Acknowledgement: Wayne Stark EE455 Lecture Notes, University of Michigan
David Wentzloff EE522 Lecture Notes, University of Michigan
Path Loss

Radiated Power Density \([W/m^2]\]

\[ P_{TX\text{density}} = \frac{P_t}{4\pi R^2} \]

Effective capture area of a receiving antenna

\[ A_{RX} = \frac{\lambda^2}{4\pi} \]

Friis transmission formula

\[ P_r = \frac{P_t}{4\pi R^2} A_{RX} = P_t \left( \frac{\lambda}{4\pi R} \right)^2 \]

\[ \frac{P_r}{P_t} = \left( \frac{\lambda}{4\pi R} \right)^2 \]
Antenna Gain

For the point radiator

\[ \frac{P_r}{P_t} = \left( \frac{\lambda}{4\pi R} \right)^2 \]

Antennas have non-spherical radiation patterns

Friis transmission formula incorporating radiation pattern

\[ \frac{P_r}{P_t} = G_t G_r \left( \frac{\lambda}{4\pi R} \right)^2 \]

Antenna Gain (G)
Directions where power density higher
Than that of point radiator
Link Budget

- A link budget is used to evaluate system performance

- Accounts for the following
  - Antenna radiation (gain)
  - Path loss
  - Circuit performance (NF)
  - Architecture performance \( (E_b/N_o) \)

- Calculates if there is sufficient SNR at the receiver for reliable communication
  - Excess SNR is called “link margin”
Link Budget

Friis Equation:

\[ \frac{P_r}{P_t} = G_t G_r \left( \frac{\lambda}{4\pi R} \right)^2 \]

\[ P_r = P_t + G_t + G_r + 20 \log_{10} \left( \frac{\lambda}{4\pi R} \right) \]

Gain has units of dB and power has units of dBm or dBW.
### Link Budget

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_t$ (Transmit power)</td>
<td>-5 dBm (316.2μW)</td>
</tr>
<tr>
<td>$G_t$ (Transmit antenna Gain)</td>
<td>0 dB</td>
</tr>
<tr>
<td>$G_r$ (Receive antenna Gain)</td>
<td>0 dB</td>
</tr>
<tr>
<td>$\lambda$ (2.45 GHz)</td>
<td>0.12491 m</td>
</tr>
<tr>
<td>$P_r$ (Received power)</td>
<td>-75 dBm (31.6pW)</td>
</tr>
<tr>
<td>Link Margin</td>
<td>70 dB</td>
</tr>
<tr>
<td>Distance (d)</td>
<td>~17 m</td>
</tr>
</tbody>
</table>

\[
NF < R_{ss} - SNR_{min} + 174 - 10 \log_{10}(BW)
\]
Receiver NF

\[ NF < R_{ss} - SNR_{\text{min}} + 174 - 10 \log_{10} (BW) \]

For BLE: \( P_e = 0.1\% = 10^{-3} \)

\[
SNR = \frac{\text{Signal}}{\text{Noise}} = \frac{E_b \times R}{N_o \times B} = \frac{E_b}{N_o}
\]

\( E_b/N_o \) depends on the type of modulation and demodulator.
Matlab Signal & Noise Modeling

According to the sampling theorem we can represent $x(t)$ by samples at rate $2f_{\text{max}}$.

In addition, assume that $x(t)$ is causal so that $x(t) = 0$ for $t < 0$.

The Fourier transform of $x(t)$ is given by

$$X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft} dt$$

Notice that when $x(t)$ is real then $X(-f) = X^*(f)$. 
Matlab Signal & Noise Modeling

The inverse Fourier transform is

\[ x(t) = \int_{-\infty}^{\infty} X(f) e^{j2\pi ft} \, df \]

\[ = \int_{-f_{\text{max}}}^{f_{\text{max}}} X(f) e^{j2\pi ft} \, df \]

Now we can approximate these integrals with finite sums.
Matlab Signal & Noise Modeling

- Since the highest frequency is $f_{\text{max}}$ we sample at a rate $2f_{\text{max}}$.
- Thus the samples are separated in time by $\Delta t = 1/(2f_{\text{max}})$.
- Suppose we have $N$ samples in the time domain.
- We can reconstruct these $N$ samples in the time domain with just $N$ samples in the frequency domain.
- The spacing in the frequency domain is $\Delta f = 2f_{\text{max}}/N$.
- Since $\Delta t = 1/(2f_{\text{max}})$ and $\Delta f = 2f_{\text{max}}/N$

$$\Delta f \Delta t = \frac{1}{N}$$

- That is, we are dividing the interval of the frequency response from $-f_{\text{max}}$ to $f_{\text{max}}$ into $N$ discrete frequencies and we are dividing time from 0 to $(N - 1) \Delta t$ into $N$ discrete time samples.
Matlab Signal & Noise Modeling

Then for $|f| < f_{\text{max}}$

$$X(f) = \int_{0}^{\infty} x(t) e^{-j2\pi ft} \, dt$$

$$\approx \sum_{l=0}^{N-1} x(l\Delta t) e^{-j2\pi f l \Delta t} \Delta t$$

$$X_m = X(m\Delta f) = \Delta t \sum_{l=0}^{N-1} x(l\Delta t) e^{-j2\pi m \Delta f l \Delta t}$$

$$= \Delta t \sum_{l=0}^{N-1} x(l\Delta t) e^{-j2\pi ml/N}$$
Matlab Signal & Noise Modeling

• Thus the above expression for $X_m$ is valid for $0 \leq |m| \leq N/2$.
• Notice that $X_{-m} = X_{N-m}$.
• In MATLAB the operation FFT takes an input vector (indexed from 1 to N) and produces an output vector (indexed from 1 to N).
• The input-output relation is

$$X_k = \sum_{n=1}^{N} x_n e^{-j2\pi (k-1)(n-1)/N} \quad k = 1, \ldots, N$$

The input should correspond to the samples starting at time 0 up to time $(N-1)\Delta t$. Notice that in order to get the correct magnitude of the frequency response we must multiply what MATLAB produces by $\Delta t$. 
Matlab Signal & Noise Modeling

• The output samples from 1 to $N/2$ (when normalized appropriately) correspond to the frequency response $X(0), X(Δf), ..., X((N/2)Δf) = X(f_{max} - Δf)$.
• The samples from $N/2 + 2$ to $N$ correspond to the frequency response from $X(-f_{max} + Δf), ..., X(-Δf)$.
• The order can be reversed into the logical order with the MATLAB command fftshift.
# Matlab Signal & Noise Modeling

<table>
<thead>
<tr>
<th>Matlab Index</th>
<th>Frequency</th>
<th>Matlab Index</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>N/2+1</td>
<td>-f_{max}</td>
</tr>
<tr>
<td>2</td>
<td>Δf</td>
<td>N/2+2</td>
<td>-f_{max}+Δf</td>
</tr>
<tr>
<td>3</td>
<td>2Δf</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>N/2+k</td>
<td>-f_{max}+(k-1) Δf</td>
</tr>
<tr>
<td>m</td>
<td>(m-1) Δf</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>N/2+(f_{max}-f2)/Δf+1</td>
<td>-f_{2}</td>
</tr>
<tr>
<td>(f1/Δf)+1</td>
<td>f_{1}</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>N-1</td>
<td>-2Δf</td>
</tr>
<tr>
<td>N/2</td>
<td>f_{max}-Δf</td>
<td>N</td>
<td>-Δf</td>
</tr>
</tbody>
</table>
Matlab Signal & Noise Modeling

\[ N/2+1 \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad N/2 \]

\[-f_{\text{max}} \quad -f_2 \quad -\Delta f \quad 0 \quad -\Delta f \quad -f_1 \quad f_{\text{max}} \quad -\Delta f\]
Matlab FFT Caveat

$x(t)$

Sample

$x_l = x((l-1)\Delta t), \ l = 1, ..., N$

Matlab

Inverse Fourier Transform

$(\text{fft} \ast \Delta t)$

$X(f)$

Sample

$X_k = X((k-1)\Delta f), \ k = 1, ..., N/2$

$X_k = X((k-1-N)\Delta f), \ k = N/2 + 1, ..., N$

Fourier Transform

$(\text{ifft}/\Delta t)$
Simulating Noise

- In a computer simulation of a communication system we represent a continuous time signal with samples according to the sampling theorem.
- A signal can be represented if the highest frequency content is less than half the sampling rate.
- Thus if $f_s$ represents the sampling rate the simulation bandwidth is $f_s/2$.
- If we consider white Gaussian noise the noise has equal power at all frequencies up to the simulation bandwidth.
Simulating Noise

- Thus the power spectral density of the noise in the simulation is

\[ S_n(f) = \frac{N_0}{2} \quad -\frac{f_s}{2} < f < \frac{f_s}{2} \]

- The noise samples (with this finite bandwidth) have variance

\[ \sigma^2 = \int_{-\frac{f_s}{2}}^{\frac{f_s}{2}} S_n(f) \, df = \int_{-\frac{f_s}{2}}^{\frac{f_s}{2}} \frac{N_0}{2} \, df = N_0 f_{\text{max}} \]
Power Management

DC/DC Converter

RF LDO
- 0.8V
- 250\mu A

Digital LDO
- 0.8V
- 500\mu A

Analog LDO
- 0.8V
- 200\mu A

PTAT IRef
- 10\mu A±15%
- 1\mu A±15%
- 10nA±15%

BG Vref
- 0.8V±20%
- Tunable

Temp. Sensor

To ADC

POR BDet.
Reset Signals

Nordic: nRF52840 Datasheet.
Terminology

- **Band** - entire spectrum for a standard
  - GSM 935-960 MHz
  - FM 87.5-108 MHz

- **Channel** – bandwidth of 1 user in the system
  - GSM 200 kHz
  - FM 100 kHz

- **Band-select filter**
- **Channel select filter**
- **Out of band Interference**
- **In-band Interference**
- **Adjacent-channel interference**
- **Alternate-channel interference**
Terminology

In-Band Blocker Levels

Desired Channel

-70 dBm

-43 dBm

-30 dBm

-23 dBm

Out-of-Band Blocker Levels

0 dBm

Edge of Band

f
Rx Signal Flow

Band Select Filter

Gain: 15dB
NF: 1-2dB

LO

LNA

LPF

PGA

Channel Select Filter

ADC

2400-2483 MHz

\[ F_{total} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \frac{F_4 - 1}{G_1 G_2 G_3} + \ldots \]
RF Filtering

Filtering requirements impact the RF design of the transceiver.

Diagram:
- PA
- BPF
- Transmitted Channel
- Adjacent Channel
- Desired Channel
- Alternate Channel
- LNA
RF Filtering

- Receive chain preceding channel-selection filtering must be sufficiently linear.
- Channel selection filtering is extremely difficult at the RF frequencies.

\[
Q_{BPF} \approx \frac{f_o}{BW}
\]

=2450MHz/1MHz
=2450

In-Band interferers are typically removed near the end of the receive chain.
Frequency Translation Mixers

\[
(A \cos \omega_1 t)(B \cos \omega_2 t) = \frac{AB}{2} \left[ \cos(\omega_1 - \omega_2) t + \cos(\omega_1 + \omega_2) t \right]
\]
TRx
Signal Flow

Fs = 128 MHz
N = 2^12
Time = 32 μS
**Baseband Signal**

$F_s = 128\ MHz$
$N = 2^{12}$
$Time = 32\ \mu S$

![Graph showing a baseband signal with time-domain and frequency-domain representations. The graph illustrates the signal and RF carrier, with amplitude and frequency data.](image)
Up Conversion

\( x(t) \times LO(t) \)
Up Conversion

\[ x(t) \ast LO(t) \]
Down Conversion

\[ RF(t) \times LO(t) \]
Down Conversion

$RF(t) \ast LO(t)$
Down Conversion

\( RF(t) \times LO(t) \)
AWGN

\[ RF(t) \ast LO(t) + n(t) \]
Down Conversion

$RF(t) \times LO(t) + n(t)$
LPF

$3^{rd}$ order Butterworth

$F_c = 2$ MHz
Down Conversion

\( RF(t) \times LO(t) + n(t) \)
Rx Architecture

- Disadvantages
  - High-Q RF, tunable RF filter required
  - Amplifier should also be tunable, high Q