Lecture 13: Beam Combos I

- Announcements:
  - HW#3 due Tuesday, 3/10, at 8 a.m.
  - Midterm Exam about 2 weeks away, Thursday, March 19, 9:30-11:00 a.m., 521 Cory (right here)
- Reading: Senturia, Chpt. 9
- Lecture Topics:
  - Bending of beams
  - Cantilever beam under small deflections
  - Combining cantilevers in series and parallel
  - Folded suspensions
  - Design implications of residual stress and stress gradients
- Last Time:
  - Finished stress gradients
  - Now, move into beam combos

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Bending Due to Stress/Strain Gradients

Find the radius of curvature:

Prior to release, axial stress: \( \sigma = \sigma_0 - \frac{\sigma_1}{(H/2)^2} \)

The internal moment:

\[
M_x = \int_{-\frac{H}{2}}^{\frac{H}{2}} \left( W \frac{d^2 z}{dx^2} \right) \text{d}x
\]

\[
= W \left( \frac{1}{2} \sigma_0 z^2 - \frac{2 \sigma_1 z^4}{3H} \right) \bigg|_{-\frac{H}{2}}^{\frac{H}{2}}
\]

\[
= W \left( \frac{1}{2} \sigma_0 \frac{H^2}{4} - \frac{2}{3} \sigma_1 \frac{H^2}{8} - \frac{1}{2} \sigma_0 \frac{H^2}{4} - \frac{2}{3} \sigma_1 \frac{H^2}{8} \right)
\]

\[
= \frac{1}{6} \sigma_1 WH^2
\]

Thus, the radius of curvature:

\[
R = \frac{1}{E} \frac{M_x}{I} = \frac{1}{2} \frac{E}{I} \frac{H}{\sigma_1}
\]

Bi-axial Modulus

\[
R = \frac{1}{2} \frac{E}{(1-\nu) \sigma_1}
\]

Radius of Curvature for a Cantilever with Stress Gradient
... and the result:

We just quantified this
Folded-Beam Suspension

1. During deposition @ high T by stress free
2. Cool to room temp (RT) & stress

How to defend against this?
1. Adjust process parameters - e.g., the deposition recipe
   - Problem: can’t always do this
2. Design a folded-beam!

Typical Questions: all demand that we know $x = f(F)$
1. How does the structure move in response to a force at a specific location?
2. What is the frequency response to an AC force applied at a specific location?
3. Noise?
4. Response to environmental stimuli? (e.g., vibration)
5. How does stress affect the behavior of the structure?
Procedure: (to get stiffness)

1. Build the clt. (Extract the clt.) → in the x-direction
   (for this example)

\[ \begin{align*}
    &k_1 \quad m_1 \quad k_2 \quad m_2 \quad k_3 \quad m_3 \quad k_4 \quad m_4 \quad k_5 \quad m_5 \quad \ldots \\
    &k_6 \quad m_6 \quad k_7 \quad m_7 \quad k_8 \quad m_8 \quad k_9 \quad m_9 \quad \ldots
\end{align*} \]

2. Analyze to get \( x = f(F) \)
   Force
   Displacement

\[ F = kx \Rightarrow x = \frac{F}{k} \]

\[ F = \frac{E}{k} \]

\[ x = \frac{E}{k} \]

(a) Case 1: Series
\[ \begin{align*}
    &x_1 = \frac{E}{k_1} \quad x_2 = \frac{E}{k_2} \\
    &\text{across variable}
\end{align*} \]

(b) Case 2: Parallel
\[ \begin{align*}
    &x_1 = \frac{E}{k_1} + \frac{E}{k_2} \\
    &x_2 = \frac{E}{k_2} \\
    &\text{across variable}
\end{align*} \]

Note: Variables because one must go thru both
\( k_1 \) \& \( k_2 \) to get from anchor to the forces pt.

\[ k_{tot} = k_1 k_2 \] (for \( k_1 \) \& \( k_2 \) in series)

\[ k_{tot} : k_1 k_2 \]

For EE's: some or capacitors (springs combo
like capacitor)

\[ \frac{1}{C_1} + \frac{1}{C_2} \equiv \frac{1}{C_{11} + C_{22}} \]

\[ \text{m} + \text{F} \rightarrow \text{F} = k_{tot} \text{X}_{tot} \]

\[ \text{F} = k_{tot} \text{X}_{tot} \]

\[ \text{want K}_{tot} \]

\[ k_1 \quad m \quad k_2 \quad m \]

\[ F = k_1 \quad X_{tot} \quad k_2 \quad X_{tot} \]

\[ \text{inductors parallel} \]

or \( \text{only need to go through one of the springs to get} \]
\( \text{from the anchor to the} \]
\( \text{forcing pt.} \)
\[ F = F_1 + F_2 = (k_1 + k_2) \frac{y_1 y_2}{y_{tot}} \]

\[ k_{tot} = k_1 + k_2 \] (for \( k_1 \) and \( k_2 \) in parallel)

**Series Combination of Springs**

1. Anchor: Clamped B.C.
2. \( y_1 \) and \( y_2 \) are series
3. \( y_{tot} = y_1 + y_2 \) is series
4. Free B.C. (just like constrains)
5. \( F = \text{force}\) at point

Guided B.C. (maintains \( R^2 \))

- \( L = 2L_c \)
- \( L_c \)
- \( L \)
- \( y_{tot} \)
- \( y_1 \)
- \( y_2 \)
- \( y_{fre} \)
- \( F \)

\[ k_{tot} = k_1 k_2 \frac{1}{k_1 + k_2} = \frac{k_1 k_2}{k_1 + k_2} \]

- \( k_2 = k_c \)
- \( k_1 = k_c \)

\( k_1 k_2 k_c \Rightarrow k_{tot} = \frac{k_c}{2} \) constrains stiffness
Parallel Combinations of Beams

\[ k_{\text{tot}} = k_a + k_b = \frac{k_a L}{2} + \frac{k_b L}{2} = k_c \]

Series: \[ k_c = \frac{k_a}{4} \]

Shifter of Folded-Beam Suspension

\[ F/4 = F_c \]

\[ L_c = \frac{L}{2} \]

Insert: \[ l_c = \frac{L}{2} \]

\[ k_c = \frac{3EI_c}{(l/2)^3} = \frac{24EI_c}{L^3} \]

\[ k_c = \frac{3EI_c}{L^3} \]

Full Beam Length
Get $k_b$:

- $k_A = k_c + k_{\text{combined}}$
- $k_c + k_{\text{parallel}} = k_c + k_c/2 = k_A$
  where $k_c = \frac{24E I_1}{L_1^3}$
- $k_{\text{series}} = k_c/2$

Assume: shuttle + folding truss are rigid.
beams are small: neglect their moiré

Find the stiffer at point A.

Apply F @ A → what is $X_A$?

$X_A = k_{\text{series}}$

$X_A = k_{\text{series}}$

$X_A = \frac{E}{k_{\text{series}}}$