

## Lecture 4

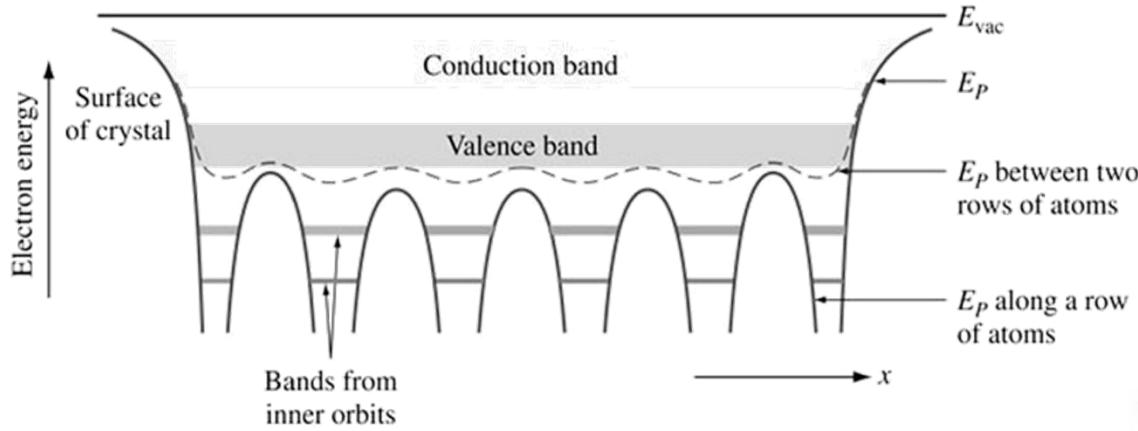
- MOSFET Transport Issues
  - semiconductor band structure
  - quantum confinement effects
  - low-field mobility and high-field saturation

Reading:

- M. Lundstrom, “[Fundamentals of Carrier Transport](#),” 2<sup>nd</sup> edition, *Cambridge University Press*, 2000.
- multiple research articles (reference list at the end of this lecture)

# Dispersion Relationship: $E$ vs. $k$

- $E$  vs. distance in semiconductors



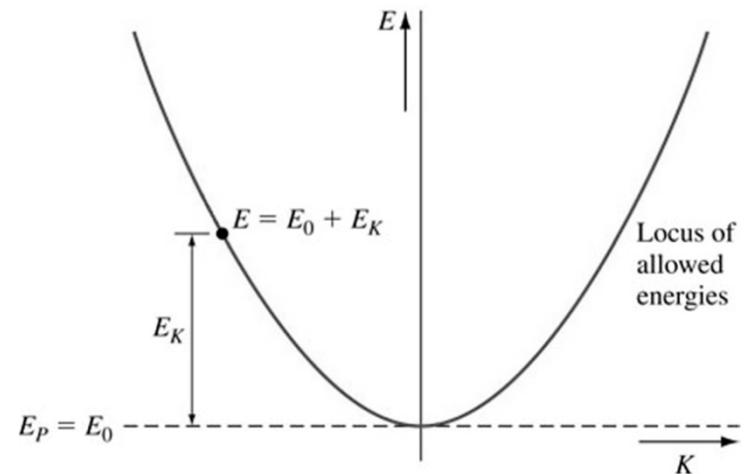
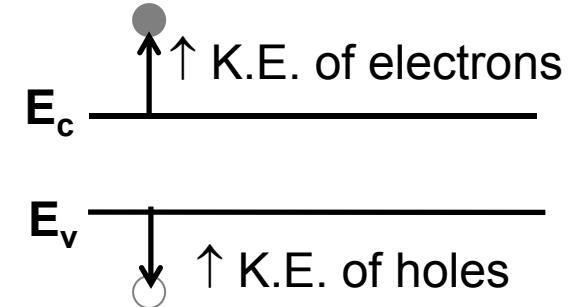
- Schrödinger equation

$$\frac{\hbar^2}{2m_0} \nabla^2 \psi + V(r) \psi = E \psi$$

$\psi \sim e^{ikx}$  called the plane wave solution  
 $k$  is called the wave vector

$$E = E_0 + \frac{\hbar^2 k^2}{2m}$$

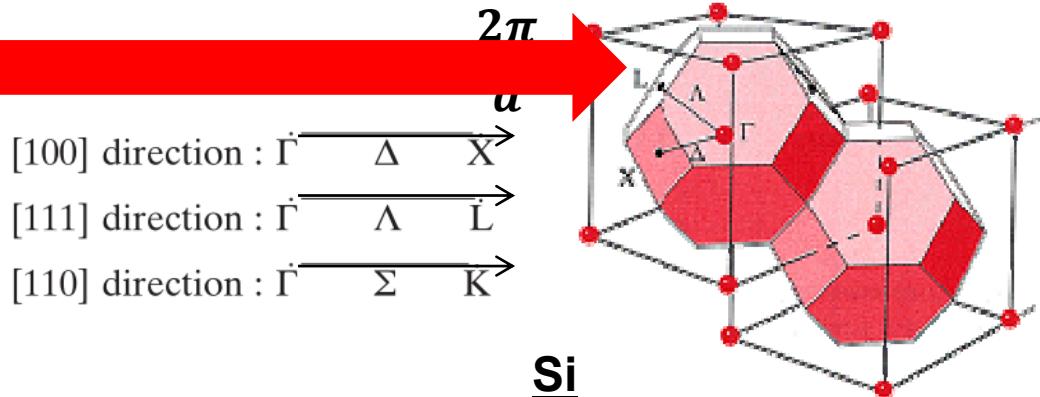
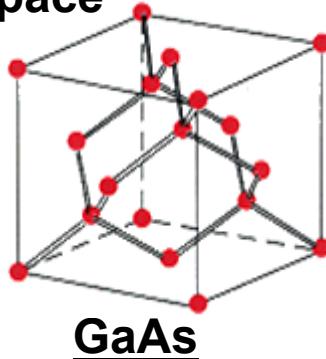
If  $V(r) = 0$  (free electron):  $m \rightarrow m_0$   
If  $V(r) = U(\vec{r})$  (electron in crystal):  $m \rightarrow m^*$



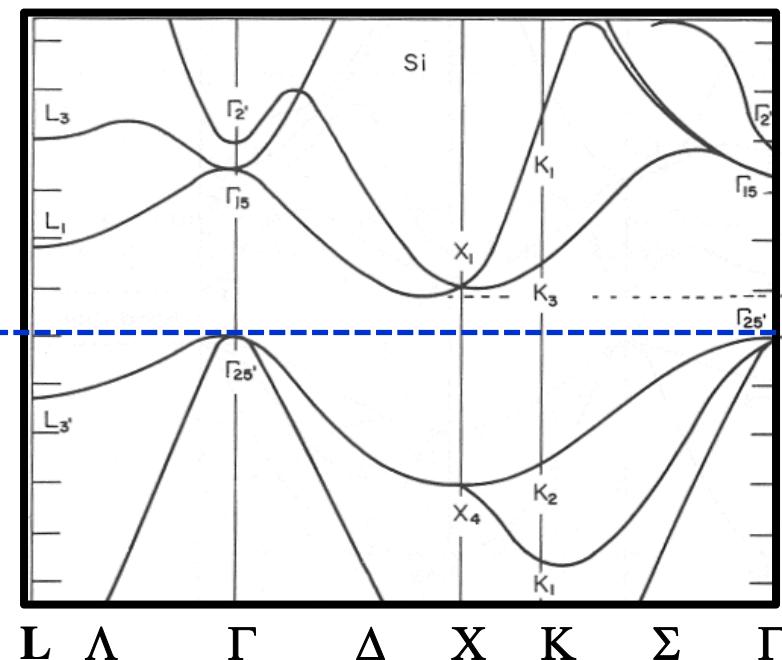
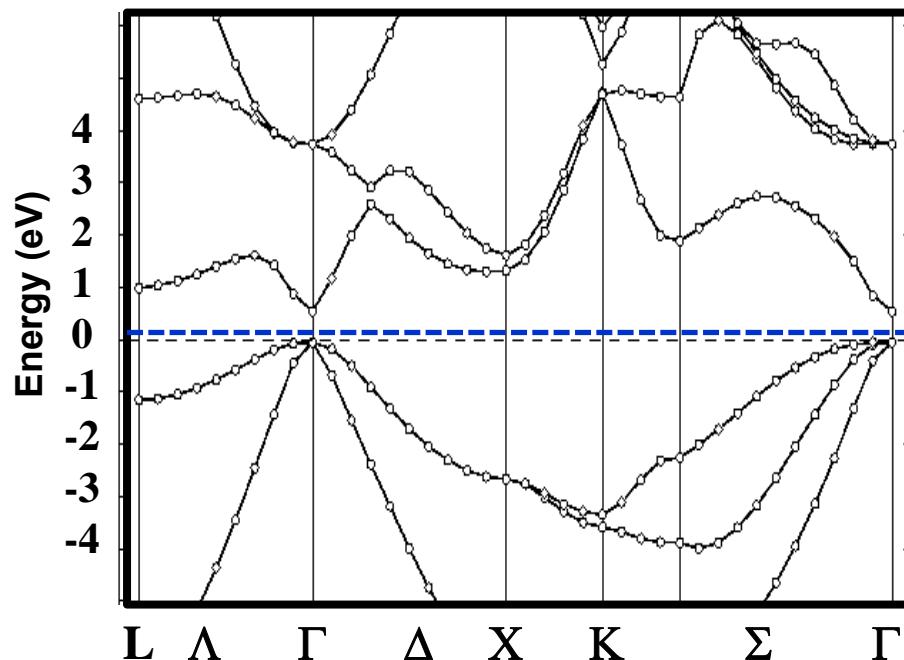
# Semiconductor Band Structure

- Reciprocal space

Zinc-blende structure

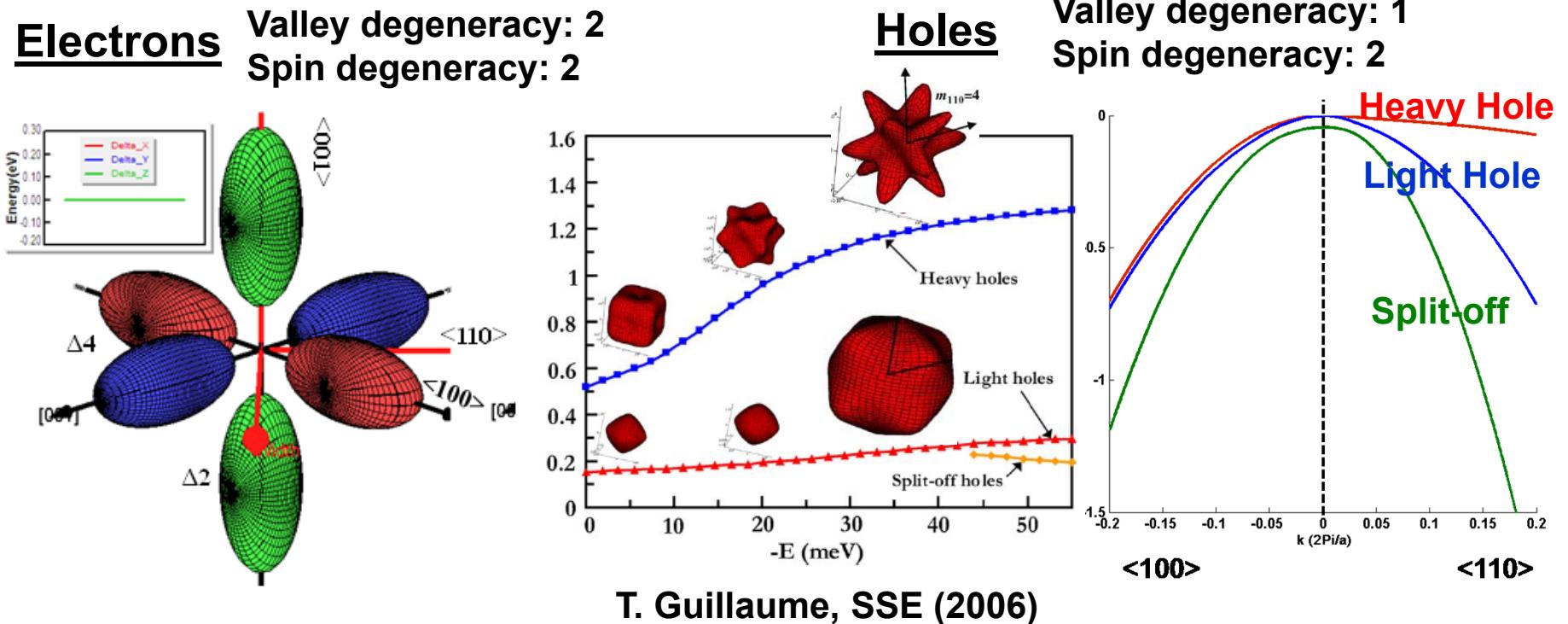


- $E$  vs.  $k$



# Silicon Band Structure

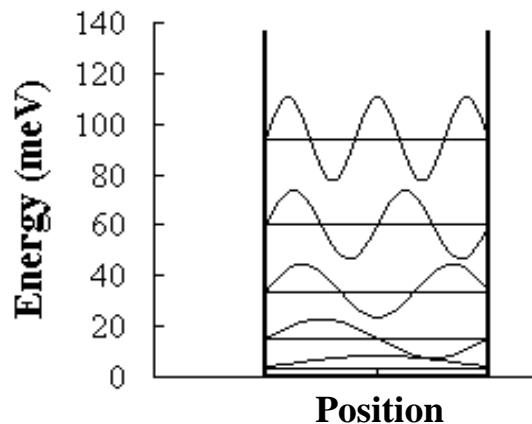
- Equi-energy contours



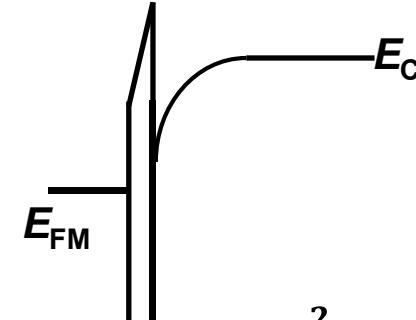
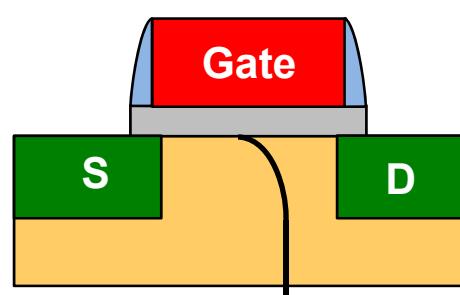
- $E$ - $k$  relationship is direction-dependent for electrons in solids.
- In Si, electrons have a more parabolic  $E$ - $k$  relationship than holes.

# Quantum Confinement (QC) Effects

- Sub-bands



In a MOS Inversion Layer:



Sub-band top/valley energies due to QC:  $E_i = \left( \frac{3\pi q \hbar}{2\sqrt{2m^*}} F_S \right)^{\frac{2}{3}} \cdot \text{AiryRoots}(i)$

F. Stern, PRB (1972)

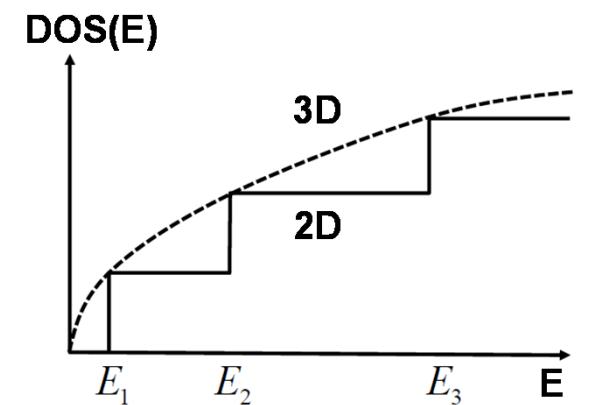
- Reduced density-of-states (DOS)

3D Bulk:

$$DOS(E) \equiv \frac{\# \text{ of } k \text{ states in } \Delta E}{\Delta E \cdot 3D \text{ Volume}}$$

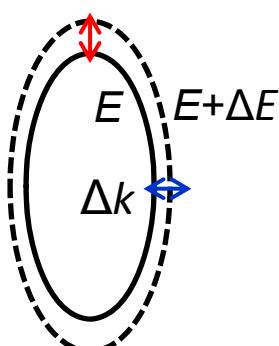
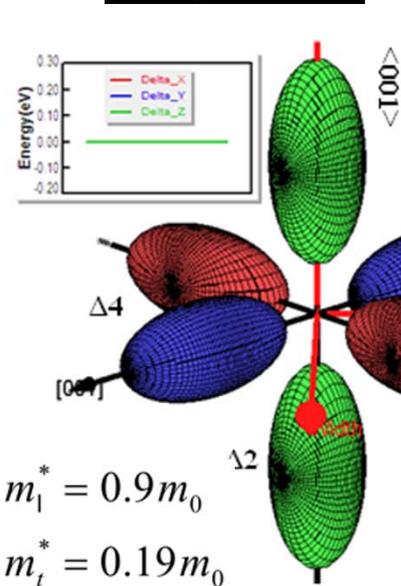
2D Inversion Layer:

$$DOS(E) \equiv \frac{\# \text{ of } k \text{ states in } \Delta E}{\Delta E \cdot 2D \text{ Area}}$$

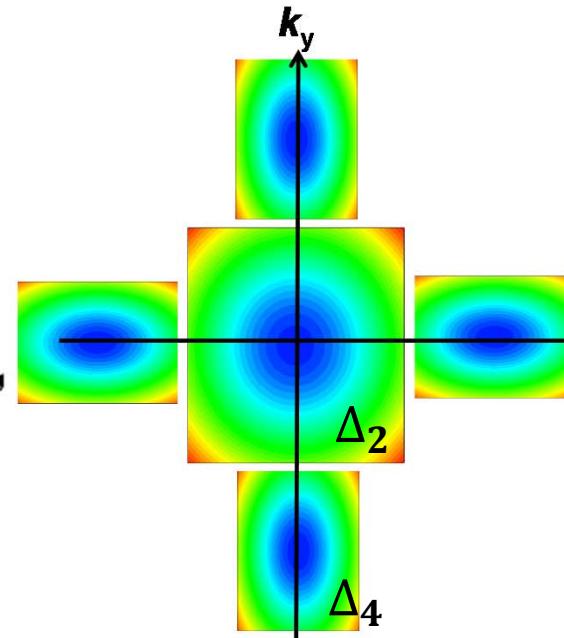


# QC Effect on Si Band Structure: Electrons

(100) Bulk Si

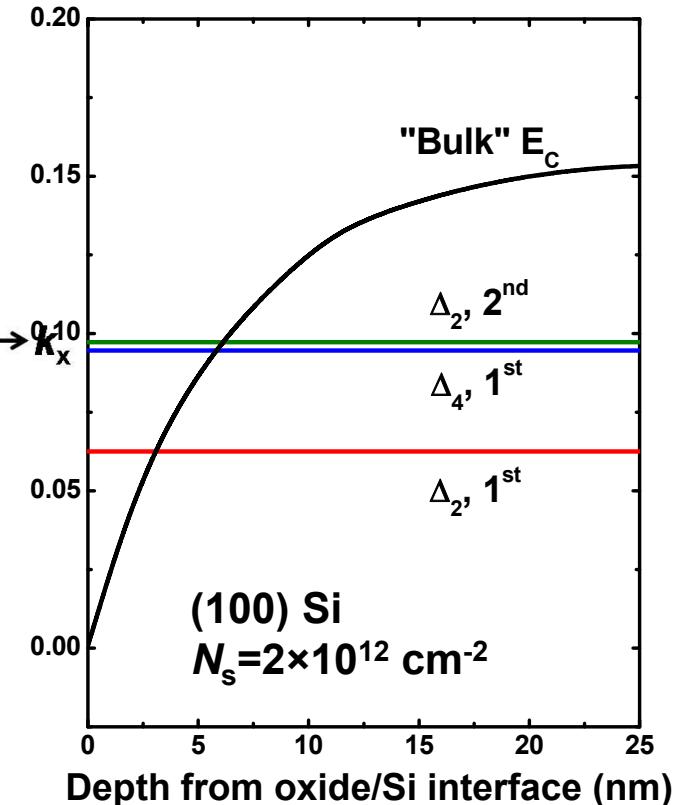


(100) Si Inversion Layer

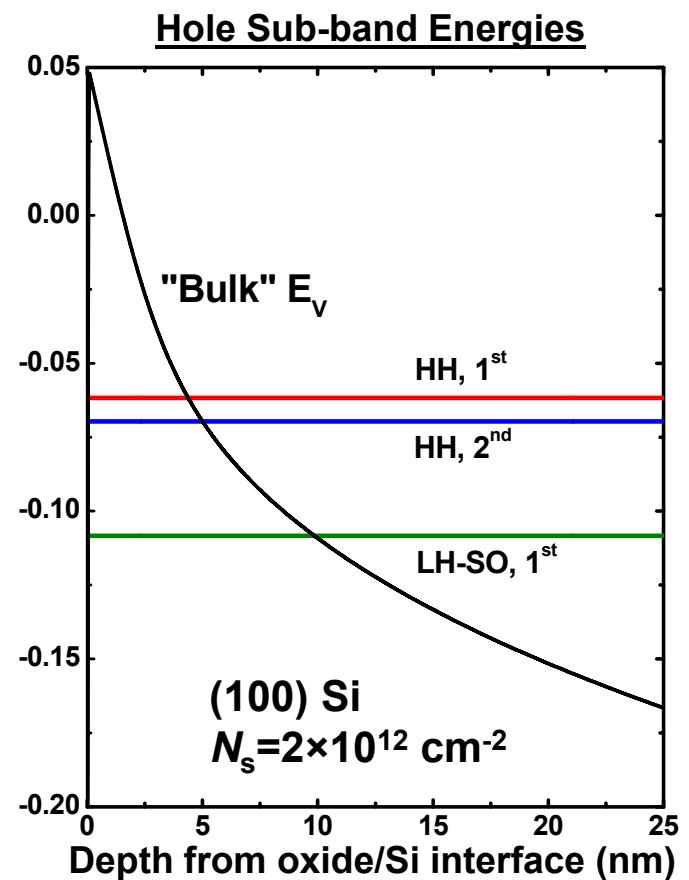
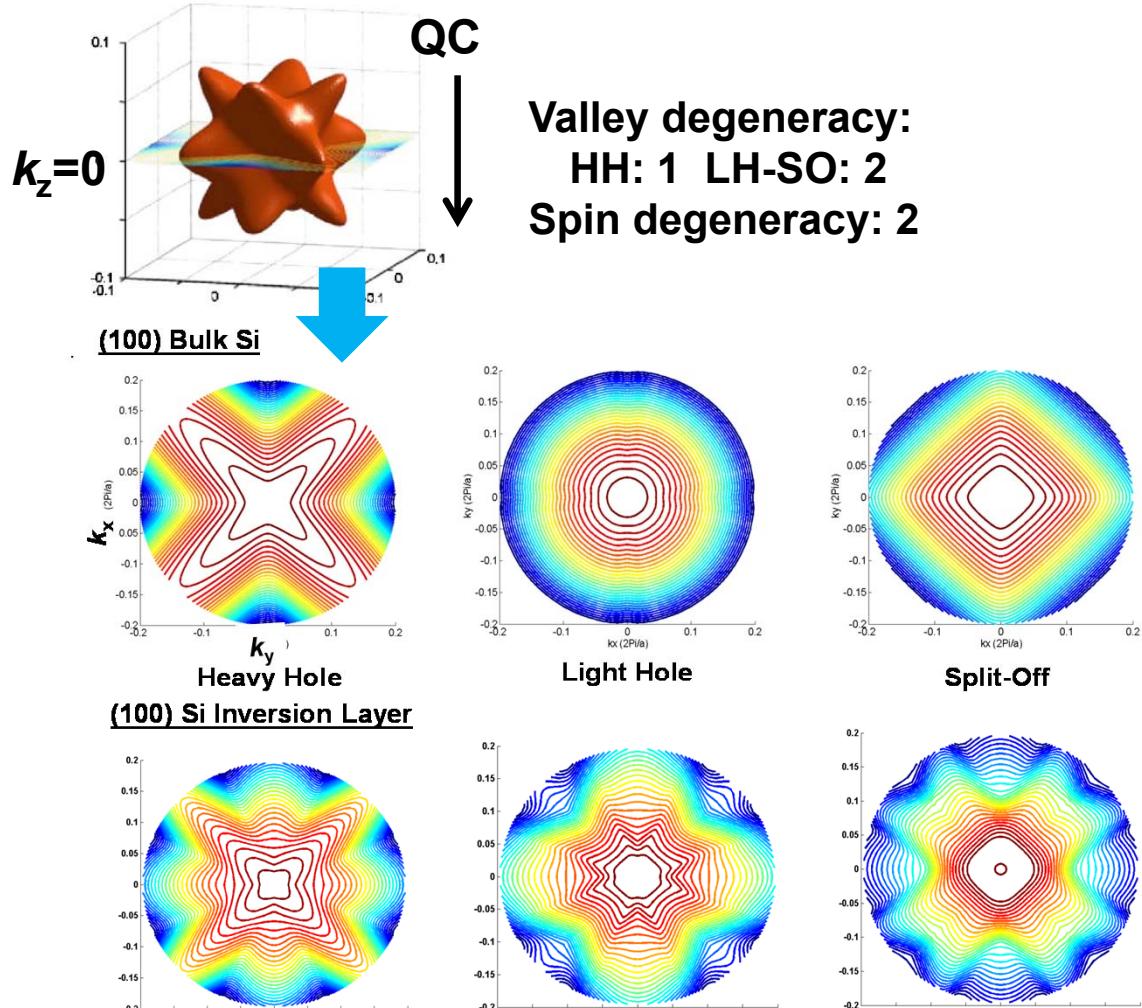


Valley degeneracy:  
 $\Delta_2$ : 2    $\Delta_4$ : 4  
 Spin degeneracy: 2

Electron Sub-band Energies



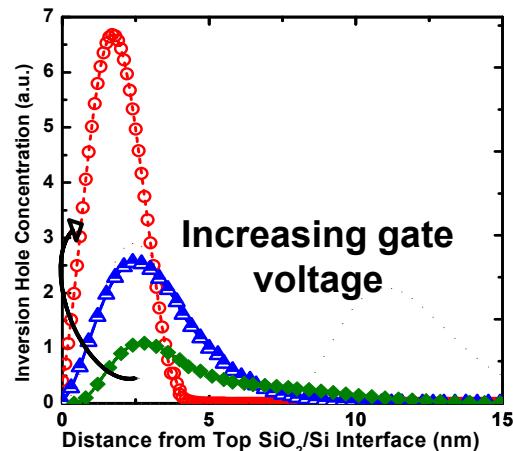
# QC Effect on Si Band Structure: Holes



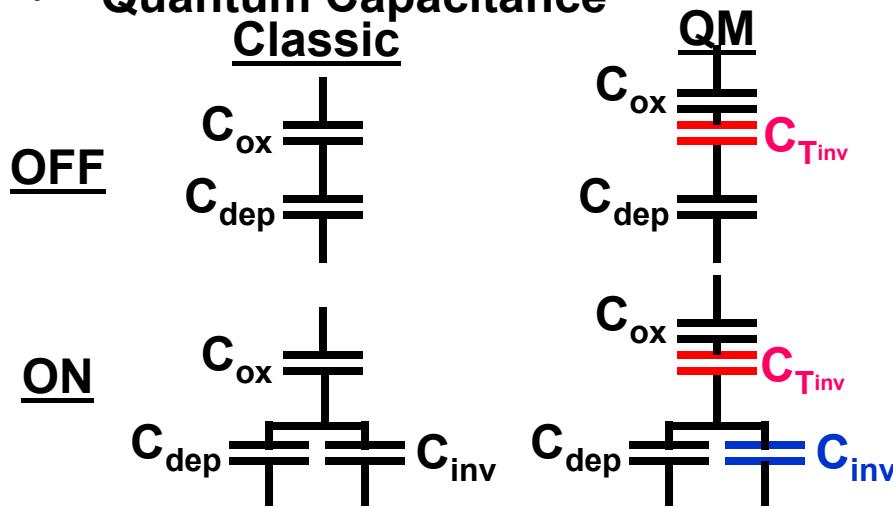
N. Xu, Ph.D. thesis (2012)

# Impacts of QC on MOS Electrostatics

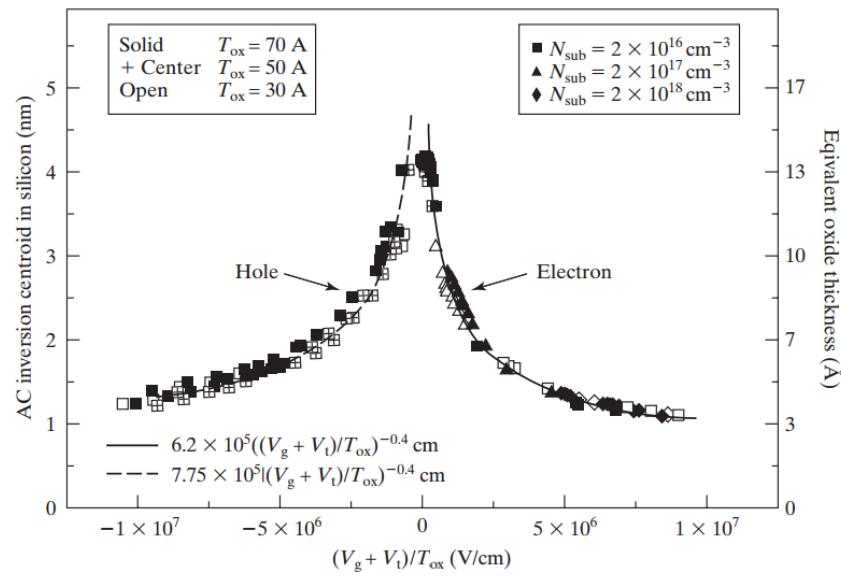
- Inversion Thickness ( $T_{inv}$ )



- Quantum Capacitance



### Measured $T_{inv}$ in Si Bulk CMOSFETs



K. Yang, VLSI-T (1999)

Hence there are 3 metrics to characterize  $C_{gate}$  ...

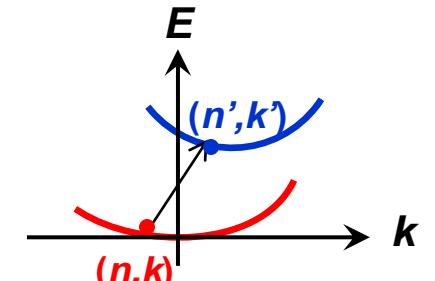
- Physical Oxide Thickness ( $t_{ox}$ )
- Effective Oxide Thickness (EOT)
- Electrical Effective Oxide Thickness (EOT<sub>elec</sub>)

# Carrier Mobility: A Quantum Mechanical View

- **Fermi Golden Rule:** transition rates between two quantum states

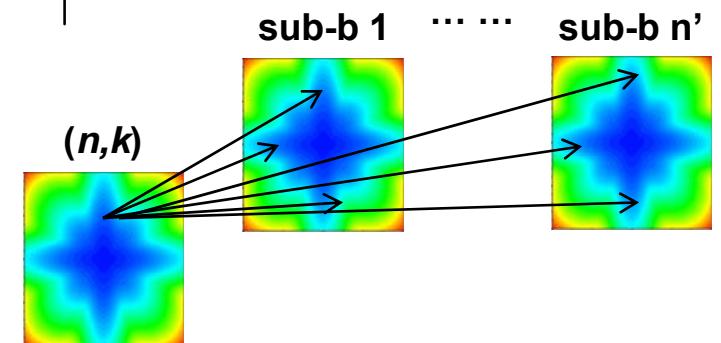
$$S_{n\vec{k},n'\vec{k}'} = \frac{2\pi}{\hbar} \left| M(n,\vec{k},n',\vec{k}') \right| \cdot \delta(E_n(\vec{k}) - E_{n'}(\vec{k}') \pm \hbar\omega)$$

wavefunction overlap along confinement direction  $F_{n,n'} = \int_z dz \cdot \left| \vec{\xi}_n(z) \cdot \vec{\xi}_{n'}^*(z) \right|^2$



- **mobility:** scattering-induced quantum state transitions

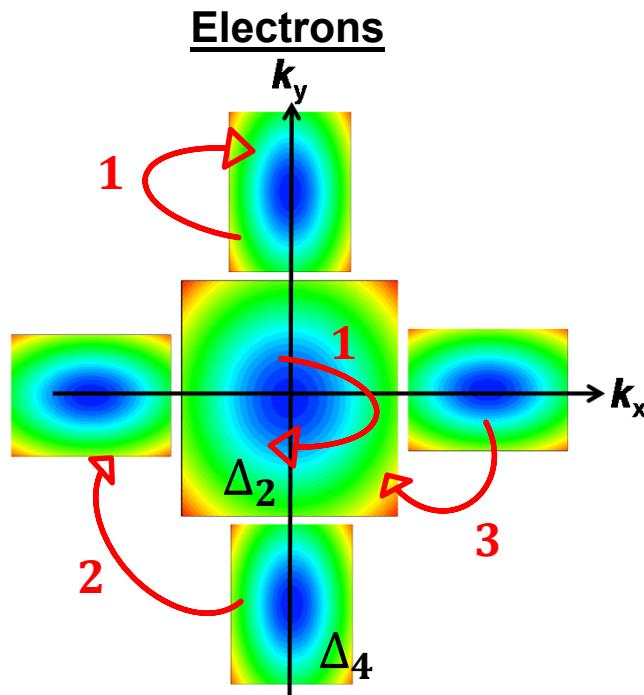
scattering rates for sub-band  $n$  at  $k$   $\frac{1}{\tau_n(\vec{k})} = \sum_{n'} \int_{\vec{k}} \frac{d\vec{k}'}{(2\pi)^2} \cdot S_{n\vec{k},n'\vec{k}'} \times \Phi(n\vec{k},n'\vec{k}')$



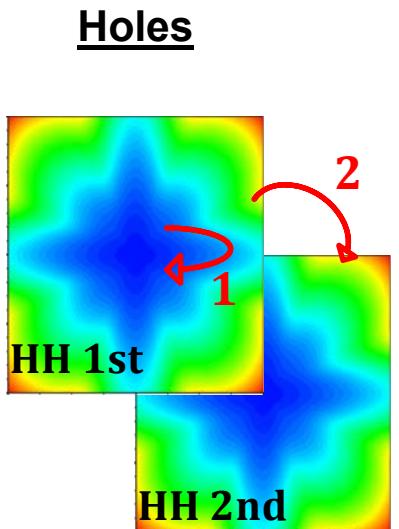
$$\mu_{i,j}^n = \frac{e}{\hbar^2} \cdot \frac{1}{k_B T} \cdot \frac{g_n}{n_n} \int_{\vec{k}} \frac{d\vec{k}}{(2\pi)^2} \cdot \tau_{i,j}^n \cdot \frac{\partial E_n}{\partial k_i} \cdot \frac{\partial E_n}{\partial k_j} \cdot f(E_n) \cdot (1 - f(E_n)) \quad \text{for the } n^{\text{th}} \text{ sub-band}$$

$$\mu_{i,j}^{tot} = \frac{1}{n_{tot}} \sum_n n_n \cdot \mu_{i,j}^n \quad \text{for the overall inversion layer}$$

# Transition Types of Carriers



- 1. intra-valley scattering
- 2.  $g$ -type inter-valley scattering
- 3.  $f$ -type inter-valley scattering



- 1. intra-sub-band scattering
- 2. Inter-sub-band scattering

$$\frac{1}{\tau_n(\vec{k})} = \sum_{n'} \int \frac{d\vec{k}'}{(2\pi)^2} \cdot S_{n\vec{k}, n'\vec{k}'} \times \Phi(n\vec{k}, n'\vec{k}')$$

Momentum Relaxation Factor

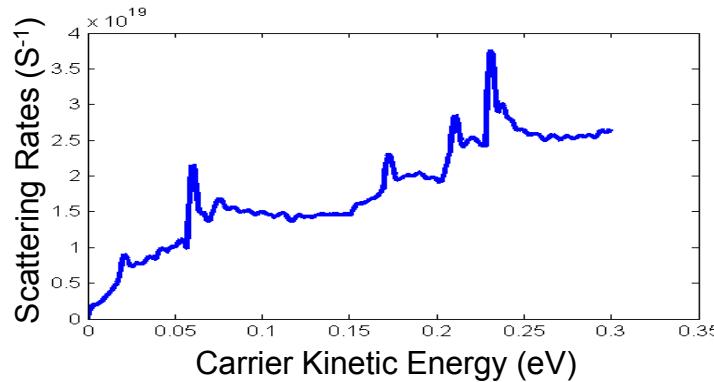
# Acoustic and Optical Phonon Scatterings

## Acoustic Phonon Scattering

AP: coherent movements of atoms, *i.e.* adjacent atoms move together.

### elastic scattering

$$M_{elast.}(n, \vec{k}, n', \vec{k}') = \frac{k_B T \Xi^2}{\rho u_l^2} \cdot F_{n, \vec{k}, n', \vec{k}'}$$

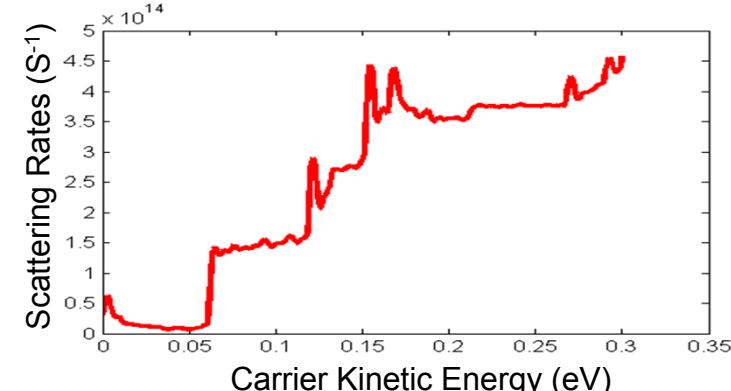


## Optical Phonon Scattering

OP: out-of-phase movements of atoms, *i.e.* adjacent atoms move in opposite directions.

### inelastic scattering

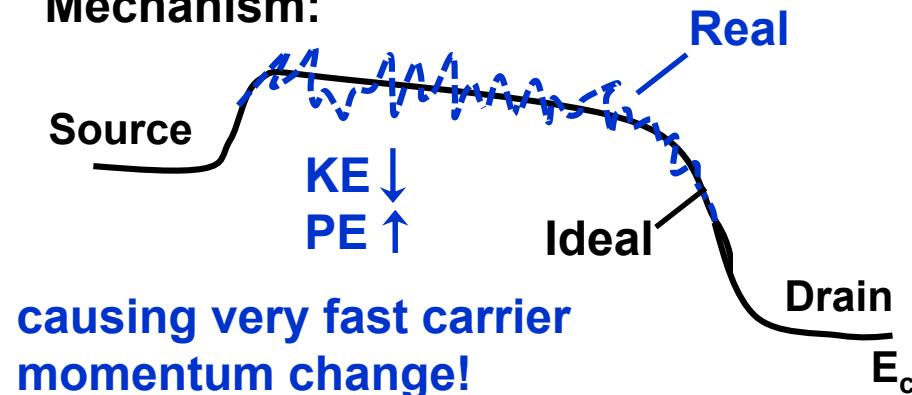
$$M_{inelast.}(n, \vec{k}, n', \vec{k}') = \frac{\hbar^2 \pi (D_t k)^2}{2 \rho(\hbar \omega)} \cdot F_{n, \vec{k}, n', \vec{k}'} \cdot \left( n_{op}(\hbar \omega) + \frac{1}{2} \mp \frac{1}{2} \right)$$



# Surface Roughness Scattering

L. Donetti, JAP (2009)

- Type: elastic scattering, mostly intra-valley/band
- Mechanism:



$$M_{surf.rough.}(n, \vec{k}, n', \vec{k}') = \int_z \xi_n(z) \cdot \vec{E} \cdot \xi_{n'}(z) dz + (E_n^0 - E_{n'}^0) \int_z \frac{d\xi_{n'}(z)}{dz} \left| \frac{\xi_n(z) dz}{2} \right|^2 \cdot S(\vec{q})$$

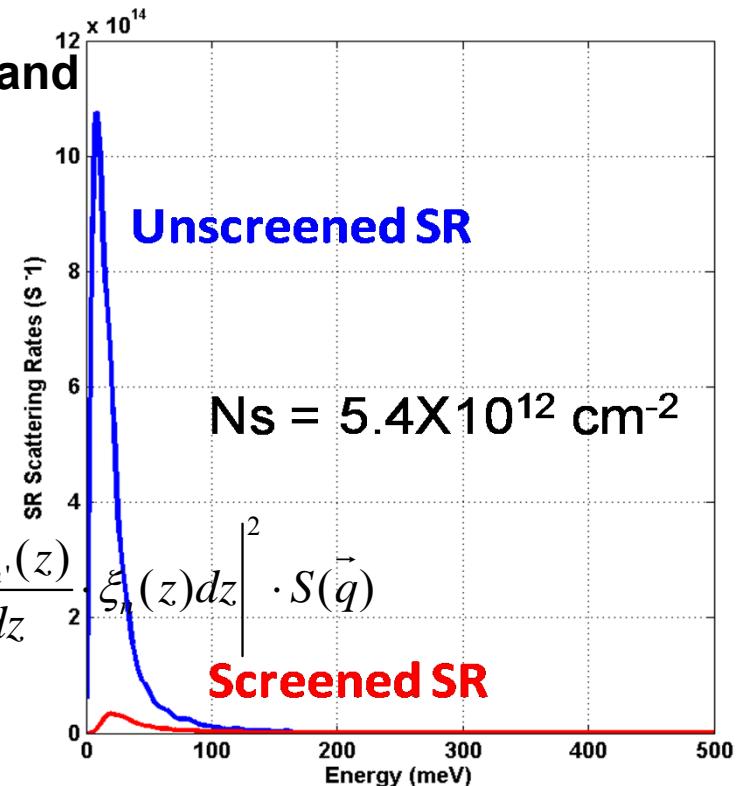
Power Spectrum of Surface Roughness

$$S(\vec{q}) = \frac{\pi(\Delta\lambda)^2}{[1 + (q\lambda)^2/2]^{3/2}}$$

Considering screening effect (important at high gate voltage):

$$\left| M_{screen}(n, \vec{k}, n', \vec{k}') \right|^2 = \frac{\left| M_{unscr.}(n, \vec{k}, n', \vec{k}') \right|^2}{\varepsilon^2(\vec{q})}$$

Dielectric function

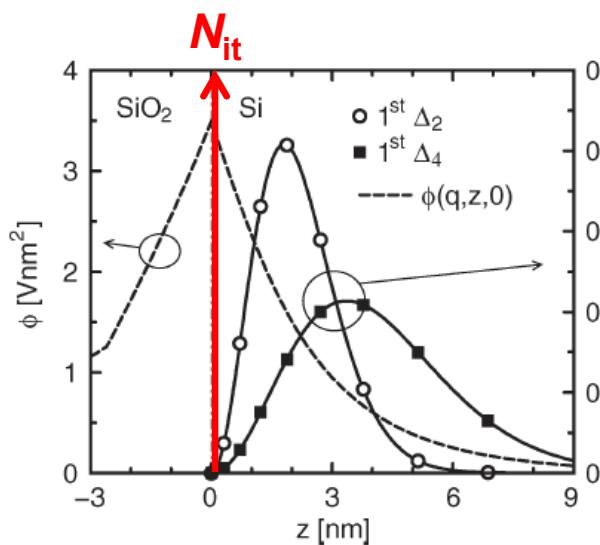


# Coulomb Scatterings

- Type: elastic scattering, mostly intra-band/valley
- Mechanism:

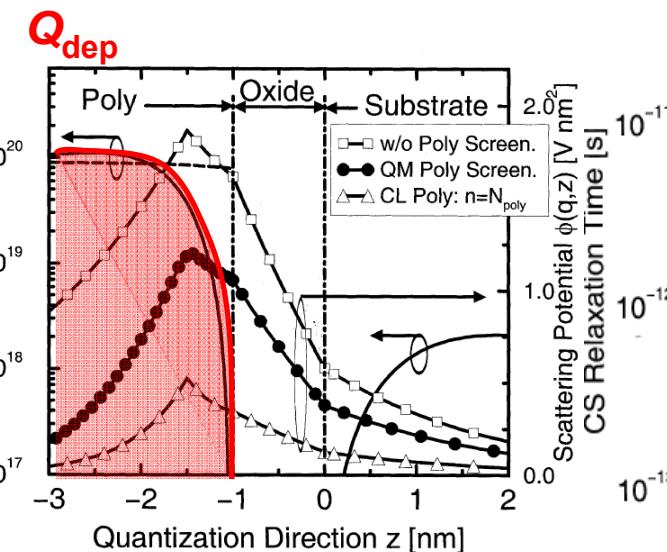
$$\phi_{unscr}(\vec{q}, z) = \frac{e}{\vec{q}(\epsilon_{Si} + \epsilon_{ox})} e^{-\vec{q}|z|} \quad M_{n,n'}(\vec{q}, z_0) = \frac{e}{A} \int_0^{\infty} \phi(\vec{q}, z, z_0) \xi_n(z) \xi_{n'}(z) dz$$

SiO<sub>2</sub>/Si Interface Charge



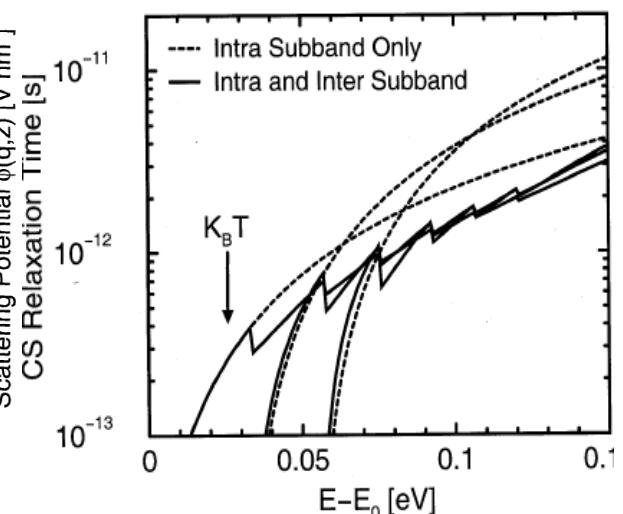
F. Driussi, TED (2009)

Depletion Charge



D. Esseni, TED (2003)

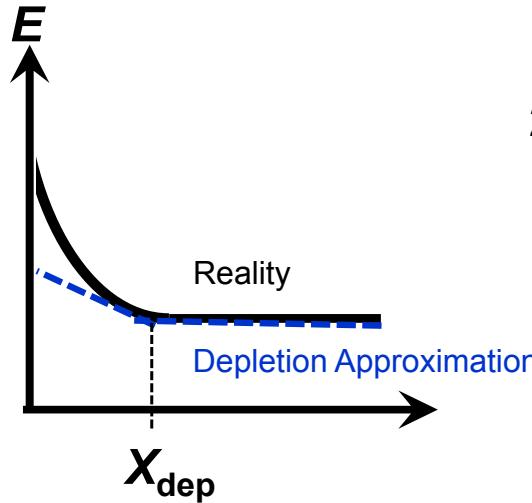
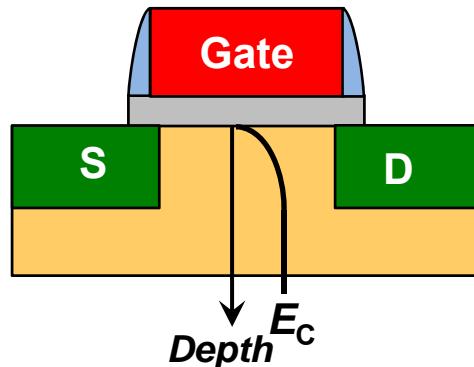
MRT (1/Scattering Rates)



F. Driussi, TED (2009)

# Effective Transverse Field ( $E_{\text{eff}}$ )

In a MOS Inversion Layer:



2D Sheet Charge Density:

$$Q_{\text{inv}} = \int_0^{T_{\text{inv}}} qn(x)dx$$

$$Q_{\text{dep}} = \int_0^{X_{\text{dep}}} qN_B(x)dx$$

Surface (oxide/Si interface) field:

$$E_s = \frac{Q_{\text{dep}} + Q_{\text{inv}}}{\epsilon_{Si}}$$

Bottom (of inversion layer) field:

$$E_b = \frac{Q_{\text{dep}}}{\epsilon_{Si}}$$

Average field:

$$E_{\text{ave}} = \frac{E_s + E_b}{2} = \frac{Q_{\text{dep}} + Q_{\text{inv}}/2}{\epsilon_{Si}} \equiv E_{\text{eff}}$$

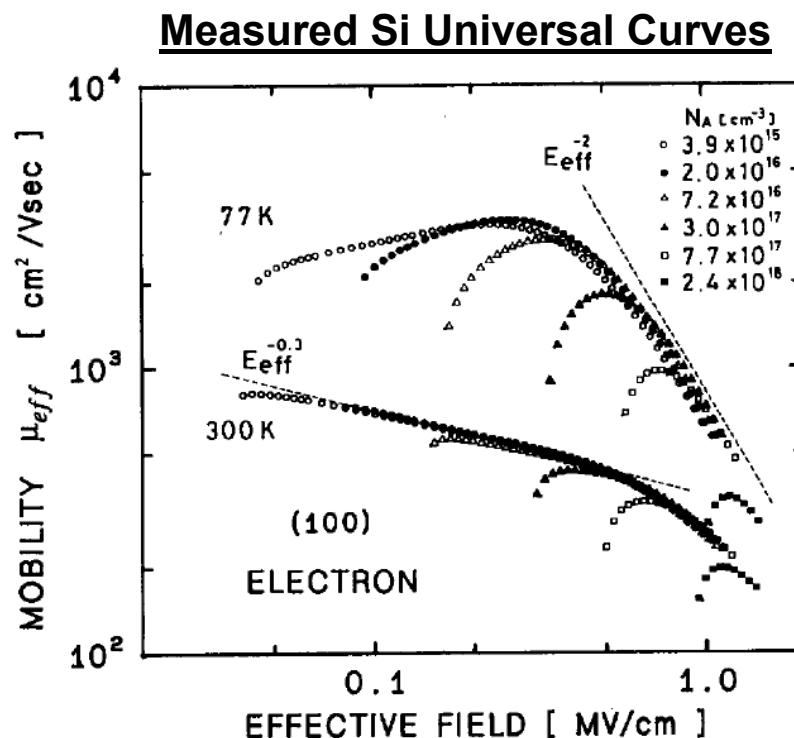
More generally:

$$E_{\text{eff}} = \frac{Q_{\text{dep}} + \alpha \cdot Q_{\text{inv}}}{\epsilon_{Si}}$$

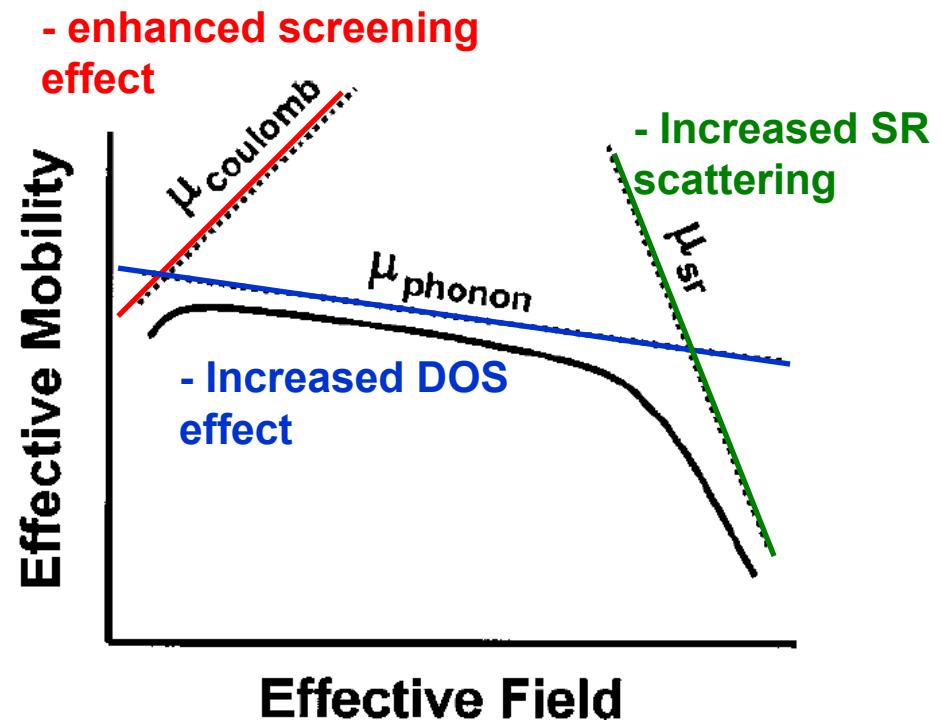
For (100) bulk Si MOSFET:  
Electrons:  $\alpha = 0.5$   
Holes:  $\alpha = 0.33$

# Universal Mobility Curve: $\mu$ vs. $E_{\text{eff}}$

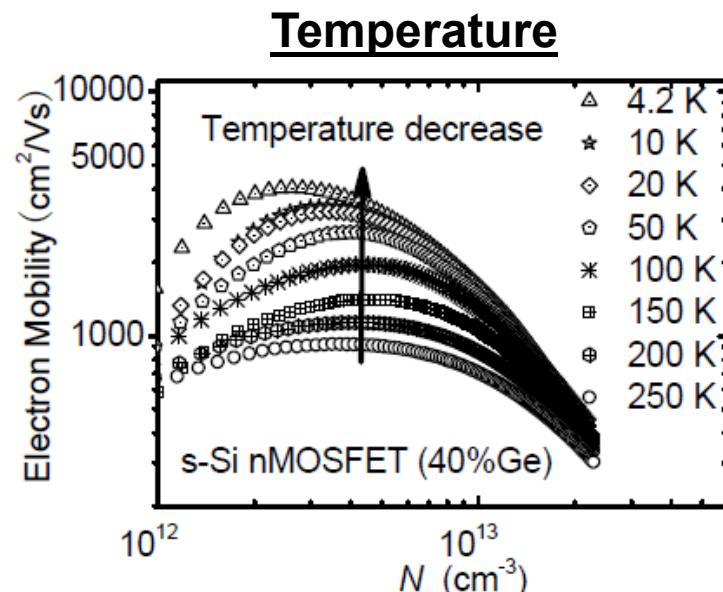
- Unify the electric field value by including the oxide thickness (compared to  $\mu$  vs.  $V_G$ ) and depletion charge effect (compared to  $\mu$  vs.  $N_{\text{inv}}$ ).



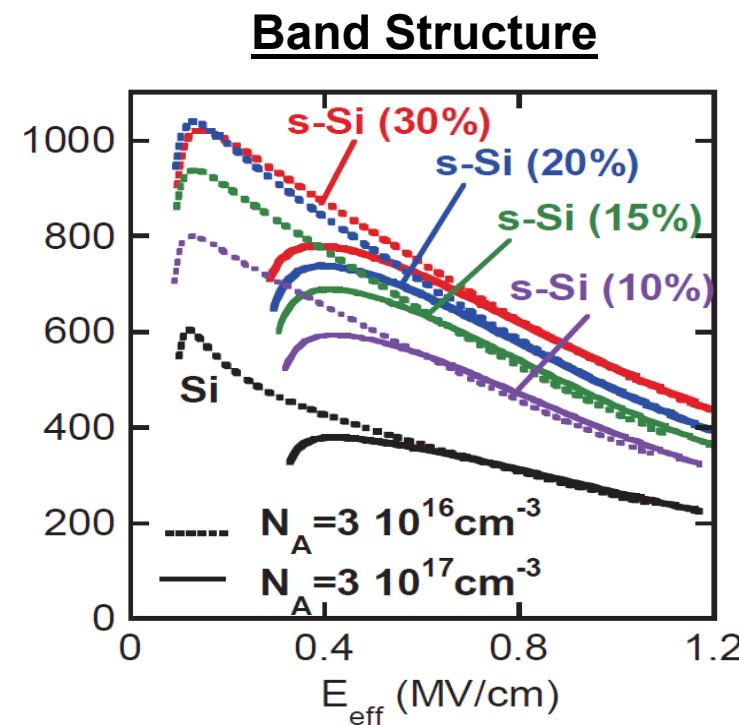
S. Takagi, TED (1994)



# Universal Mobility Curve: Dependent Factors



Y. Zhao, IEDM (2008)

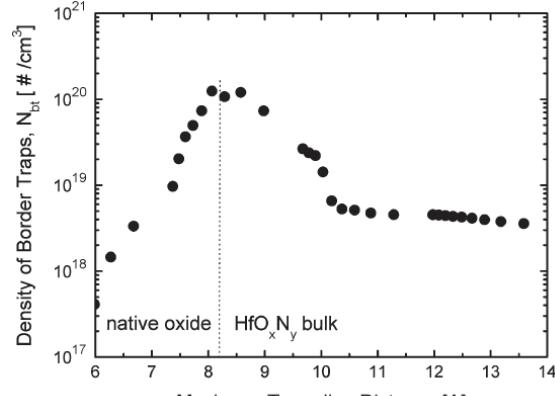


O. Weber, VLSI-T (2007)

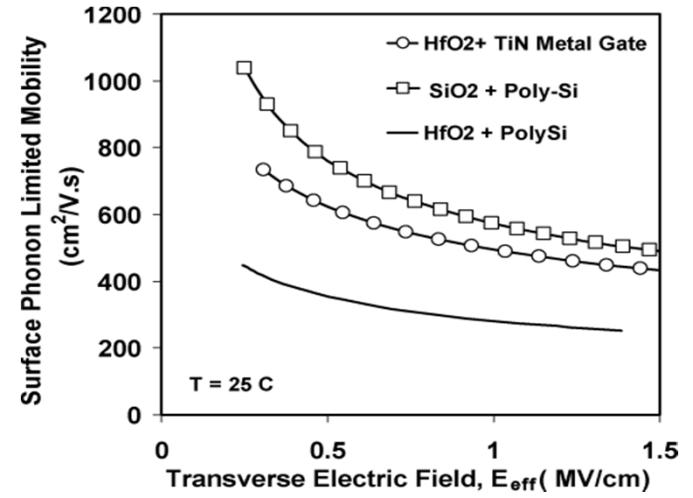
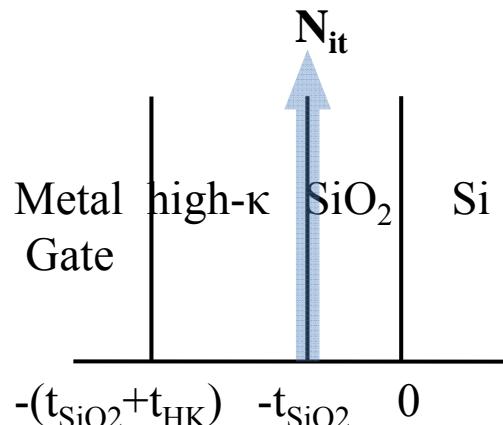
- Si universal mobility curves are often used to show a new technology's enhancement.

# High- $\kappa$ -induced Scatterings

- Remote Coulomb Scattering

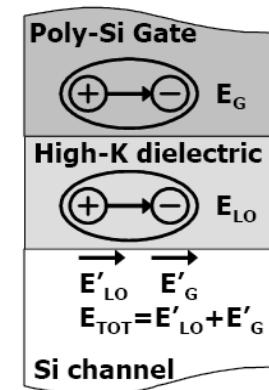
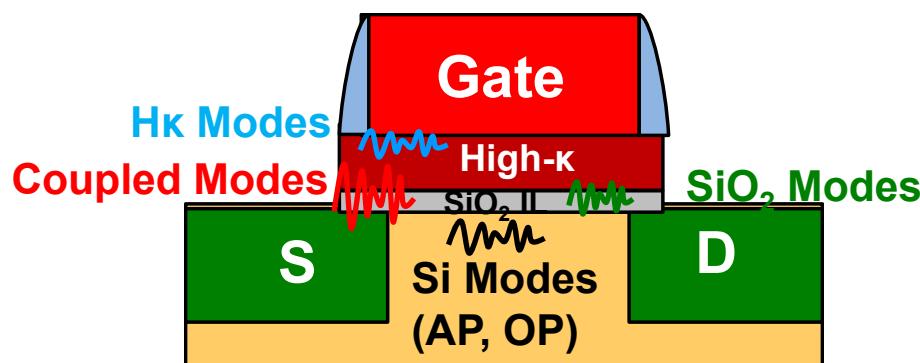


C.-Y. Lu, EDL (2006)

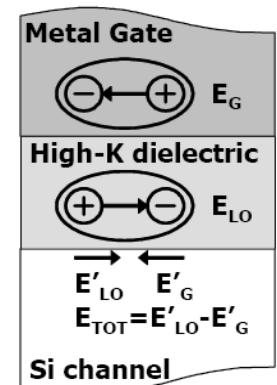


R. Chau, EDL (2004)

- Remote (Surface Optical) Phonon Scatterings



(a) In resonance

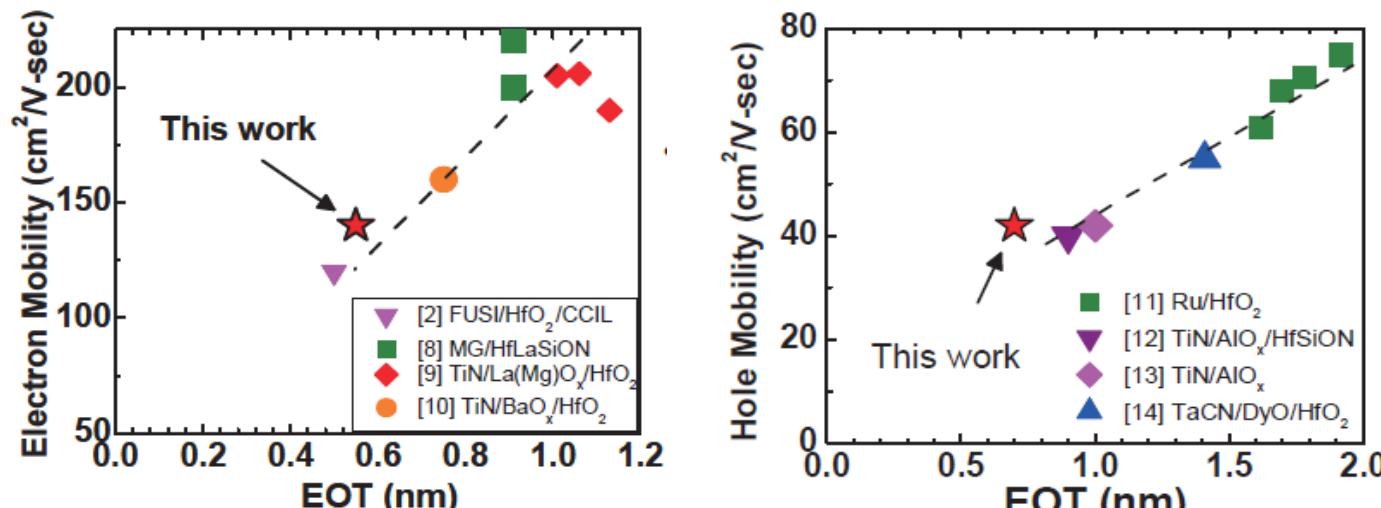


(b) Off resonance

Courtesy of D. Vesileska (ASU)

# Dependence of High- $\kappa$ Thickness

- Mobility doesn't follow universal curve.

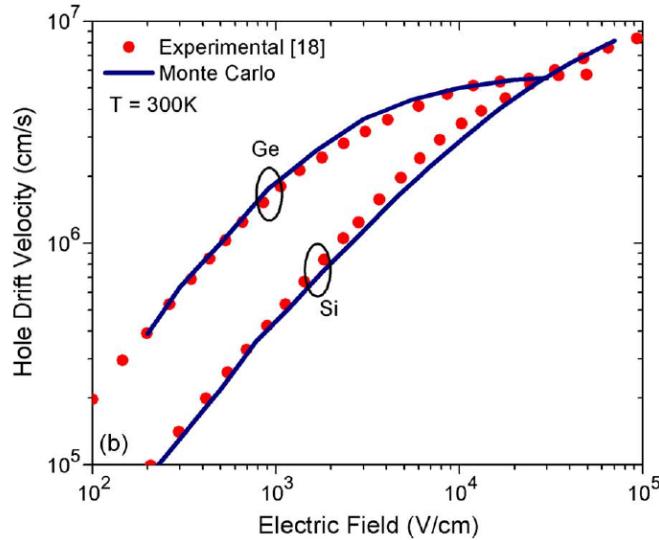


K. Choi, VLSI-T (2009)

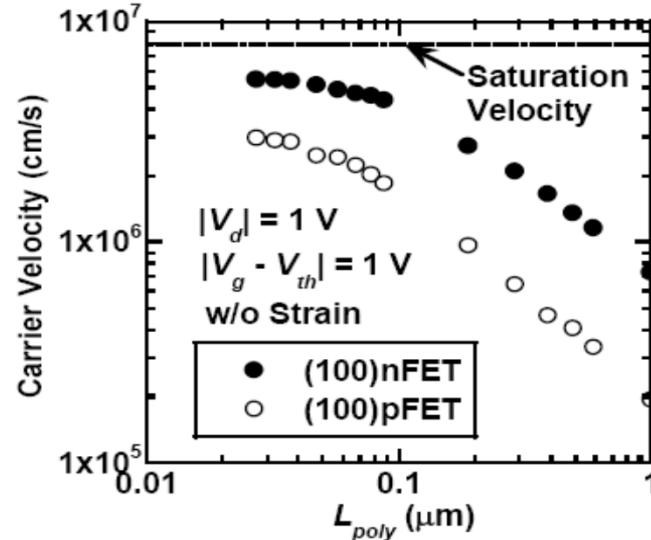
- By using metal-gate technology, RCS becomes the limiting factor in state-of-the-art MOSFET's mobility.

# Si Carrier Velocity Saturation

- Under high lateral electric field, carrier velocity saturates to a constant value, due to dramatically enhanced optical phonon scatterings.



B. Ho, TED (2011)



M. Saitoh, IEDM (2009)

- State-of-the-art technologies actually pushes away MOSFETs from velocity saturation, due to:
  - Increased doping/junction defects
  - High- $\kappa$ -induced scatterings
  - Ballistic transport (will be discussed in Lec. 5)
  - Reduced  $V_{DD}$

# References

## Band Structure

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4. K. Yang, Y.-C. King, C. Hu, "Quantum Effect in Oxide Thickness Determination From Capacitance Measurement," *Symposium on VLSI Technology Digest*, pp. 77-78, 1999.

## Mobility

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6. (SRS) L. Donetti, F. Gamiz, N. Rodriguez, A. Godoy, C. Sampedro, “The Effect of Surface Roughness Scattering on Hole Mobility in Double Gate Silicon-on-Insulator Devices,” *Journal of Applied Physics*, Vol.106, 023705, 2009.
7. (CS) F. Diussi, D. Esseni, “Simulation Study of Coulomb Mobility in Strained Silicon,” *IEEE Transactions on Electron Devices*, Vol.56, no.9, pp. 2052-2059, 2009.
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# References

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11. O. Weber, S. Takagi, “New Findings on Coulomb Scattering Mobility in Strained-Si nFETs and its Physical Understanding,” *Symposium on VLSI Technology Digest*, pp. 130-131, 2007.
12. (high- $\kappa$  trap) C.-Y. Lu, K.-S. C.-Liao, P.-H. Tsai, T.-K. Wang, “Depth Profiling of Border Traps in MOSFET With High- $\kappa$  Gate Dielectric by Charge-Pumping Technique,” *IEEE Electron Device Letters*, Vol. 27, no.10, pp. 859-861, 2006.
13. (high- $\kappa$ ) R. Chau, S. Datta, M. Doczy, B. Doyle, J. Kavalieros, M. Metz, “High- $\kappa$ /Metal Gate Stack and Its MOSFET Characteristics,” *IEEE Electron Device Letters*, Vol.25, no.6, pp. 408-410, 2004.
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