Lecture 2: Benefits of Scaling I

Announcements:
- The notes from last time are online in the Lecture link table; video also already up
- Modules 1 & 2 are also online (also, in the Lecture link table)
- HW#1 online and due Feb. 11 at 8 a.m.
- Get your computer accounts by following the instructions at the end of the Course Info Sheet
- You all have received invites to join the class Piazza group

Today:
- Reading: Senturia, Chapter 1
- Lecture Topics:
  - Benefits of Miniaturization
  - Examples
    - GHz micromechanical resonators
    - Chip-scale atomic clock
    - Micro gas chromatograph

Last Time: Going through Module 1
- Finish Module 1, then start going through Module 2

Scaling of Guitar Strings

\[
\text{guitar string} = \text{transverse, vibrating, stretched wire}
\]

\[
\text{simple support allows any angle here}
\]

\[
\text{deflected line is the mode shape}
\]

\[
\text{Free Body Diagram}
\]

\[
\frac{\partial^2 z}{\partial t^2} = \frac{m}{I} \frac{\partial^2 z}{\partial x^2}
\]

\[
\text{moment per unit length}
\]

\[
\text{inertial force} = ma
\]

\[
\text{tension} = \text{axial force}
\]
Introduction to MEMS

Lecture 2w: Benefits of Scaling I

\[
S \left( \frac{d^2 u}{dr^2} + \frac{2}{r} \frac{du}{dr} - \frac{1}{r^2} u \right) = S \left( \frac{d^2 y}{dx^2} + \frac{2}{w} \frac{dy}{dx} - \frac{1}{w^2} y \right) = 0
\]

Solve

\[
f_r = \frac{1}{2L} \sqrt{\frac{S}{m}}
\]

If \( n \) \rightarrow \infty

\( n \) = mode: 1, 2, 3, ...

Clamped-Clamped (or Fixed-Fixed) Beam

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\[
f_0 = \frac{1}{2\pi} \sqrt{\frac{k}{m}} = 1.03 \sqrt{\frac{E}{\rho}} \frac{h}{L^2}
\]

Where

- \( E \) = Young's modulus of elasticity [GPa]
- \( \rho \) = density [kg/m³]
- \( h \) = thickness [m]
- \( L \) = length [m]

Example

\( L = 40 \mu m, h = 3 \mu m \)

\( \text{polysilicon} \rightarrow E = 150 \text{GPa}, \rho = 2380 \text{ kg/m}^3 \)

\[
f_0 = (1.03) \sqrt{\frac{150 \times 2}{2380 \times (40 \mu m)^2}} = \frac{1}{160} \text{MHz}
\]

Why?

- Not quite as simple
- Infinitely many modes
- Why?

Near the anchor reduces \( f_0 \) by \( \sim 1 \text{MHz} \)

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Remarks:
- Eq. (1) not accurate when \( L \approx h \)
- Anchor loss when \( L \approx h \) → beam becomes too stiff → lowers \( Q \)