Get Potential Energy & Frequency

Folded-beam suspension

\[ K_{eq(truss)} = 19.2 \, \text{N/m} \]
\[ M_{eq(truss)} = 8.64 \times 10^{-11} \, \text{kg} \]
\[ C_{eq(truss)} = 4.08 \times 10^{-10} \, \text{kg/s} \]

Shuttle w/ mass \( M_s \)

\[ K_{eq(shuttle)} = 4.8 \, \text{N/m} \]
\[ M_{eq(shuttle)} = 2.16 \times 10^{-11} \, \text{kg} \]
\[ C_{eq(shuttle)} = 1.02 \times 10^{-10} \, \text{kg/s} \]

Folding truss w/ mass \( M_t \)

\[ K_{eq(truss)} = 4.8 \, \text{N/m} \]
\[ M_{eq(truss)} = 8.64 \times 10^{-11} \, \text{kg} \]
\[ C_{eq(truss)} = 4.08 \times 10^{-10} \, \text{kg/s} \]

3CC 3\( \lambda \)/4 Bridged \( \mu \)Mechanical Filter

Performance:

\( f_o = 9 \, \text{MHz} \), \( BW = 20 \, \text{kHz} \), \( PBW = 0.2\% \)
\( I.L. = 2.79 \, \text{dB} \), Stop. Rej. = 51 dB

20dB S.F. = 1.95, 40dB S.F. = 6.45

Design:

\( L_r = 40 \, \mu \text{m} \)
\( W_r = 6.5 \, \mu \text{m} \)
\( h_r = 2 \, \mu \text{m} \)
\( L_c = 3.5 \, \mu \text{m} \)
\( L_b = 1.6 \, \mu \text{m} \)
\( V_P = 10.47 \, \text{V} \)
\( P = -5 \, \text{dBm} \)
\( R_Q = R_{Qo} = 12 \, \Omega \)

Micromechanical Filter Circuit

Bridging Beam

Coupling Beam

Resonator

Input

Output

\[ R_Q \]

\[ V_i \]

\[ V_P \]

\[ V_o \]

\[ V_i \]

\[ V_P \]

\[ V_o \]

Micromechanical Filter Circuit

Bridging Beam

Coupling Beam

Resonator

Input

Output

\[ R_Q \]

\[ V_i \]

\[ V_P \]

\[ V_o \]
Micromechanical Filter Circuit

Beam Resonator Equivalent Circuits
(Pretty Much the Same Stuff)
Equivalent Dynamic Mass

- Once the mode shape is known, the lumped parameter equivalent circuit can then be specified.
- Determine the equivalent mass at a specific location \( x \) using knowledge of kinetic energy and velocity.

\[
\text{Equivalent Mass} = M_{eq}(x) = \frac{K.E.}{\frac{1}{2}V^2} = \frac{1}{2} \rho A \int_0^l V^2(x) \, dx
\]

Maximum Kinetic Energy

\[
\text{Density}
\]

Maximum Velocity @ location \( x \)

Maximum Velocity Function

Equivalent Dynamic Stiffness & Damping

- Stiffness then follows directly from knowledge of mass and resonance frequency.

\[
\omega_0: \frac{K_{eq}(x)}{M_{eq}(x)} \rightarrow K_{eq}(x) = \omega_0^2 M_{eq}(x)
\]

- And damping also follows readily.

\[
Q: \frac{\omega_0 M_{eq}(x)}{C_{eq}(x)} \rightarrow C_{eq}(x) = \frac{\omega_0 M_{eq}(x)}{Q} = \frac{K_{eq}(x) M_{eq}(x)}{Q}
\]

Equivalent Lumped Mechanical Circuit
Equivalent Lumped Mechanical Circuit

Example: Polysilicon w/ $l=14.9 \mu m$, $W=6 \mu m$, $h=2 \mu m \rightarrow 70$ MHz

$K_{eq}(0) = 19,927 \text{ N/m}$

$M_{eq}(0) = 1.03 \times 10^{-13} \text{ kg}$

$C_{eq}(0) = 5.66 \times 10^{-9} \text{ kg/s}$

$K_{eq}(\ell/2) = 53,938 \text{ N/m}$

$M_{eq}(\ell/2) = 2.78 \times 10^{-13} \text{ kg}$

$C_{eq}(\ell/2) = 1.53 \times 10^{-8} \text{ kg/s}$