FINAL EXAM

Must be turned in 5:00PM on Friday, December 14, 2001 at 508 Cory. LATE PAPERS WILL NOT BE GRADED. No exceptions this time!

This is a take-home, open-book exam. You may consult any sources you wish, however, only results from class notes, or standard references (integral tables, physical constants, etc.) may be cited without derivation. You must work strictly individually. No collaboration with anyone else, either in or out of this class is permitted.

If you have questions regarding the exam, you can submit them via email. I will also post office hours to be held during the week of Dec. 10-14 on the web site. Any clarifications, corrections, or other information of importance to everyone will be posted to the web site. Check it frequently.

1. [30 points total] Consider a thin periodic grating with period $L$.
   a) [5 points] Let the grating be translated in the transverse direction perpendicular to the grating periods. Show that for a translation distance of $L/2$, there is a relative phase shift of $\pi$ that is introduced between any two adjacent diffraction orders.
   b) [10 points] Let the amplitude transmittance function of the grating be as shown in Fig. 1.

   ![Figure 1](image)

   What is the fraction of light diffracted into the first order? What is the value of $t_m$ that maximizes this efficiency?

   c) [15 points] Suppose the grating is a pure square-wave phase grating such that the phase of the transmitted light jumps between 0 and $\phi$ radians. Find the diffraction efficiency of this grating for the first order, and then find the value of $\phi$ that maximizes this efficiency. If we want to maximize the efficiency of diffraction into the third order, do we choose a different value of $\phi$? Explain.
2. [35 points total] Consider the scanning microscope shown in Fig. 2. Light from a point source P is collected and focused by lens 1 onto the partially transmitting sample. Light transmitted through the sample is collected by lens 2 and focused onto the detector. The detector is not an imaging detector, but simply integrates the intensity of light falling onto it’s surface to produce a single proportionate output level. The sample is placed on an x-y stage and an image is formed by scanning the stage and then displaying the detector signal as a function of the stage position.

Assume that all the distances, d, are equal. Assume that the detector has a large enough area so that all of the light collected by lens 2 reaches the detector. Further assume that the detector sensitivity is uniform over it’s full area.

a) [10 points] Take the limit where lens 2 is infinitely large, so that all light transmitted through the sample is detected. Show that the image formed is equivalent to incoherent imaging in a conventional microscope. That is:

\[ I(x_s, y_s) = |h_1|^2 \otimes |t|^2 \]  

where \( h_1 \) is the point-spread function for lens 1 at the sample plane, \( t \) is the amplitude transmission function for the sample, and \((x_s, y_s)\) represent the coordinates of the scanning stage.

b) [10 points] Now take the limit where lens 2 has a very small collection numerical aperture. Show that the image formed is equivalent to coherent imaging in a conventional microscope. That is:

\[ I(x_s, y_s) = |h_1 \otimes t|^2 \]  

Notice that in this microscope, lens 1 behaves as the imaging lens, while lens 2 plays a role similar to the condenser lens in a conventional microscope!

c) [15 points] Now assume that lens 1 and lens 2 are identical. Furthermore, a very small pinhole is placed in front of the detector at the focus of lens 2. Show that the image formed in this mode (confocal scanning microscope) is:

\[ I(x_s, y_s) = |h_1^2 \otimes t|^2 \]
3. [35 points total] Consider the Fourier transform holography configuration shown in Fig. 3.

Assume that the object consists of a square transparency of width $L$. The detector is a square imaging array of width $X$, and has $N*N$ detector pixels. The object is relatively small and $d$, the distance to the detector, is far enough to be in the Fraunhofer far-field.

a) [15 points] Show how the reconstruction of the object can be calculated by taking a suitable Fourier transform of the image recorded on the detector.

b) [10 points] How far from the center of the object transparency should the reference point source be placed in order to assure that the reconstructed twin images are separated from the on-axis terms?

c) [10 points] What determines the spatial resolution with which the object can be reconstructed in this geometry?