Outline

- Reading: Senturia, Chpt. 8
- Lecture Topics:
  - Stress, strain, etc., for isotropic materials
  - Thin films: thermal stress, residual stress, and stress gradients
  - Internal dissipation
  - MEMS material properties and performance metrics

Vertical Stress Gradients

- Variation of residual stress in the direction of film growth
- Can warp released structures in z-direction

Elasticity
Normal Stress (1D)

If the force acts normal to a surface, then the stress is called a normal stress.

\[ \sigma = \frac{F}{A} \text{ [N/m}^2\text{, Pa]} \]

Microscopic definition: force per unit area acting on the surface of a differential volume element of a solid body.

\[ \varepsilon = \frac{\Delta L}{L} \text{, \(\varepsilon_{\text{strain}}\)} \]

Strain (1D)

In the elastic regime (i.e., for "small" stresses at "low" temperatures), strain is found to be proportional to stress.

\[ \sigma = E \varepsilon \]

The Poisson Ratio

Apply normal stress to a free-standing object but also get contraction in directions transverse to the uniaxial strain.

\[ \varepsilon_y = \frac{W - W'}{W'} = -\gamma \varepsilon_x \]

Shear Stress & Strain (1D)

Shear stress:

\[ \tau_{xy} = \frac{F_{\text{shear}}}{A_{\text{parallel}}} \text{ [Pa]} \]

Shear strain:

\[ \gamma = \frac{\Delta L}{L} \text{, \(\gamma_{\text{shear}}\)} \]

\[ \tau = \frac{E}{2(1+\nu)} \]

\[ G = \frac{E}{2(1+\nu)} \]

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2D and 3D Considerations

* Important assumption: the differential volume element is in static equilibrium → no net forces or torques (i.e., rotational movements)
  - Every σ must have an equal σ in the opposite direction on the other side of the element
  - For no net torque, the shear forces on different faces must also be matched as follows:
    \[ \tau_{xy} = \tau_{yx}, \tau_{xz} = \tau_{zx}, \tau_{yz} = \tau_{zy} \]

Stresses acting on a differential volume element

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2D Strain

* In general, motion consists of
  - rigid-body displacement (motion of the center of mass)
  - rigid-body rotation (rotation about the center of mass)
  - Deformation relative to displacement and rotation

Area element experiences both displacement and deformation

* Must work with displacement vectors

* Differential definition of axial strain:
  \[ \varepsilon_x = \frac{u_x(x + \Delta x) - u_x(x)}{\Delta x} = \frac{\partial u_x}{\partial x} \]

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2D Shear Strain

For shear strains, must remove any rigid body rotation that accompanies the deformation

* Use a symmetric definition of shear strain:
  \[ \tau_{xy} = \theta_z + \theta_1 \approx \frac{\Delta u_y}{\Delta x} + \frac{\Delta u_x}{\Delta y} \]

For small amplitude deformations:

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Isotropic Elasticity in 3D

* Isotropic = same in all directions

* The complete stress-strain relations for an isotropic elastic solid in 3D: (i.e., a generalized Hooke’s Law)
  \[
  \begin{align*}
  \varepsilon_x &= \frac{1}{E} \left[ \sigma_x - \nu(\sigma_y + \sigma_z) \right] \\
  \gamma_{xy} &= \frac{1}{G} \tau_{xy} \\
  \varepsilon_y &= \frac{1}{E} \left[ \sigma_y - \nu(\sigma_z + \sigma_x) \right] \\
  \gamma_{yz} &= \frac{1}{G} \tau_{yz} \\
  \varepsilon_z &= \frac{1}{E} \left[ \sigma_z - \nu(\sigma_x + \sigma_y) \right] \\
  \gamma_{zx} &= \frac{1}{G} \tau_{zx}
  \end{align*}
\]

Basically, add in off-axis strains from normal stresses in other directions
Cubic symmetry implies that $\alpha$ is independent of direction.

[Madou, Fundamentals of Microfabrication, CRC Press, 1998]