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Gaussian Filtering of a Rectangular Pulse

\[ x(t) = P_T(t) \]

\[ h(t) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left\{ -\frac{t^2}{2\sigma^2} \right\} \]

The parameter \( \sigma \) is related to the 3dB bandwidth \( B \) by

\[ \sigma^2 = \frac{\ln(2)}{\pi^2 B} \quad \beta = BT \]

This filter has the advantage of reducing sideband power, reducing interference with neighboring channels, at the cost of increasing intersymbol interference (ISI)
Gaussian Filtering of a Rectangular Pulse

Fig 1: Inter Symbol Interference.
Gaussian Filtering of a Rectangular Pulse
Noise

- Unwanted electric signals come from a variety of sources, both from
  - Naturally occurring noise (Atmospheric disturbances, extraterrestrial radiation, random electron motion)
  - Human interference. (Other communication systems, Lighting, ignition etc.)

- One unavoidable electrical noise is the thermal motion of electrons in conducting media
  - Modeled as white noise, a useful model in communication. (AWGN channel)
Thermal Noise

For all practical purposes, the mean square voltage spectral density of thermal noise is constant at:

\[ G_v(f) = 2RkT \quad \text{Units: V}^2/\text{Hz} \]

\[ P_a = \frac{\left\langle \frac{[v_s(t)/2]^2}{R_s} \right\rangle}{4R_s} = \frac{\left\langle v_s^2(t) \right\rangle}{4R_s} \]

\[ G_a(f) = \frac{G_v(f)}{4R} = \frac{kT}{2} = \frac{N_o}{2} \quad \text{Units: W/Hz} \]

All frequency components in equal proportion and is therefore called \textit{white noise}. 
$S_{th} = -174 \text{ dBm} / \text{Hz}$
Noise Bandwidth

\[ \Delta f = \frac{1}{|H(pk)|^2} \int_{0}^{\infty} |H(f)|^2 df \]

\[ H(f) = \frac{1}{1 + j2\pi fRC} \quad \Delta f = \frac{1}{4RC} = \frac{\pi}{2} f_{3dB} \]
Notes on Thermal Noise

- Noise Power depends on temperature and bandwidth

\[ P_{NA} = kT \Delta f \]

- Only ways to reduce thermal noise are cooling and filtering.

- To calculate total noise power, multiply by the noise bandwidth

\[ \bar{e}_n^2 = 4kTR\Delta f = \frac{kT}{C} \]

- Thermal noise only exists in real resistances
Noise Figure (NF)

Noise figure (NF) measure the degradation of Signal-to-Noise ratio (SNR) caused by components in a signal chain.

\[ F = \frac{SNR_{in}}{SNR_{out}} \quad \text{Units: Dimensionless} \]

\[ NF = 10 \log_{10} (F) = 10 \log_{10} \left( \frac{SNR_{in}}{SNR_{out}} \right) = SNR_{in,dB} - SNR_{out,dB} \quad \text{Units: Dimensionless} \]

It is a number by which the performance of a radio receiver can be specified. Lower the better.