_{x2} L_{x2}
R_{x3} C_{x3} L_{x3}
C_{12}
C_{23}
$\left|\frac{i_o}{v_i}\right|$
ω_o
ω
• In resonator-based filters: high tank Q ⇔ low insertion loss
• At right: a 0.1% bandwidth, 3-res filter @ 1 GHz (simulated)
↳ heavy insertion loss for resonator Q < 10,000
Increasing Insertion Loss
Tank Q
30,000
20,000
10,000
5,000
4,000
Transmission [dB]
0
-5
-10
-15
-20
-25
-30
-35
-40
998
999
1000
1001
1002
Frequency [MHz]
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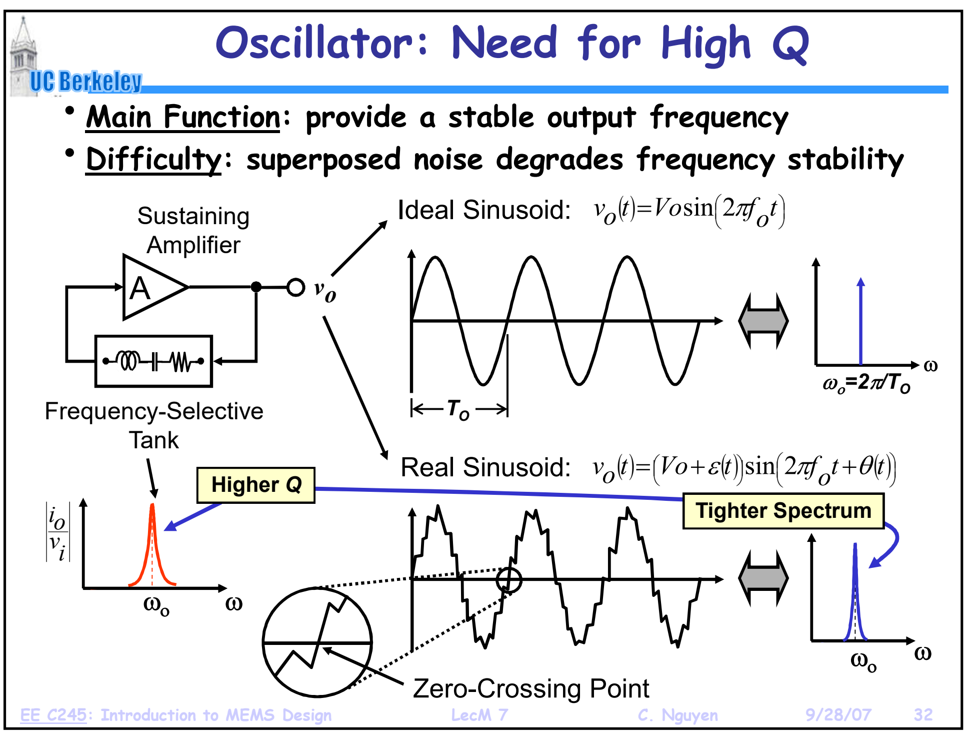
Oscillator: Need for High Q
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• Main Function: provide a stable output frequency
• Difficulty: superposed noise degrades frequency stability
Sustaining Amplifier
A
v_o
Frequency-Selective Tank
Ideal Sinusoid: $v_o(t)=Vo\sin(2\pi f_o t)$
T_o
$\omega_o=2\pi/T_o$
ω
Real Sinusoid: $v_o(t)=(Vo+\varepsilon(t))\sin(2\pi f_o t+\theta(t))$
Higher Q
Tighter Spectrum
$\left|\frac{i_o}{v_i}\right|$
ω_o
ω
Zero-Crossing Point
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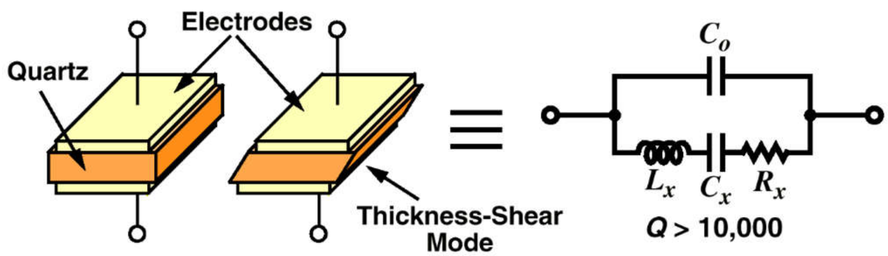
Attaining High Q

- Problem: IC's cannot achieve Q's in the thousands
  - transistors ⇨ consume too much power to get Q
  - on-chip spiral inductors ⇨ Q's no higher than ~10
  - off-chip inductors ⇨ Q's in the range of 100's
- Observation: vibrating mechanical resonances ⇨ Q > 1,000
- Example: quartz crystal resonators (e.g., in wristwatches)
  - extremely high Q's ~ 10,000 or higher (Q ~ 10^6 possible)
  - mechanically vibrates at a distinct frequency in a thickness-shear mode



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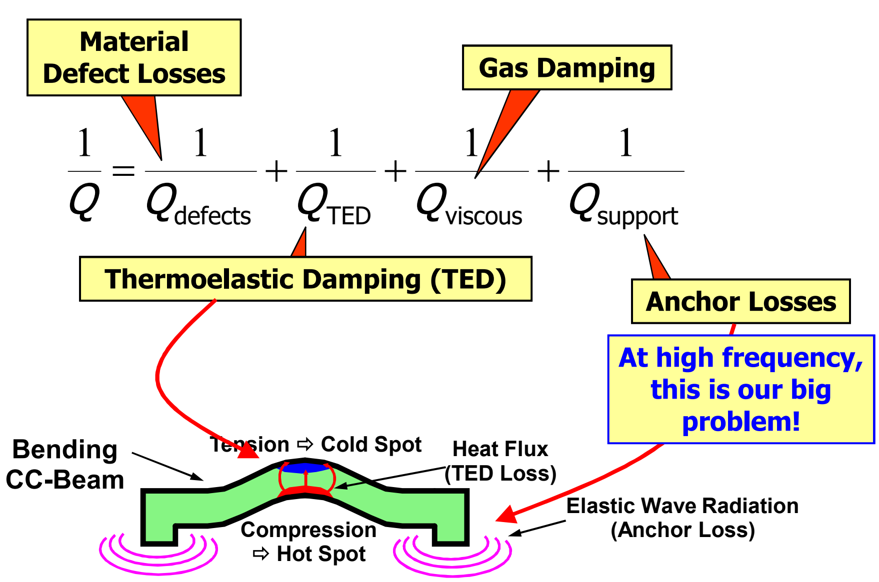
Energy Dissipation and Resonator Q

$$\frac{1}{Q} = \frac{1}{Q_{defects}} + \frac{1}{Q_{TED}} + \frac{1}{Q_{viscous}} + \frac{1}{Q_{support}}$$



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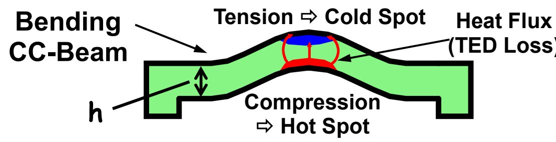
Thermoelastic Damping (TED)

- Occurs when heat moves from compressed parts to tensioned parts → heat flux = energy loss



$$\varsigma = \Gamma(T)\Omega(f) = \frac{1}{2Q}$$

$$\Gamma(T) = \frac{\alpha^2 TE}{4\rho C_p}$$

$$\Omega(f_o) = 2\left[\frac{f_{TED} f}{f_{TED}^2 + f^2}\right]$$

$$f_{TED} = \frac{\pi K}{2\rho C_p h^2}$$

ζ = thermoelastic damping factor
α = thermal expansion coefficient
T = beam temperature
E = elastic modulus
ρ = material density
C_p = heat capacity at const. pressure
K = thermal conductivity
f = beam frequency
h = beam thickness
f_{TED} = characteristic TED frequency

TED Characteristic Frequency

$$f_{TED} = \frac{\pi K}{2\rho C_p h^2}$$

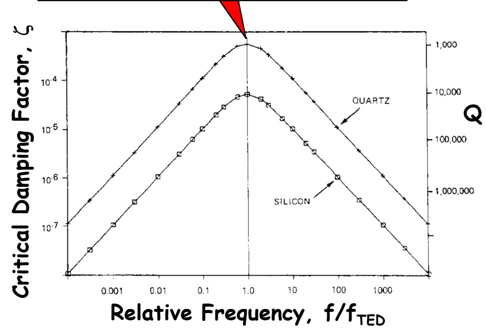
ρ = material density
C_p = heat capacity at const. pressure
K = thermal conductivity
h = beam thickness
f_{TED} = characteristic TED frequency

- Governed by
  - Resonator dimensions
  - Material properties

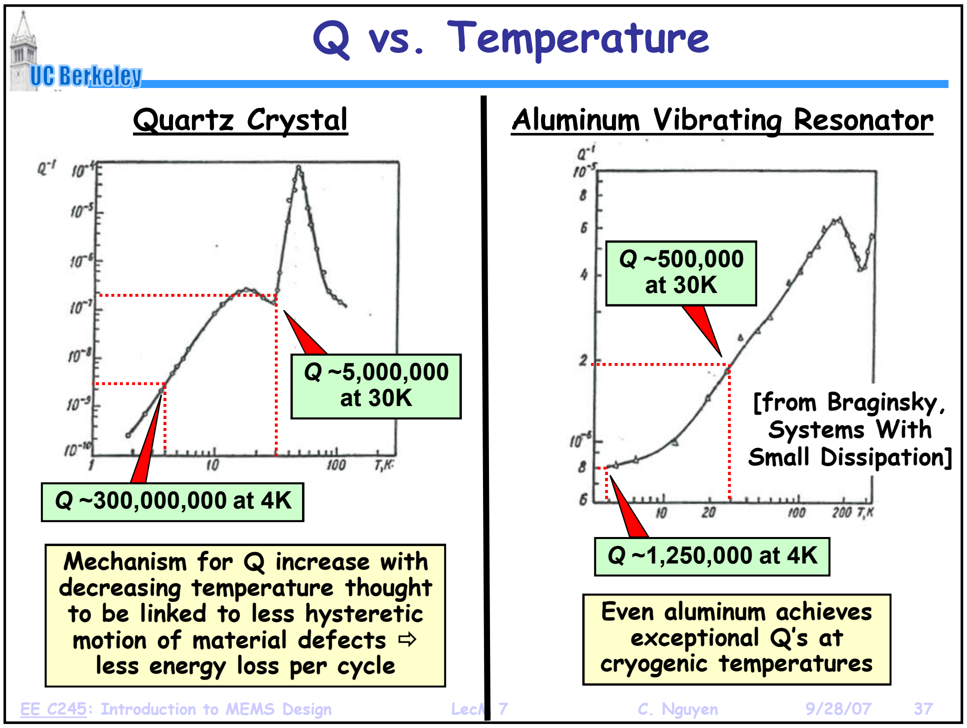
TABLE 1. MATERIAL PROPERTIES

Property	Silicon	Quartz	Units
Thermal expansion	2.60	13.70	ppm/°K
Elastic modulus	1.70	0.78	10^{12} dyne/cm^2
Material density	2.33	2.60	g/cm^3
Heat capacity	0.70	0.75	J/g/°K
Thermal conductivity	1.50	0.10	10^7 dyne/°K/s
Peak damping @ 300°k	1.06	11.34	10^{-4}

Peak where Q is minimized



[from Roszhart, Hilton Head 1990]


Q vs. Temperature
UC Berkeley
Quartz Crystal
Aluminum Vibrating Resonator
Q ~500,000 at 30K
Q ~5,000,000 at 30K
Q ~300,000,000 at 4K
[from Braginsky, Systems With Small Dissipation]
Q ~1,250,000 at 4K
Mechanism for Q increase with decreasing temperature thought to be linked to less hysteretic motion of material defects ⇨ less energy loss per cycle
Even aluminum achieves exceptional Q's at cryogenic temperatures
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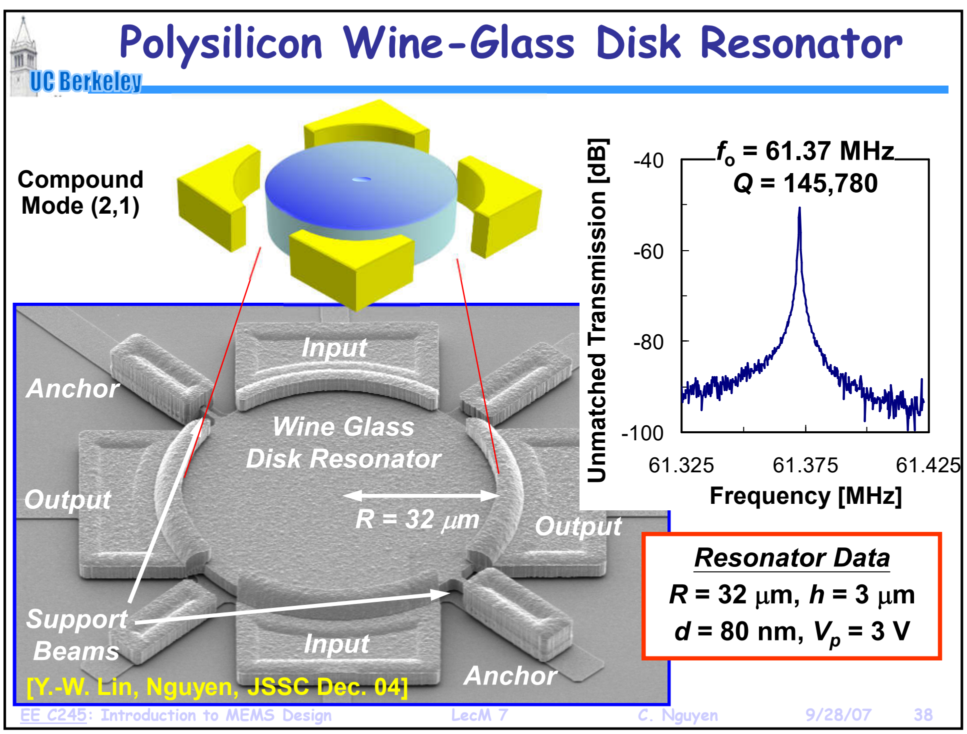
Polysilicon Wine-Glass Disk Resonator
UC Berkeley
Compound Mode (2,1)
f_o = 61.37 MHz
Q = 145,780
Unmatched Transmission [dB]
-40
-60
-80
-100
61.325
61.375
61.425
Frequency [MHz]
Input
Anchor
Wine Glass Disk Resonator
Output
R = 32 μm
Output
Support Beams
Input
Anchor
[Y.-W. Lin, Nguyen, JSSC Dec. 04]
Resonator Data
R = 32 μm, h = 3 μm
d = 80 nm, V_p = 3 V
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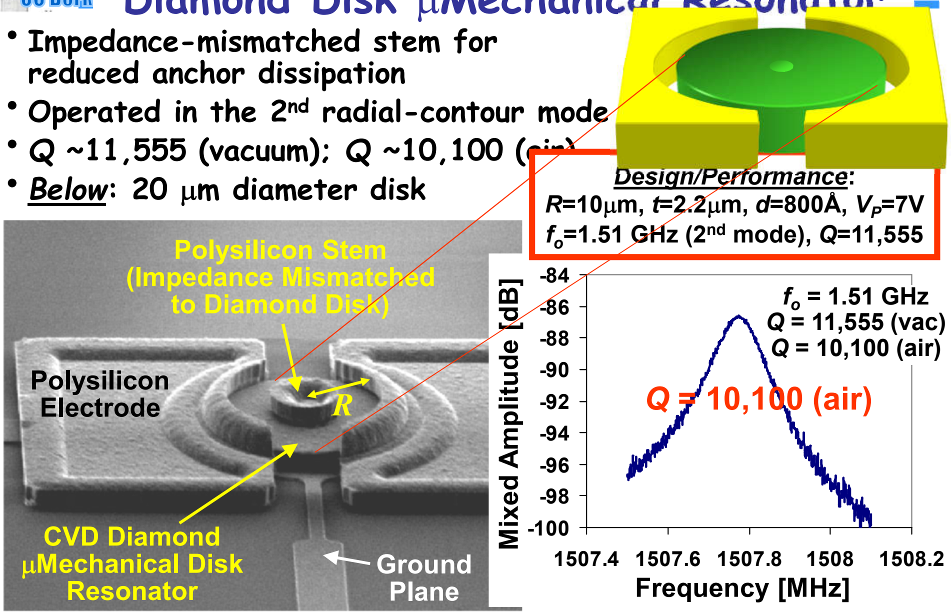
1.51-GHz, Q=11,555 Nanocrystalline Diamond Disk μMechanical Resonator

- Impedance-mismatched stem for reduced anchor dissipation
- Operated in the 2nd radial-contour mode
- Q ~11,555 (vacuum); Q ~10,100 (air)
- *Below*: 20 μm diameter disk



[Wang, Butler, Nguyen MEMS'04]

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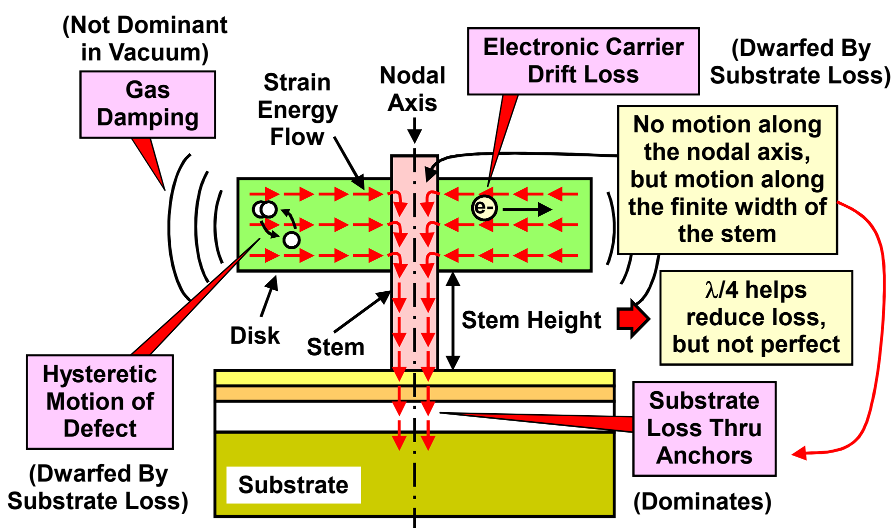
Disk Resonator Loss Mechanisms



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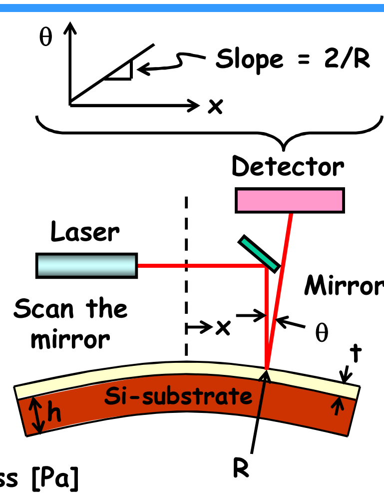
MEMS Material Property Test Structures

Stress Measurement Via Wafer Curvature

- Compressively stressed film → bends a wafer into a convex shape
- Tensile stressed film → bends a wafer into a concave shape
- Can optically measure the deflection of the wafer before and after the film is deposited
- Determine the radius of curvature R, then apply:



$$\sigma = \frac{E' h^2}{6Rt}$$

σ = film stress [Pa]
E′ = E/(1-ν) = biaxial elastic modulus [Pa]
h = substrate thickness [m]
t = film thickness
R = substrate radius of curvature [m]

MEMS Stress Test Structure

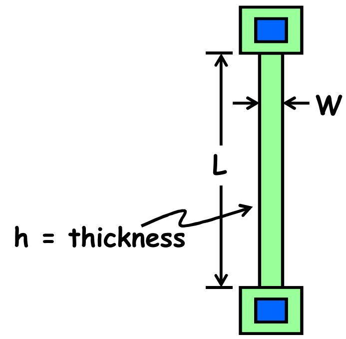
UC Berkeley

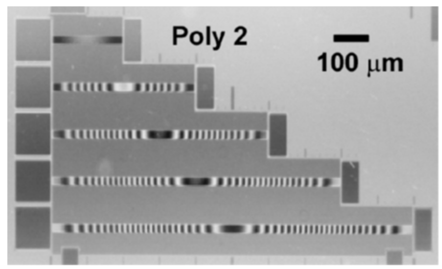
- Simple Approach: use a clamped-clamped beam
  - Compressive stress causes buckling
  - Arrays with increasing length are used to determine the critical buckling load, where

$$\sigma_{critical} = -\frac{\pi^2}{3}\frac{Eh^2}{L^2}$$

E = Young's modulus [Pa]
I = (1/12)Wh^3 = moment of inertia
L, W, h indicated in the figure

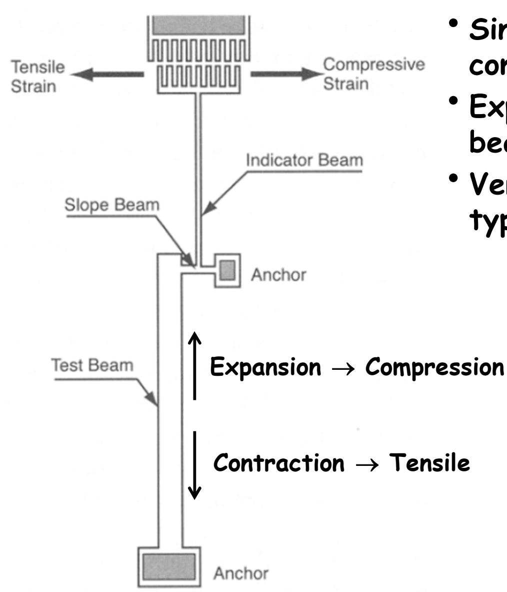
- Limitation: Only compressive stress is measurable



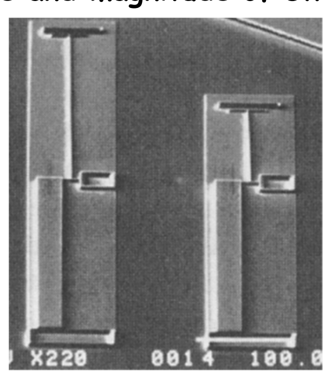


More Effective Stress Diagnostic

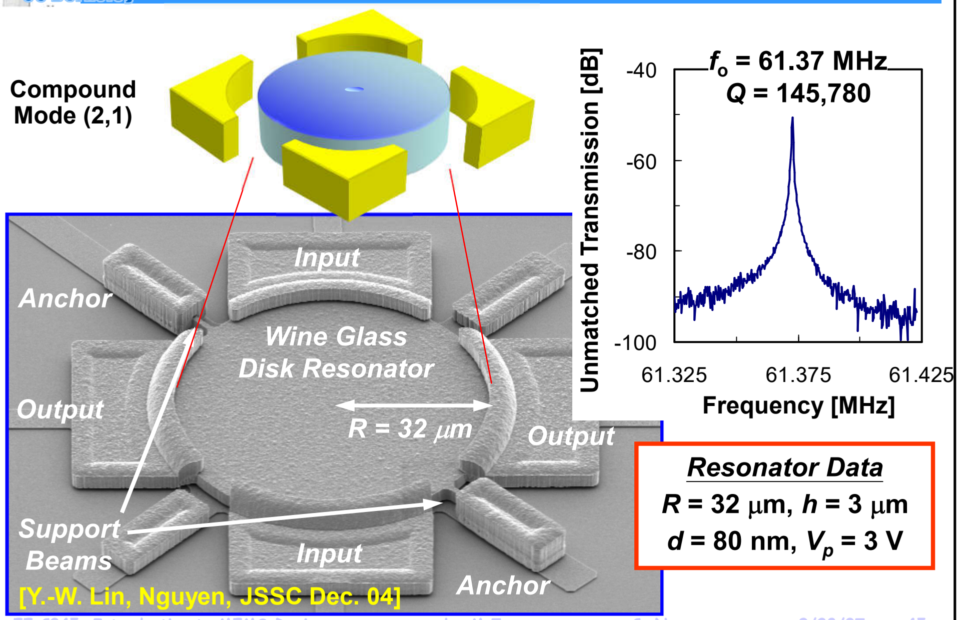
UC Berkeley



- Single structure measures both compressive and tensile stress
- Expansion or contraction of test beam → deflection of pointer
- Vernier movement indicates type and magnitude of stress



Q Measurement Using Resonators



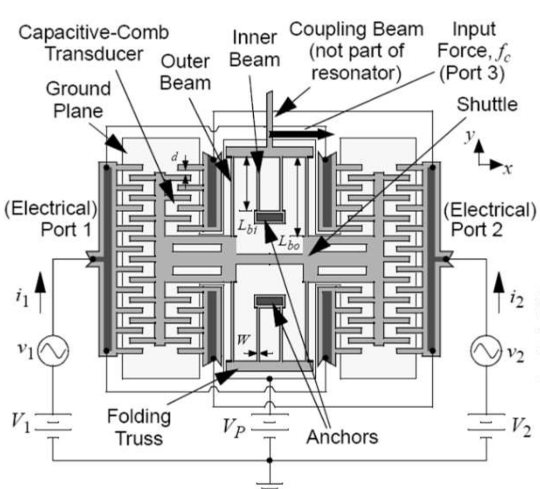
Resonator Data

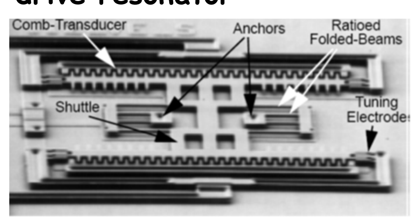
R = 32 μm, h = 3 μm

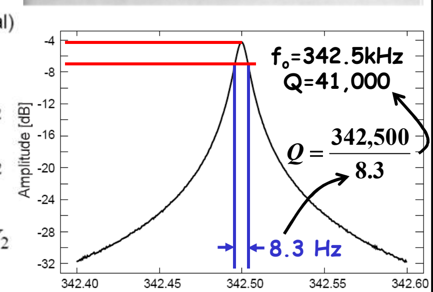
d = 80 nm, V_p = 3 V

Folded-Beam Comb-Drive Resonator

- Issue w/ Wine-Glass Resonator: non-standard fab process
- Solution: use a folded-beam comb-drive resonator

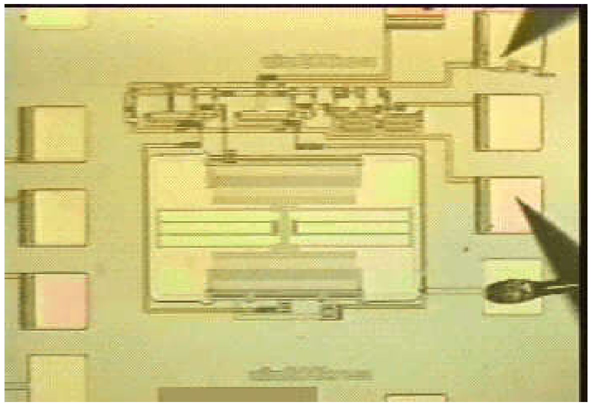






Comb-Drive Resonator in Action

- Below: fully integrated micromechanical resonator oscillator using a MEMS-last integration approach



Folded-Beam Comb-Drive Resonator

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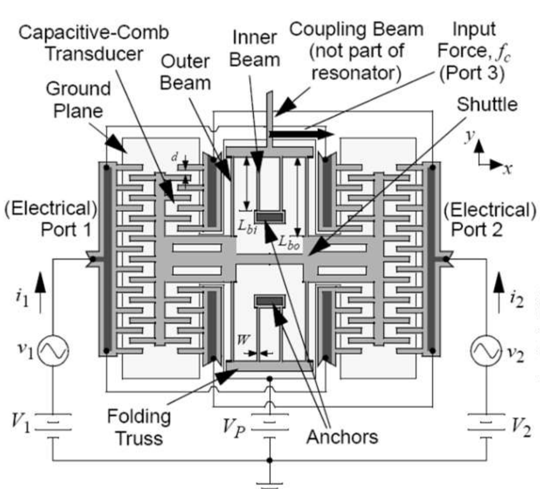
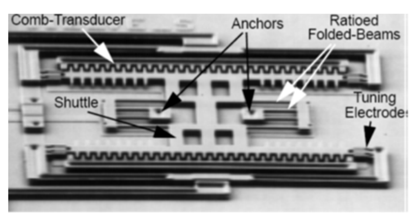
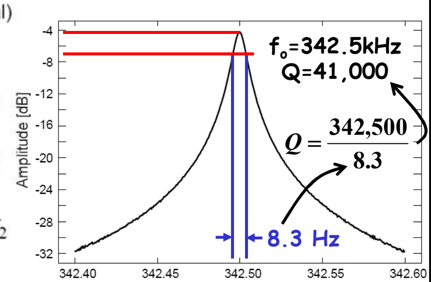
- Solution: use a folded-beam comb-drive resonator







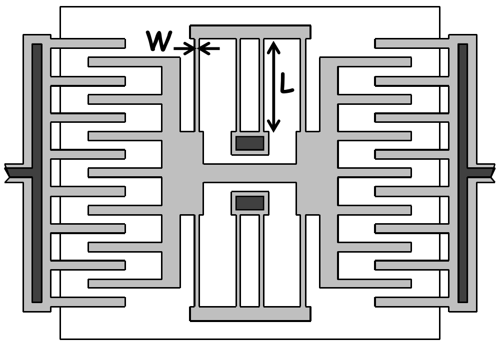
Measurement of Young's Modulus

- Use micromechanical resonators
  - Resonance frequency depends on E
  - For a folded-beam resonator:

$$\text{Resonance Frequency} = f_o = \left[\frac{4Eh(W/L)^3}{M_{eq}}\right]^{1/2}$$

(E: Young's modulus; M_{eq}: Equivalent mass)

h = thickness



- Extract E from measured frequency f_o
- Measure f_o for several resonators with varying dimensions
- Use multiple data points to remove uncertainty in some parameters

Anisotropic Materials

Elastic Constants in Crystalline Materials

- Get different elastic constants in different crystallographic directions → 81 of them in all
  - Cubic symmetries make 60 of these terms zero, leaving 21 of them remaining that need be accounted for
- Thus, describe stress-strain relations using a 6x6 matrix

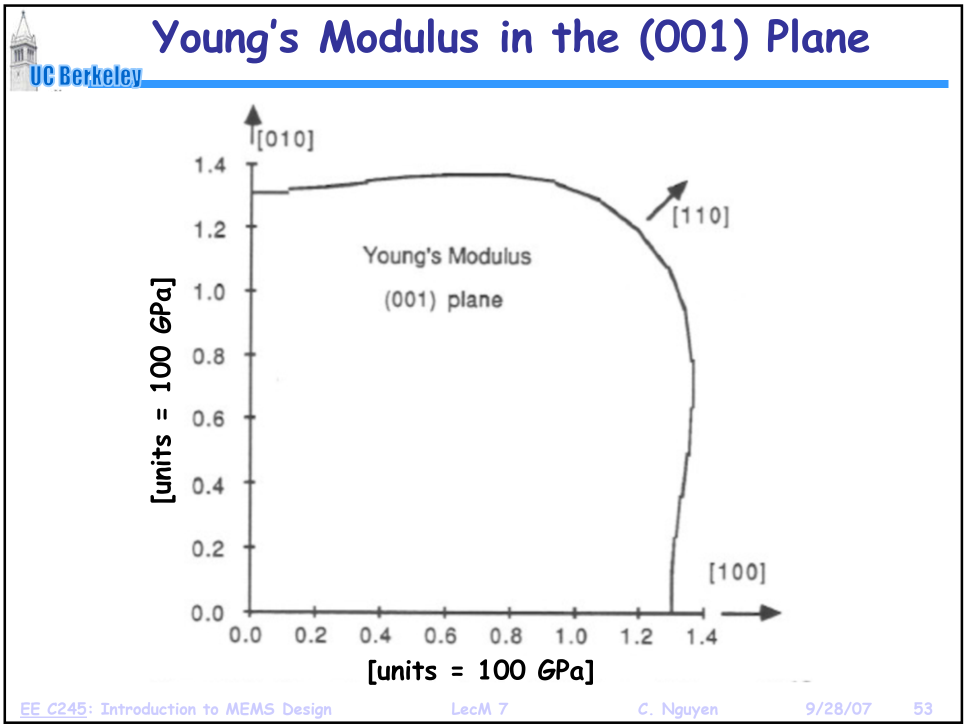
$$\underbrace{\begin{bmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{yz}\\ \tau_{zx}\\ \tau_{xy}\end{bmatrix}}_{\text{Stresses}} = \underbrace{\begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16}\\ C_{12} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26}\\ C_{13} & C_{23} & C_{33} & C_{34} & C_{35} & C_{36}\\ C_{14} & C_{24} & C_{34} & C_{44} & C_{45} & C_{46}\\ C_{15} & C_{25} & C_{35} & C_{45} & C_{55} & C_{56}\\ C_{16} & C_{26} & C_{36} & C_{46} & C_{56} & C_{66}\end{bmatrix}}_{\text{Stiffness Coefficients}} \underbrace{\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \gamma_{yz}\\ \gamma_{zx}\\ \gamma_{xy}\end{bmatrix}}_{\text{Strains}}$$

Stiffness Coefficients of Silicon

- Due to symmetry, only a few of the 21 coefficients are non-zero
- With cubic symmetry, silicon has only 3 independent components, and its stiffness matrix can be written as:

$$\begin{bmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{yz}\\ \tau_{zx}\\ \tau_{xy}\end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0\\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0\\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0\\ 0 & 0 & 0 & C_{44} & 0 & 0\\ 0 & 0 & 0 & 0 & C_{55} & 0\\ 0 & 0 & 0 & 0 & 0 & C_{66}\end{bmatrix} \begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \gamma_{yz}\\ \gamma_{zx}\\ \gamma_{xy}\end{bmatrix}$$

where
$$\begin{cases} C_{11} = 165.7\ \text{GPa}\\ C_{12} = 63.9\ \text{GPa}\\ C_{44} = 79.6\ \text{GPa}\end{cases}$$


UC Berkeley
Young's Modulus in the (001) Plane
[010]
[110]
[100]
Young's Modulus
(001) plane
[units = 100 GPa]
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
[units = 100 GPa]
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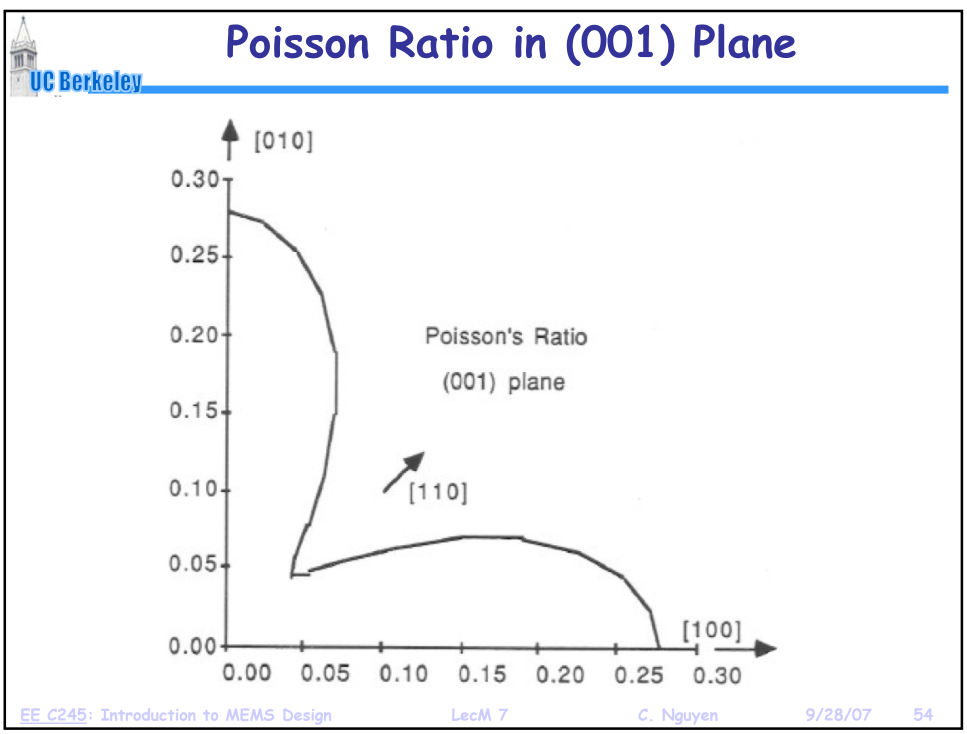
UC Berkeley
Poisson Ratio in (001) Plane
[010]
[110]
[100]
Poisson's Ratio
(001) plane
0.00
0.05
0.10
0.15
0.20
0.25
0.30
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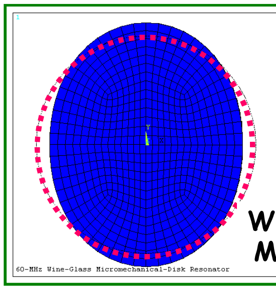
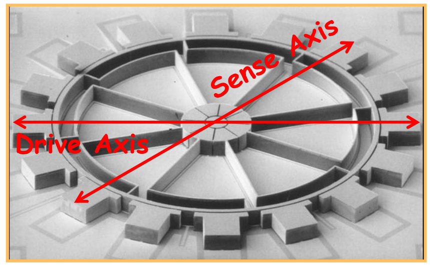
Anisotropic Design Implications

UC Berkeley

- Young's modulus and Poisson ratio variations in anisotropic materials can pose problems in the design of certain structures
- E.g., disk or ring resonators, which rely on isotropic properties in the radial directions
  - Okay to ignore variation in RF resonators, although some Q hit is probably being taken
- E.g., ring vibratory rate gyroscopes
  - Mode matching is required, where frequencies along different axes of a ring must be the same
  - Not okay to ignore anisotropic variations, here



Wine-Glass Mode Disk



Ring Gyroscope

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