Parallel-Plate Capacitive Nonlinearity

- Example: clamped-clamped laterally driven beam with balanced electrodes
- Nomenclature:
  - Total Value
  - DC Component (upper case variable; upper case subscript)
  - AC or Signal Component (lower case variable; lower case subscript)

Voltage-Controllable Center Frequency

- Quadrature force voltage-controllable electrical stiffness:
  \[ k_e = \frac{\varepsilon_0 A_o V_f^2}{d^3} \]
  \[ f_o = \frac{1}{2 \pi} \frac{m_k - k_e}{m} \]

Microresonator Thermal Stability

- Thermal stability of poly-Si micromechanical resonator is 10X worse than the worst case of AT-cut quartz crystal

Geometric-Stress Compensation

- Use a temperature dependent mechanical stiffness to null frequency shifts due to Young's modulus thermal dep.
- Problems:
  - Stress relaxation
  - Compromised design flexibility
Voltage-Controllable Center Frequency

Voltage-Controllable Center Frequency

Excellent Temperature Stability

Can One Cancel $k_e$ with Two Electrodes?

- What if we don’t like the dependence of frequency on $V_P$?
- Can we cancel $k_e$ via a differential input electrode configuration?
- If we do a similar analysis for $F_{d2}$ at Electrode 2:

$$F_{d2} = -V_P\frac{C_\alpha^2}{d_2} |v_2| \cos \omega_d t + V_P^2\frac{C_\alpha^2}{d_2^2} |x| \sin \omega_d t$$

- Adds to the quadrature term → $k_e$’s add, no matter the electrode configuration!

Measured $\Delta f/f$ vs. $T$ for $k_e$-Compensated μResonators

- Slits help to release the stress generated by lateral thermal expansion: linear $T_C$ curves $\approx -0.24$ ppm/°C!
Problems With Parallel-Plate C Drive

- Nonlinear voltage-to-force transfer function
  - Resonance frequency becomes dependent on parameters (e.g., bias voltage $V_p$)
  - Output current will also take on nonlinear characteristics as amplitude grows (i.e., as $x$ approaches $d_o$)
  - Noise can alias due to nonlinearity
- Range of motion is small
  - For larger motion, need larger gap ... but larger gap weakens the electrostatic force
  - Large motion is often needed (e.g., by gyroscopes, vibromotors, optical MEMS)

Electrostatic Comb Drive