Lecture Outline

- Reading: Senturia, Chpt. 14, Chpt. 16, Chpt. 21
- Lecture Topics:
  - Gyroscopes
  - Gyro Circuit Modeling
  - Minimum Detectable Signal (MDS)
    - Noise
    - Angle Random Walk (ARW)

Classic Spinning Gyroscope

- A gyroscope measures rotation rate, which then gives orientation → very important, of course, for navigation
- Principle of operation based on conservation of momentum
- Example: classic spinning gyroscope

Rotor will preserve its angular momentum (i.e., will maintain its axis of spin) despite rotation of its gimbled chassis
Vibratory Gyroscopes

- Generate momentum by vibrating structures
- Again, conservation of momentum leads to mechanisms for measuring rotation rate and orientation
- Example: vibrating mass in a rotating frame

\[
\begin{align*}
\text{Mass at rest} & \quad \text{Driven into vibration along the } y\text{-axis} \\
y\text{-displaced mass} & \quad \text{Capacitance between mass and frame = constant} \end{align*}
\]

\[
C(t_2) > C(t_1)
\]

Basic Vibratory Gyroscope Operation

**Principle of Operation**

- Tuning Fork Gyroscope:
  - Input Rotation
  - Driven Vibration @ \( f_0 \)

\[
\begin{align*}
\vec{a}_c &= 2\vec{v} \times \vec{\Omega} \\
\text{Coriolis Acceleration} &\text{ (Sense)} \\
\text{Coriolis Torque} &\text{Drive} \\
\text{Beam Mass} &\text{Beam Stiffness} \\
\text{Sense Frequency} &\text{Driven Velocity}
\end{align*}
\]

**Drive/Sense Response Spectra:**

\[
\begin{align*}
\text{Amplitude} &\text{ Drive Response} \\
f_0 (@ T_1) &\text{ Sense Response} \\
\omega &\text{ Rotation Rate}
\end{align*}
\]

Vibratory Gyroscope Performance

**Principle of Operation**

- Tuning Fork Gyroscope:
  - Input Rotation
  - Driven Vibration @ \( f_0 \)

\[
\begin{align*}
\vec{a}_c &= \frac{\vec{F}_d}{k} \\
\text{Beam Mass} &\text{ Beam Stiffness} \\
\text{Beam Frequency} &\text{ Driven Velocity} \\
\end{align*}
\]

\[
\begin{align*}
\vec{a}_c &= 2\vec{v} \times \vec{\Omega} \\
\text{Coriolis Force} &\text{ (Sense) Response} \\
\text{Coriolis Torque} &\text{Drive} \\
\text{Sensitivity} &\text{Driven Velocity}
\end{align*}
\]

- To maximize the output signal \( x \), need:
  - Large sense-axis mass
  - Small sense-axis stiffness
  - (Above together mean low resonance frequency)
  - Large drive amplitude for large driven velocity (so use comb-drive)
  - If can match drive freq. to sense freq., then can amplify output by \( Q \) times
MEMS-Based Gyroscopes

Vibrating Ring Gyroscope

[Draper Labs., Ayazi, GA Tech.]

Nuclear Magnetic Resonance Gyro [NIST]

MEMS-Based Tuning Fork Gyroscope

- In-plane drive and sense modes pick up z-axis rotations
- Mode-matching for maximum output sensitivity

* From [Zaman, Ayazi, et al, MEMS'06]

MEMS-Based Tuning Fork Gyroscope

* Drive and sense axes must be stable or at least track one another to avoid output drift

Problem: if drive frequency changes relative to sense frequency, output changes ⇔ bias drift

Need: small or matched drive and sense axis temperature coefficients to suppress drift
**Mode Matching for Higher Resolution**

- For higher resolution, can try to match drive and sense axis resonance frequencies and benefit from Q amplification.

![Diagram](https://example.com/diagram1.png)

**Issue: Zero Rate Bias Error**

- Imbalances in the system can lead to zero rate bias error.

![Diagram](https://example.com/diagram2.png)

**Nuclear Magnetic Res. Gyroscope**

- The ultimate in miniaturized spinning gyroscopes?
  - Better if this is a noble gas nucleus (rather than e-), since nuclei are heavier → less susceptible to B field.
  - Soln: Spin polarize Xe\(^{139}\) nuclei by first polarizing e- of Rb\(^{87}\) (a la CSAC), then allowing spin exchange.

![Diagram](https://example.com/diagram3.png)

**MEMS-Based Tuning Fork Gyroscope**

- Drive Voltage Signal
  - Drive Oscillation Sustaining Amplifier
  - Differential TransR Sense Amplifier

![Diagram](https://example.com/diagram4.png)
Determining Sensor Resolution