Lecture Outline

* Reading: Senturia, Chpt. 5, Chpt. 6
* Lecture Topics:
  - Energy Conserving Transducers
    - Charge Control
    - Voltage Control
  - Parallel-Plate Capacitive Transducers
    - Linearizing Capacitive Actuators
    - Electrical Stiffness
  - Electrostatic Comb-Drive
    - 1st Order Analysis
    - 2nd Order Analysis
Basic Physics of Electrostatic Actuation

- **Goal**: Determine gap spacing $g$ as a function of input variables
- **First**, need to determine the energy of the system
- **Two ways to change the energy**:
  - Change the charge $q$
  - Change the separation $g$

$$\Delta W(q, g) = Vq + F_e \Delta g$$

$$dW = Vdq + F_e dg$$

**Note**: We assume that the plates are supported elastically, so they don't collapse

Stored Energy

- Here, the stored energy is the work done in increasing the gap after charging capacitor at zero gap

$$W = 0 + \int_0^g F_e dg'$$

- No change in charge: $dq = 0$

$$F_e = \frac{q}{2} \varepsilon \frac{g^2}{e A}$$

- (independent of $g$)

$$W = \int_0^g F_e dg' = \frac{q}{2} \varepsilon \frac{g^2}{e A}$$

- For a capacitor $C$:

$$W(q) = \int_0^q Vdq = \frac{q}{2} \varepsilon \frac{g^2}{e A}$$

**Work done to change $C$ to $q$ at fixed gap**

$$W = \frac{1}{2} \varepsilon \frac{g^2}{e A}$$
Charge-Control Case

* Having found stored energy, we can now find the force acting on the plates and the voltage across them:

\[ \frac{\partial W}{\partial g} = \frac{1}{2} \frac{q^2}{\varepsilon A} \]

\[ F_e = \frac{1}{2} \frac{q^2}{\varepsilon A} \]

\[ V = \frac{q}{\varepsilon A} \]

\[ \Rightarrow \text{independent of gap spacing!} \]

Voltage-Control Case

* Practical situation: We control \( V \)

\[ \text{Charge control on the typical sub-pF MEMS actuation capacitor is difficult} \]

\[ \text{Need to find } F_e \text{ as a partial derivative of the stored energy } W = W(V, g) \text{ with respect to } g \text{ with } V \text{ held constant? But can’t do this with present } W(q, g) \text{ formula} \]

\[ \text{Solution: Apply Legendre transformation and define the co-energy } W'(V, g) \]

\[ \text{Energy} \]

\[ \text{Displacement (e.g., displacement, charge)} \]

\[ \Rightarrow \text{Can define co-energy as: } W'(e) = e q - W(q) \]

(from this plot)
Co-Energy Formulation

* For our present problem (i.e., movable capacitive plates), the co-energy formulation becomes

\[
\mathcal{W}'(V, g) = qV - \mathcal{W}(q, g)
\]

Differentially, this becomes:

\[
d\mathcal{W}'(V, g) = (qgV + Vdq) - d\mathcal{W}(q, g)
\]

But [\(d\mathcal{W}(q, g) = F_e dg + Vdq\)]

Working Co-Energy Expression

From which:

\[
\text{Charge, } Q = \left. \frac{d\mathcal{W}'(V, g)}{dV} \right|_{g = \text{const.}}
\]

\[
\text{Force, } F_e = -\left. \frac{d\mathcal{W}'(V, g)}{dg} \right|_{V = \text{const.}}\]

\[\Rightarrow\text{ this gives force as a function of applied voltage}\]

Electrostatic Force (Voltage Control)

* Find co-energy in terms of voltage (with gap held constant)

\[
\mathcal{W}' = \int_0^V q(g, V')dV' = \int_0^V \left( \frac{\varepsilon A}{g} \right) V'dV' = \frac{1}{2} \left( \frac{\varepsilon A}{g} \right) V^2 = \frac{1}{2} CV^2
\]

(as expected)

* Variation of co-energy with respect to gap yields electrostatic force:

\[
F_e = \left. -\frac{d\mathcal{W}'(V, g)}{dg} \right|_V = -\left( \frac{\varepsilon A}{g^2} \right) V^2 = \frac{1}{2} C V^2
\]

strong function of gap!

* Variation of co-energy with respect to voltage yields charge:

\[
q = \left. \frac{d\mathcal{W}'(V, g)}{dV} \right|_g = \left( \frac{\varepsilon A}{g} \right) V = CV
\]

as expected
Spring-Suspended Capacitive Plate

Charge Control of a Spring-Suspended Capacitive Plate

Force generated by charge $q$ supplied by current $I$:

$$F_e = \frac{\partial V(q, g)}{\partial q} = \frac{q^2}{2 \varepsilon A}$$

Restoring force of spring:

$$F_{spring} = kZ = F_e$$

@ equilibrium:

$$g = g_0 - \frac{F_e}{k} = g_0 - \frac{\frac{1}{2} q^2}{2 \varepsilon A k}$$

Initial gap:

$$V = \frac{q}{C} = \frac{q}{\varepsilon A} g = \frac{q}{\varepsilon A} \left( g_0 - \frac{1}{2} \frac{q^2}{2 \varepsilon A k} \right) \Rightarrow V \propto q^2$$

Can increase $q$ and drive $g \to 0$. 

Power as $g^2$. 

Voltage Control of a Spring-Suspended C

\[ F_{\text{spring}} = kz = Fe \]

But now:

\[ F_e = \frac{\partial W(V,g)}{\partial g} \bigg|_{V} = \frac{1}{2} \frac{\varepsilon A V^2}{g^2} \]

And the gap:

\[ g = g_0 - z = g_0 - \frac{Fe}{k} = \left[ g_0 - \frac{1}{2} \frac{\varepsilon A V^2}{g^2 - k} \right] \Rightarrow g \text{ shows up on both sides!} \]

Charge: (for a stable gap)

\[ q = \frac{\partial W(V,g)}{\partial V} \bigg|_{g} = CV = \frac{\varepsilon A V}{g} = q \]

Stability Analysis

* Net attractive force on the plate:

\[ F_{\text{net}} = \frac{\varepsilon A V^2}{2g^2} - k(g_0 - g) \]

* An increment in gap \( dg \) leads to an increment in net attractive force \( dF_{\text{net}} \)

\[ dF_{\text{net}} = \frac{3F_{\text{net}}}{\partial g} \cdot dg = \left[ -\frac{\varepsilon A V^2}{g^3} + k \right] \cdot dg \]

\[ F_{\text{net}} \rightarrow dF_{\text{net}} \rightarrow (\text{stability condition}) \]

Thus, read:

\[ k > \frac{\varepsilon A V^2}{g^3} \]

(for a stable uncollapsed state)
**Pull-In Voltage \( V_{PI} \)**

- \( V_{PI} \) = voltage at which the plates collapse
- The plate goes unstable when

\[
 k = \frac{\varepsilon AV_{PI}^2}{g_{PI}^3} \quad (1) \quad \text{and} \quad F_{net} = 0 = \frac{\varepsilon AV_{PI}^2}{2g_{PI}^2} - k(g_o - g_{PI}) \quad (2)
\]

- Substituting (1) into (2):

\[
 0 = \frac{\varepsilon AV_{PI}^2}{2g_{PI}^2} - \frac{\varepsilon AV_{PI}^2}{g_{PI}^3}(g_o - g_{PI})
\]

\[
 V_{PI} = \frac{\sqrt{kg_{PI}^3}}{\varepsilon A} \quad \text{pull-in gap}
\]

\[
 g_{PI}^* = \frac{1}{2} g_o \quad \Rightarrow \quad g_{PI} = \frac{2}{3} g_o
\]

When a gap is driven by a voltage to \( 2/3 \) its original spacing, collapse will occur!

---

**Voltage-Controlled Plate Stability Graph**

- Below: Plot of normalized electrostatic and spring forces vs. normalized displacement \( 1 - (g/g_o) \)

- Spring Force
- Electrical Forces
- Stable Equilibrium Points

Increasing \( V \)

\[
 \text{Normalized Displacement} \quad 0 \quad 0.2 \quad 0.4 \quad 0.6 \quad 0.8 \quad 1
\]

\[
 \text{Forces} \quad 0 \quad 0.2 \quad 0.4 \quad 0.6 \quad 0.8 \quad 1
\]
Advantages of Electrostatic Actuators

- Easy to manufacture in micromachining processes, since conductors and air gaps are all that's needed → low cost!
- Energy conserving → only parasitic energy loss through $I^2R$ losses in conductors and interconnects
- Variety of geometries available that allow tailoring of the relationships between voltage, force, and displacement
- Electrostatic forces can become very large when dimensions shrink → electrostatics scales well!
- Same capacitive structures can be used for both drive and sense of velocity or displacement
- Simplicity of transducer greatly reduces mechanical energy losses, allowing the highest Q's for resonant structures

Problems With Electrostatic Actuators

- Nonlinear voltage-to-force transfer function
- Relatively weak compared with other transducers (e.g., piezoelectric), but things get better as dimensions scale
Linearizing the Voltage-to-Force Transfer Function

- Apply a DC bias (or polarization) voltage $V_p$ together with the intended input (or drive) voltage $v_i(t)$, where $V_p \gg v_i(t)$

\[ v(t) = V_p + v_i(t) \]

\[ F_e(t) = \frac{\partial}{\partial x} \left( \frac{1}{2} C \left[ \frac{v(t)}{g_0} \right]^2 \right) \]

\[ = \frac{1}{2} \frac{C}{g_0^2} \left[ \frac{v(t)}{g_0} \right]^2 + \frac{1}{2} \left( V_p + v_i(t) \right)^2 \frac{2C}{g_0^2} \]

\[ = \frac{1}{2} \left( V_p^2 + 2V_p v_i(t) + \left[ \frac{v_i(t)}{g_0} \right]^2 \right) \frac{2C}{g_0^2} \]

\[ [V_p \gg \frac{v_i(t)}{g_0}] \Rightarrow F_e(t) \approx \frac{1}{2} \frac{C}{g_0^2} \frac{V_p^2}{2g_0} + \frac{V_p^2}{2g_0} \frac{2C}{g_0} \]

\[ \frac{C(x)}{g_0} \approx C_0 \left( 1 - \frac{x^2}{g_0^2} \right) \approx C_0 \left( 1 + \frac{x^2}{g_0^2} \right) \]

\[ \left[ x \ll g_0 \right] \approx \text{linear} \]

\[ \frac{2C}{g_0^2} \ll \frac{C}{g_0} \Rightarrow F_e(t) \approx \frac{1}{2} \frac{C}{g_0^2} V_p^2 + \frac{V_p}{g_0} \frac{C}{g_0} \frac{v_i(t)}{g_0} \]
Differential Capacitive Transducer

- The net force on the suspended center electrode is

\[ F_{net} = F_{er}(t) - F_{e}(t) \]

Do the math.

\[
F_{net}(t) = \frac{1}{2} \frac{\partial}{\partial x} \left\{ \left[ V_R(t) \right]^2 - \left[ V_L(t) \right]^2 \right\} = \left( \frac{V_p^2}{2} + 2V_p N(t) + \left[ V(t) \right] \right) - \left( \frac{V_p^2}{2} - 2V_p N(t) + \left[ V(t) \right] \right) = \frac{\partial}{\partial x} \left[ \frac{V_p^2}{2} N(t) + 2V_p \right] \]

Linear \( \approx \) \( \frac{V(t)}{N(t)} \) (gap match limited)

Remaining Nonlinearity

(Electrical Stiffness)
Parallel-Plate Capacitive Nonlinearity

- Example: clamped-clamped laterally driven beam with balanced electrodes

- Nomenclature:
  - $V_a$ or $v_A$:
  - $v_a = |v_a| \cos \omega t$
  - $V_a$ or $v_A = V_A + v_a$

- Conductive Structure:
  - Electrode
  - $k_m$
  - $F_{dl}$

- Expression for $\frac{\partial C}{\partial x}$:
  - $C(x) \cdot \frac{dA}{dx} \cdot \frac{C_0 (1 + \frac{x}{d_1})^{-1}}{d_1} \rightarrow \frac{dC}{dx} = \frac{C_0}{d_1} \left(1 + \frac{x}{d_1}\right)^{-2}$

- Expand the Taylor series further:
  - $\frac{dC}{dx} = \frac{C_0}{d_1} \left(1 + A_1 x + A_2 x^2 + A_3 x^3 + \cdots \right)$

- Where:
  - $A_1 = \frac{2}{d_1}$
  - $A_2 = \frac{3}{d_1^2}$
  - $A_3 = \frac{4}{d_1^3}$
  - $\cdots$
Parallel-Plate Capacitive Nonlinearity

* Thus, the expression for force from the left side becomes:

\[ F_{di} = \frac{1}{2} \frac{dC}{dx} (V_p - v_i) \left( V_p^2 - 2V_p v_i + v_i^2 \right) \]

\[ + \frac{1}{2} \frac{dC}{dx} \left( V_p^2 - 2V_p N_i + N_i^2 \right) \]

\[ = \frac{1}{2} \left( \frac{C_{oi}}{d_1} \right) \left\{ \left( V_p + A_i x \right) \left( V_p^2 - 2V_p N_i + N_i^2 \right) \right. \]

\[ + A_i V_p^2 \left( x - 2A_i V_p x N_i + A_i x N_i^2 \right) \}

@ resonance: \( \chi = \frac{dF_{di}}{dk} \approx \frac{d}{dk} \frac{dC}{dx} V_p V_i \)

Thus:

\[ N_i = (V_i) \cos \omega t \rightarrow x = \frac{1}{2} x \sin \omega t \]

\[ = 90^\circ \text{ phase-shifted from } N_i \]

• Retaining only terms at the drive frequency:

\[ F_{di} \bigg|_{\omega_o} = V_p \frac{C_{oi}}{d_1} | v_1 | \cos \omega_o t + V_p^2 \frac{C_{oi}}{d_1^2} | x \sin \omega_o t \]

Drive force arising from the input excitation voltage at the frequency of this voltage

Proportional to displacement

90° phase-shifted from drive, so in phase with displacement

* These two together mean that this force acts against the spring restoring force!

\( \epsilon \) A negative spring constant

\( \epsilon \) Since it derives from \( V_p \), we call it the electrical stiffness, given by:

\[ k_e = V_p^2 \frac{C_{oi}}{d_1^2} = V_p^2 \frac{\epsilon A}{d_1} \]
Electrical Stiffness, $k_e$

- The electrical stiffness $k_e$ behaves like any other stiffness.
- It affects resonance frequency:

$$\omega' = \sqrt{\frac{k}{m} - \frac{k_m}{k_e}}$$

$$= \sqrt{\frac{k_m}{m} \left(1 - \frac{k_e}{k_m}\right)^{1/2}}$$

$$\omega_o' = \omega_o \left(1 - \frac{V_{p1}^2 \varepsilon A}{k_m d_1^3}\right)^{1/2}$$

Frequency is now a function of dc-bias $V_{p1}$.

Voltage-Controllable Center Frequency

- Quadrature force voltage-controllable electrical stiffness:

$$k_e = \varepsilon_0 \frac{A}{d^3} \frac{V}{P}$$

$$f_o = \frac{1}{2\pi}\sqrt{\frac{k_m - k_e}{m}}$$

Graph showing frequency vs. DC-bias ($V_p$)
1. Thermal stability of poly-Si micromechanical resonator is 10X worse than the worst case of AT-cut quartz crystal.

2. Use a temperature dependent mechanical stiffness to null frequency shifts due to Young's modulus thermal dep. [Hsu et al, IEDM'00]

3. **Problems:**
   - stress relaxation
   - compromised design flexibility

[EE C245: Introduction to MEMS Design] (LecM 12) C. Nguyen 11/18/08
Voltage-Controllable Center Frequency

\[ k_e = \frac{\varepsilon A o V^2}{d^3} \]

\[ f_o = \frac{1}{2\pi}\sqrt{\frac{k_m - k_e}{m}} \]

Excellent Temperature Stability

Electrode Overlap Area

\[ k_e \sim 1/d^2 \Delta \]

\[ f_o \sim (k_m - k_e)^{0.5} \Delta \]

Counteracts reduction in frequency due to Young's modulus temp. dependence

\[ -1.7\text{ppm/°C} \]

\[ -0.24\text{ppm/°C} \]

AT-cut Quartz Crystal at Various Cut Angles

On par with quartz!
Measured $\Delta f/f$ vs. $T$ for $k_e$-Compensated $\mu$Resonators

Design/Performance:
- $f_0=10\text{MHz}$, $Q=4,000$
- $V_p=8V$, $h_e=4\mu m$
- $d_o=1000\AA$, $h=2\mu m$
- $W_f=8\mu m$, $L=40\mu m$

[Hsu et al. MEMS'02]

- Slits help to release the stress generated by lateral thermal expansion $\rightarrow$ linear $TC_f$ curves $\rightarrow -0.24\text{ppm/°C}$!

Can One Cancel $k_e$ w/ Two Electrodes?

- What if we don't like the dependence of frequency on $V_p$?
- Can we cancel $k_e$ via a differential input electrode configuration?
- If we do a similar analysis for $F_{d2}$ at Electrode 2:

\[
F_{d2} \bigg|_{\omega_o} = -V_p^2 \frac{C_o}{d_2^2} \left| v_2 \right| \cos \omega_o t
\]

\[
+ V_p^2 \frac{C_o}{d_2^2} \left| x \right| \sin \omega_o t
\]

Subtracts from the $F_{d1}$ term, as expected

Adds to the quadrature term $\rightarrow k_e$'s add, no matter the electrode configuration!
Problems With Parallel-Plate C Drive

- Nonlinear voltage-to-force transfer function
  - Resonance frequency becomes dependent on parameters (e.g., bias voltage $V_P$)
  - Output current will also take on nonlinear characteristics as amplitude grows (i.e., as $x$ approaches $d_o$)
  - Noise can alias due to nonlinearity
- Range of motion is small
  - For larger motion, need larger gap ... but larger gap weakens the electrostatic force
  - Large motion is often needed (e.g., by gyroscopes, vibromotors, optical MEMS)

Electrostatic Comb Drive
Electrostatic Comb Drive

- Use of comb-capacitive transducers brings many benefits
  - Linearizes voltage-generated input forces
  - (Ideally) eliminates dependence of frequency on dc-bias
  - Allows a large range of motion

Comb-Driven Folded Beam Actuator

Comb-Drive Force Equation (1st Pass)

$$C(x) = \varepsilon \epsilon_0 \frac{2 \epsilon_0 h}{d} \frac{\partial^2}{\partial x^2} \left( \frac{2 \epsilon_0 h}{d} \right)$$

$$F_d \approx \frac{2V_i^2}{2} \frac{\partial V}{\partial x} \left( V_d x^2 + 2V \mu \alpha + \frac{a^2}{2} \right)$$

$$F_d \approx -2V_p \frac{\epsilon_0 h}{d} \frac{\partial V}{\partial x} \left( V_d x^2 + 2V \mu \alpha + \frac{a^2}{2} \right)$$

But wait! This ignores other practical effects! (No dependence on x! LINEAR!)
Lateral Comb-Drive Electrical Stiffness

- Again: 
  \[ C(x) = \frac{2N\varepsilon h x}{d} \rightarrow \frac{\partial C}{\partial x} = \frac{2N\varepsilon h}{d} \]

- No \( (\partial C/\partial x) \) \( x \)-dependence \( \rightarrow \) no electrical stiffness: \( k_e = 0! \)
- Frequency immune to changes in \( V_P \) or gap spacing!

Typical Drive & Sense Configuration

**Simple Analysis:**

\[
F_{d1} = \frac{1}{2} \frac{\partial^2}{\partial x^2} (N_i V_i) = \frac{1}{2} \left( \varepsilon \frac{\partial}{\partial x} \right) \left( N_i^2 - 2V_{P1} V_i + V_i^2 \right) (2N_i^2)
\]

\[
F_{d2} = \frac{1}{2} \frac{\partial^2}{\partial x^2} (N_i V_i) = \frac{1}{2} \left( \varepsilon \frac{\partial}{\partial x} \right) \left( N_i^2 - 2V_{P2} V_i + V_i^2 \right) (2N_i^2)
\]

\[
\therefore \quad F_{net} = F_{d1} + F_{d2} = \frac{1}{2} \left( \varepsilon \frac{\partial}{\partial x} \right) \left( N_i^2 - N_i^2 - 2(V_{P1} V_i - V_{P2} V_i) + V_i^2 \right) (2N_i^2)
\]
Comb-Drive Force Equation (2nd Pass)

• In our 1st pass, we accounted for
  - Parallel-plate capacitance between stator and rotor
• ... but neglected:
  - Fringing fields
  - Capacitance to the substrate
• All of these capacitors must be included when evaluating the energy expression!

Comb-Drive Force With Ground Plane Correction

• Finger displacement changes not only the capacitance between stator and rotor, but also between these structures and the ground plane → modifies the capacitive energy

\[
F_{e,x} = \frac{\partial W'}{\partial x} = \frac{1}{2} \frac{dC_{sp}}{dx} V_s^2 + \frac{1}{2} \frac{dC_{rp}}{dx} V_r^2 + \frac{1}{2} \frac{dC_{rs}}{dx} (V_s - V_r)^2
\]

[Gary Fedder, Ph.D., UC Berkeley, 1994]
Capacitance Expressions

- Case: $V_r = V_p = 0V$
- $C_{sp}$ depends on whether or not fingers are engaged

$$C_{sp} = N[C'_{sp,e}x + C'_{sp,fr}(L - x)]$$

$$C_{rs} = NC'_{rs}x$$

Region 2

Region 3

Comb-Drive Force With Ground Plane Correction

- Finger displacement changes not only the capacitance between stator and rotor, but also between these structures and the ground plane, modifies the capacitive energy

$$F_{e,x} = \frac{\partial W'}{\partial x} = \frac{1}{2} \frac{dC_{sp}}{dx} V_s^2 + \frac{1}{2} \frac{dC_{rs}}{dx} V_r^2 + \frac{1}{2} \frac{dC_{sp,e}}{dx} (V_s - V_r)^2$$

$$F_{e,x} = \frac{N}{2}(C'_{rs} + C'_{sp,e} - C'_{sp,fr})V_s^2$$

(for $V_r = V_p = 0$)

[Gary Fedder, Ph.D., UC Berkeley, 1994]
Simulate to Get Capacitors $\rightarrow$ Force

- Below: 2D finite element simulation

\[ F_{e,x} = \frac{N}{2} \left( C_{rs} + C'_{sp,e} - C'_{sp,r} \right) V_s^2 \]

20-40% reduction of $F_{e,x}$

Vertical Force (Levitation)

\[ F_{e,z} = \frac{\partial W'}{\partial z} = \frac{1}{2} \frac{dC_{sp}}{dz} V_s^2 + \frac{1}{2} \frac{dC_{sp}}{dz} V_r^2 + \frac{1}{2} \frac{dC_{rs}}{dz} (V_s - V_r)^2 \]

- For $V_r = 0$V (as shown):

\[ F_{e,z} = \frac{1}{2} N \frac{\frac{d(C_{sp,e} + C'_{sp})}{dz}}{d} V_s^2 \]
Simulated Levitation Force

- Below: simulated vertical force $F_z$ vs. $z$ at different $V_p$'s [f/ Bill Tang Ph.D., UCB, 1990]

See that $F_z$ is roughly proportional to $-z$ for $z$ less than $z_o \rightarrow$ it's like an electrical stiffness that adds to the mechanical stiffness

$$F_z \approx \gamma_z V_p^2 \left(\frac{z_o - z}{z_o}\right) = k_e (z_o - z)$$

Electrical Stiffness

Vertical Resonance Frequency

- Vertical resonance frequency
- Lateral resonance frequency at $V_p = 0V$

$$\frac{\omega_z}{\omega_{z_0}} = \sqrt{\frac{k_z + k_e}{k_z}} \quad \text{where} \quad k_e = \left(\frac{\gamma_z}{z_o}\right) V^2$$

- Signs of electrical stiffnesses in MEMS:
  - Comb (x-axis) $\rightarrow k_e = 0$
  - Comb (z-axis) $\rightarrow k_e > 0$
  - Parallel Plate $\rightarrow k_e < 0$

Copyright © 2020 Regents of the University of California
Suppressing Levitation

- Pattern ground plane polysilicon into differentially excited electrodes to minimize field lines terminating on top of comb
- Penalty: x-axis force is reduced

Force of Comb-Drive vs. Parallel-Plate

- Comb drive (x-direction)
  \[ F_{e,x} = \frac{1}{2} \frac{\epsilon_o h}{d_o} V_s^2 \]

- Differential Parallel-Plate (y-direction)
  \[ F_{e,y} = \frac{1}{2} \frac{\epsilon_o h L_d}{d_o} V_s^2 \]
  
  \[ F_{e,y} = \frac{1}{2} \frac{\epsilon_o h L_d}{d_o} V_s^2 + \frac{L_d}{d_o} \]
  
  Parallel-plate generates a much larger force; but at the cost of linearity

Gap = \( d_o = 1 \) \( \mu \)m
Thickness = \( h = 2 \) \( \mu \)m
Finger Length = \( L_f = 100 \) \( \mu \)m
Finger Overlap = \( L_d = 75 \) \( \mu \)m