FINAL EXAM

Must be turned in 5:00PM on Friday, December 14, 2001 at 508 Cory. LATE PAPERS WILL NOT BE GRADED. No exceptions this time!

This is a take-home, open-book exam. You may consult any sources you wish, however, only results from class notes, or standard references (integral tables, physical constants, etc.) may be cited without derivation. **You must work strictly individually. No collaboration with anyone else, either in or out of this class is permitted.**

If you have questions regarding the exam, you can submit them via email. I will also post office hours to be held during the week of Dec. 10-14 on the web site. Any clarifications, corrections, or other information of importance to everyone will be posted to the web site. Check it frequently.

1. [30 points total] Consider a thin periodic grating with period L.

   a) [5 points] Let the grating be translated in the transverse direction perpendicular to the grating periods. Show that for a translation distance of $L/2$, there is a relative phase shift of π that is introduced between any two adjacent diffraction orders.

   b) [10 points] Let the amplitude transmittance function of the grating be as shown in Fig. 1.

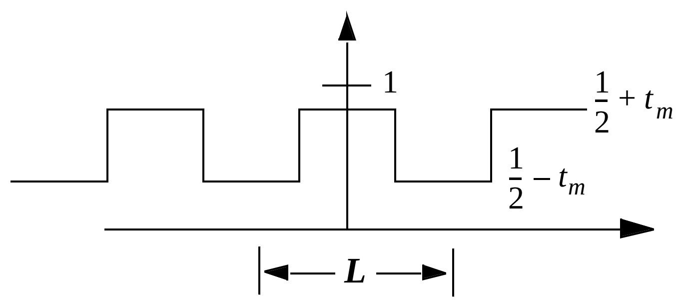

Figure 1

   What is the fraction of light diffracted into the first order? What is the value of t_m that maximizes this efficiency?

   c) [15 points] Suppose the grating is a pure square-wave phase grating such that the phase of the transmitted light jumps between 0 and ϕ radians. Find the diffraction efficiency of this grating for the first order, and then find the value of ϕ that maximizes this efficiency. If we want to maximize the efficiency of diffraction into the third order, do we choose a different value of ϕ? Explain.

2. [35 points total] Consider the scanning microscope shown in Fig. 2. Light from a point

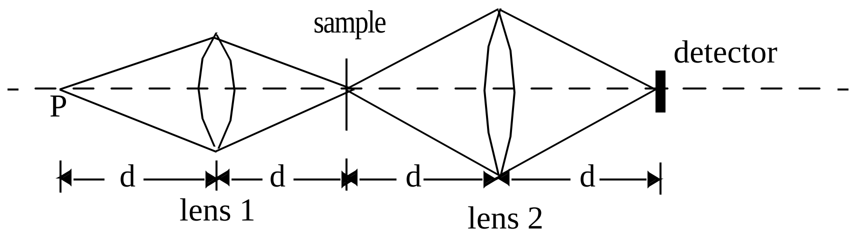

Figure 2

source P is collected and focused by lens 1 onto the partially transmitting sample. Light transmitted through the sample is collected by lens 2 and focused onto the detector. The detector is not an imaging detector, but simply integrates the intensity of light falling onto it's surface to produce a single proportionate output level. The sample is placed on an x-y stage and an image is formed by scanning the stage and then displaying the detector signal as a function of the stage position.

Assume that all the distances, d, are equal. Assume that the detector has a large enough area so that all of the light collected by lens 2 reaches the detector. Further assume that the detector sensitivity is uniform over it's full area.

a) [10 points] Take the limit where lens 2 is infinitely large, so that all light transmitted through the sample is detected. Show that the image formed is equivalent to incoherent imaging in a conventional microscope. That is:

$$I(x_s, y_s) = |h_1|^2 \otimes |t|^2 \tag{1}$$

where h_1 is the point-spread function for lens 1 at the sample plane, t is the amplitude transmission function for the sample, and (x_s , y_s) represent the coordinates of the scanning stage.

b) [10 points] Now take the limit where lens 2 has a very small collection numerical aperture. Show that the image formed is equivalent to coherent imaging in a conventional microscope. That is:

$$I(x_s, y_s) = |h_1 \otimes t|^2 \tag{2}$$

Notice that in this microscope, lens 1 behaves as the imaging lens, while lens 2 plays a role similar to the condenser lens in a conventional microscope!

c) [15 points] Now assume that lens 1 and lens 2 are identical. Furthermore, a very small pinhole is placed in front of the detector at the focus of lens 2. Show that the image formed in this mode (confocal scanning microscope) is:

$$I(x_s, y_s) = |h_1^2 \otimes t|^2 \tag{3}$$

3. [35 points total] Consider the Fourier transform holography configuration shown in Fig. 3.

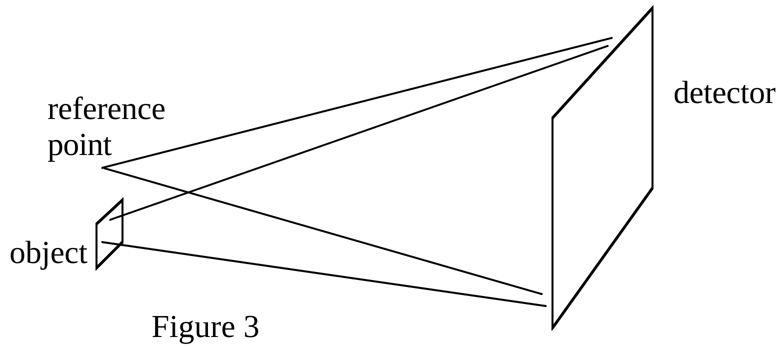

Figure 3

Assume that the object consists of a square transparency of width L. The detector is a square imaging array of width X, and has $N*N$ detector pixels. The object is relatively small and d, the distance to the detector, is far enough to be in the Fraunhofer far-field.

a) [15 points] Show how the reconstruction of the object can be calculated by taking a suitable Fourier transform of the image recorded on the detector.

b) [10 points] How far from the center of the object transparency should the reference point source be placed in order to assure that the reconstructed twin images are separated from the on-axis terms?

c) [10 points] What determines the spatial resolution with which the object can be reconstructed in this geometry?