EE 247B / ME 218 Discussion 2

Kieran Peleaux
February 7th, 2020
The circuit problems we’ve looked at have been mostly thermal
Inputs have been convenient power sources (like lasers)
Is this actually useful? How is thermal energy typically coupled into a thermal body/MEMS?

Electrical power through resistive heating!
Power Dissipated in a Resistor

- Remember Ohm's law!

\[ V = I \cdot R, \quad I = \frac{V}{R} \]

\[ P = I \cdot V \]

\[ P = \frac{V^2}{R} = I^2R \]

Most of this energy is thermal!


**REFRESHER: ELECTRICAL RESISTANCE**

\[ R = \frac{l}{\sigma \cdot A} \]

- resistance (Ω)
- conductivity (Ω m⁻¹)
- length (m)
- cross-sectional area (m²)

\[ \sigma = \frac{l}{\sigma \cdot A} \]

- resistivity

\[ R = \frac{L}{WH} \]

- number of squares
How do we handle circuits with multiple inputs (i.e., sources)?

1. Pick a source to analyze
2. Suppress (turn off) all other independent sources
   - Set either V or I to zero
     * short-circuit voltage sources & open-circuit current sources
3. Find the output of interest for the modified circuit
4. Repeat steps 1-3 for all sources
5. Sum all resultant outputs to find total output due to all sources

\[ V_R = V_{R1} + V_{R2} \]
• $V_A$ represents a step function voltage source (can think of it as switching the voltage on at some time, $t$)
• Find the time-constant of the circuit
• Find the steady-state temperature on the shuttle if the final value of $V_A$ is 1V

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THERMAL CKT. EXAMPLE

PARAMETER | VALUE | UNIT
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Young’s Modulus, $E$ | 150 | GPa
Density, $\rho$ | 2,300 | kg/m³
Poisson Ratio, $\nu$ | 0.226 | -
Sheet Resistance, $R_s$ | 10 | $\Omega$/$\square$
Specific Heat, $c_p$ | 770 | J/kg.K
Thermal Conductivity, $k$ | 30 | W/m.K
Beam Length/Width/Thickness, $L/W/H$ | 50/2/2 | $\mu$m/$\mu$m/$\mu$m
Folding Truss Area, $A_f$ | 250 | $\mu$m²
Shuttle Area, $A_s$ | 8,000 | $\mu$m²

$R_{s,b} = R_s \frac{L}{W} = \left( \frac{10 \Omega}{\square} \right) \left( \frac{50 \mu\text{m}}{2 \mu\text{m}} \right) = 250 \Omega$

$P_b = I_A \cdot V_b = \left( \frac{I_A}{2} \right) \cdot R_{s,b}$

$I_A = \frac{V_A}{(1V)} = \frac{1V}{2(250 \Omega)} = 2mA$

$P_b = (1mA)^2 \left( \frac{250 \Omega}{2} \right) = 250 \mu$W
**THERMAL Ckt. EXAMPLE**

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*(Diagram and additional text not included in this representation)*
THERMAL Ckt. EXAMPLE

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Beam Length/Width/Thickness, \( L/W/H \) | 50/2/2 | \( \mu m/\mu m/\mu m \)
Folding Truss Area, \( A_f \) | 250 | \( \mu m^2 \)
Shuttle Area, \( A_s \) | 8,000 | \( \mu m^2 \)

\[
R_{th,b} = \frac{L}{k(W \cdot H)} = \frac{50\mu m}{(30\Omega \cdot \square)(2\mu m)^2} = 4.17 \times 10^5 \frac{K}{W}
\]

```
R_{th,b}/8
1
4P_b
V_{\text{in}}
R_{th,b}/4
1
2
4P_b
R_{th,b}/8
C_{th}
```

"Fold" the ckt. across this line of symmetry: 

\[
V_{\text{in}} = 4P_b \cdot \frac{R_{th,b}}{8} = \frac{(250\mu m)(4.17 \times 10^5 K)}{W} = 52.1K = \Delta T_1
\]
**THERMAL Ckt. Example**

\[ \Delta T_2 = 4P_b \cdot \frac{3}{8} R_{th,b} = 156.3 \text{ K} \]

\[ T_{shuttle,ss} = 298 \text{ K} + \Delta T_1 + \Delta T_2 = 506.3 \text{ K} \]

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\[ C = C_{th} \cdot \frac{R_{th,b}}{2} \]

\[ C_{th} = \frac{A_s \cdot H \cdot c_p}{1} \]

\[ C_{th} = 2.83 \times 10^{-8} \text{ J/K} \]

\[ \text{Time constant} = \frac{R_{th,b}}{4} \]

\[ \tau = \left( 2.83 \times 10^{-8} \text{ J/K} \right) \left( \frac{4.17 \times 10^5 \text{ W}}{2} \right) \]

\[ \tau = 5.9 \text{ ms} \]

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EE C247B/ME C218 | K. Peleaux | Spring 2020