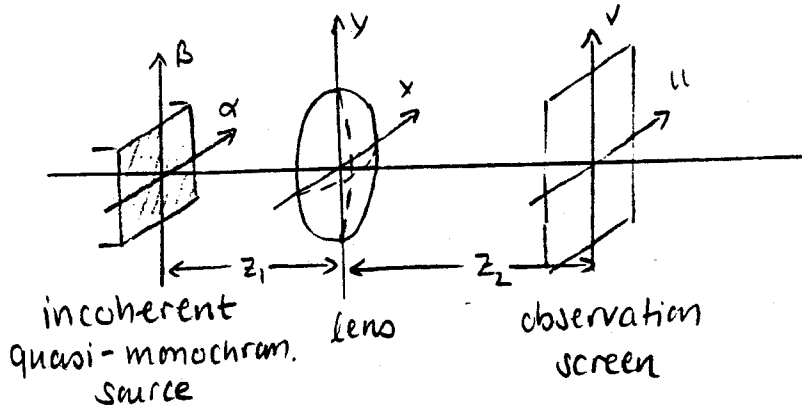
① Problem 5-5



Note that according to (eq. 5.2-35), the amplitude and phase of the observed fringes depends on the mutual intensity incident at the two pinholes.

From the van Cittert-Zernike thm, we find the mutual intensity $\underline{J}'$ incident on the lens:

$$\underline{J}'(x_1, y_1; x_2, y_2) = \frac{K}{(\lambda z_1)^2} e^{-j\frac{\pi}{\lambda z_1}(\rho_2^2-\rho_1^2)} \iint_{-\infty}^{\infty} I(\alpha,\beta)\, e^{j\frac{2\pi}{\lambda z_1}(\Delta x\alpha + \Delta y\beta)}\, d\alpha\, d\beta$$

where $\rho_1 \equiv \sqrt{x_1^2+y_1^2}$, $\rho_2 \equiv \sqrt{x_2^2+y_2^2}$. Thus the mutual intensity $\underline{J}''$ incident on the pinholes is

$$\underline{J}''(x_1, y_1; x_2, y_2) = e^{-j\frac{\pi}{\lambda f}(\rho_1^2-\rho_2^2)}\, \underline{J}'(x_1, y_1; x_2, y_2)$$

Using (eq. 5.2-37) for the spatial phase of the fringes ϕ_{12}, we find

$$\phi_{12} = \arg\{\underline{J}''(x_1, y_1; x_2, y_2)\} - \frac{\pi}{\lambda z_2}(\rho_2^2-\rho_1^2).$$

This can be written as

$$\Phi_{12} = -\frac{\pi}{\lambda}\left[\frac{1}{z_1}+\frac{1}{z_2}-\frac{1}{f}\right](\rho_2^2-\rho_1^2) + \arg\left\{\iint_{-\infty}^{\infty} I(\alpha,\beta)\, e^{j\frac{2\pi}{\lambda z_1}(\Delta x\alpha+\Delta y\beta)}\, d\alpha\, d\beta\right\}$$

If the distances z_1 and z_2 satisfy the lens law

$$\boxed{\frac{1}{z_1}+\frac{1}{z_2}=\frac{1}{f}}$$

this expression can clearly be seen to only depend on the relative separations $\Delta x, \Delta y$ of the pinholes.

(2) <u>Text</u>, <u>Prob 5-6</u> (Michelson Interferometer)

(a) Assume 50% beam splitter so that the output field is given by

$$\underline{v}(t) = \underline{u}(t) + \underline{u}(t+\tau_d)$$

where $\tau_d = 2h/c$ is relative time delay.

So,

$$\underline{\Gamma}_v(\tau) = \langle \underline{v}(t+\tau)\, \underline{v}^*(t)\rangle$$

$$= 2\underline{\Gamma}_u(\tau) + \underline{\Gamma}_u(t-\tau_d) + \underline{\Gamma}_u(t+\tau_d)$$

and,

$$\mathcal{G}_v(\nu) = \mathcal{F}\{\underline{\Gamma}_v(\tau)\} = 2\mathcal{G}_u(\nu) + e^{j2\pi\nu\tau_d}\mathcal{G}_u(\nu) + e^{-j2\pi\nu\tau_d}\mathcal{G}_u(\nu)$$

$$= 2\mathcal{G}_u(\nu)\left[1+\cos(2\pi\nu\tau_d)\right]$$

The corresponding normalized version:

$$\underline{\gamma}_v(\tau) = \frac{\underline{\Gamma}_v(\tau)}{\underline{\Gamma}_v(0)} = \frac{2\underline{\Gamma}_u(\tau) + \underline{\Gamma}_u(\tau-\tau_d) + \underline{\Gamma}_u(\tau+\tau_d)}{2\underline{\Gamma}_u(0) + \underline{\Gamma}_u(-\tau_d) + \underline{\Gamma}_u(\tau_d)}$$

Dividing each term by $2\underline{\Gamma}_u(0)$ yields,

$$\underline{\gamma}_v(\tau) = \frac{\underline{\gamma}_u(\tau) + \frac{1}{2}\underline{\gamma}_u(\tau-\tau_d) + \frac{1}{2}\underline{\gamma}_u(\tau+\tau_d)}{1 + \mathrm{Re}\{\underline{\gamma}_u(\tau_d)\}}$$

and,

$$\hat{\mathcal{G}}_v(\nu) = \mathcal{F}\{\underline{\gamma}_v(\tau)\} = \frac{\hat{\mathcal{G}}_u(\nu)\,[1 + \cos(2\pi\nu\tau_d)]}{1 + \mathrm{Re}\{\underline{\gamma}_u(\tau_d)\}}$$

For $\tau_d \gg \tau_c \sim 1/\Delta\nu$, the output spectrum $\mathcal{G}_v(\nu)$ contains spectral fringes (e.g. Text, Fig. 5-14) and is significantly different in form than input $\mathcal{G}_u(\nu)$.

(b) Consider a rectangular input spectrum.

$$\hat{\mathcal{G}}_u(\nu) = \frac{1}{\Delta\nu}\,\mathrm{rect}\left(\frac{\nu-\bar{\nu}}{\Delta\nu}\right)$$

So,

$$\underline{\gamma}_u(\tau) = \mathcal{F}^{-1}\{\hat{\mathcal{G}}_u(\nu)\} = \mathrm{sinc}(\tau\Delta\nu)\,e^{-j2\pi\bar{\nu}\tau}$$

Using the result of part (a),

$$\hat{\mathcal{G}}_v(\nu) = \frac{\frac{1}{\Delta\nu}\,\mathrm{rect}\left(\frac{\nu-\bar{\nu}}{\Delta\nu}\right)\left[1 + \cos(2\pi\nu\tau_d)\right]}{1 + \mathrm{sinc}(\tau_d\Delta\nu)\cos(2\pi\bar{\nu}\tau_d)}$$

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③ Text, Prob 5-7

Start with Eq (5.3-8) letting $\tau_d = \tau_2 - \tau_1 = \frac{r_2 - r_1}{c}$

$$\hat{\mathcal{J}}_Q(\nu) = \frac{\hat{\mathcal{J}}(\nu) + A\, \mathrm{Re}\{\hat{\underline{\mathcal{J}}}_{12}(\nu)\, e^{-j2\pi\nu\tau_d}\}}{1 + A\, \mathrm{Re}\{\underline{\gamma}_{12}(\tau_d)\}}$$

For $\tau_d \gg \tau_c$, $\underline{\gamma}_{12}(\tau_d) \simeq 0$ and

$$\hat{\mathcal{J}}_Q(\nu) \simeq \hat{\mathcal{J}}(\nu) + A\, \mathrm{Re}\{\hat{\underline{\mathcal{J}}}_{12}(\nu)\, e^{-j2\pi\nu\tau_d}\}$$

Here
$$A = \frac{2\sqrt{I^{(1)} I^{(2)}}}{I^{(1)} + I^{(2)}}.$$

Cross-spectral purity condition:

$$\underline{\gamma}_{12}(\tau) = \underline{\mu}_{12}\, \underline{\gamma}(\tau)$$

or, by Fourier transforming,

$$\hat{\underline{\mathcal{J}}}_{12}(\nu) = \underline{\mu}_{12}\, \hat{\mathcal{J}}(\nu)$$

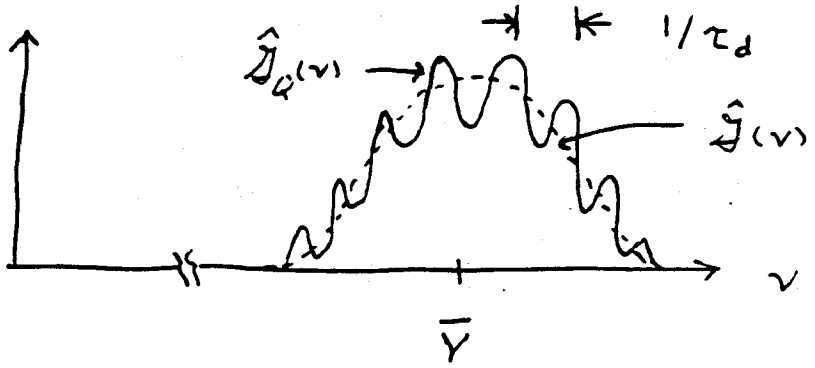
So,

$$\hat{\mathcal{J}}_Q(\nu) \simeq \hat{\mathcal{J}}(\nu)\left[1 + A\mu_{12}\cos(2\pi\nu\tau_d - \alpha_{12})\right]$$

where $\mu_{12} = |\underline{\mu}_{12}|$, $\alpha_{12} = \arg\{\underline{\mu}_{12}\}$

Since $\tau_d \gg \tau_c \sim 1/\Delta\nu$, spectrum at Q has observable fringes. For a given value of A, we can estimate μ_{12} and α_{12} from the

Visibility and phase shift of the spectral fringes.



To determine phase shift, we must first pick a reference point in the spectrum, perhaps $\bar{\nu}$.

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④ Problem 5-11

According to eq. (5.2-42) diffraction effects due to the finite pinhole size produce an envelope of the intensity with a half-width of the order

$$\Delta x_a \cong \frac{\bar{\lambda}(2f)}{\delta} = \frac{c(2f)}{\delta \bar{\nu}}$$

Note that since the pinholes are located in the pupil plane (Fourier transform plane) of the optical system, the images of the source due to the two pinholes overlap completely.

According to eq. (5.2-28) the finite bandwidth of the source produces an envelope of the fringe visibility with a half-width of the order

$$\Delta x_b \cong \frac{c(2f)}{\Delta\nu\, s}$$

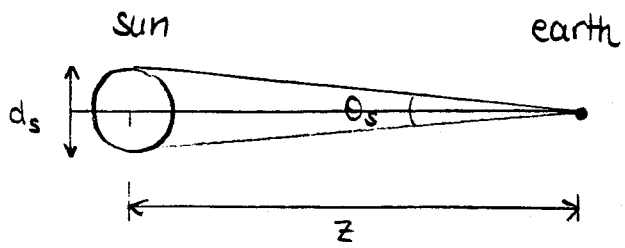
In order for effect (a) to dominate over effect (b) we must have

$$\Delta x_a \ll \Delta x_b \Rightarrow \delta\bar{\nu} \gg \Delta\nu\, s$$

or,

$$\boxed{\frac{\Delta\nu}{\bar{\nu}} \ll \frac{\delta}{s}}$$

⑤ Problem 5-13



coherence area: A_c
diameter of A_c: d_c
cross-sectional area of the sun: A_s

from eq. (5.6-12) we know

$$A_c = \frac{\bar{\lambda}^2 z^2}{A_s} \Rightarrow \frac{\pi d_c^2}{4} = \frac{4\bar{\lambda}^2 z^2}{\pi d_s^2} \cong \frac{4\bar{\lambda}^2}{\pi \theta_s^2}$$

$$d_c = \frac{4}{\pi}\frac{\bar{\lambda}}{\theta_s} = \frac{4}{\pi}\frac{0.55\,\mu m}{0.0093} = \underline{\underline{75.3\,\mu m}}$$

⑥ Problem 5-17

Use Van Cittert - Zernike theorem which holds for quasimonochromatic light under paraxial conditions. Starting with Eq (5.6-10),

$$\mu(P_1, P_2) = \frac{e^{-j\psi} \iint_{-\infty}^{\infty} I(\xi,\eta)\, e^{j\frac{2\pi}{\lambda z}(\Delta x\, \xi + \Delta y\, \eta)}\, d\xi\, d\eta}{\iint_{-\infty}^{\infty} I(\xi,\eta)\, d\xi\, d\eta}$$

Without loss of generality, place point source at origin of (ξ,η)- plane.

$$I(\xi,\eta) = I_0\, \delta(\xi,\eta)$$

Thus,

$$\boxed{\mu(P_1, P_2) = \frac{e^{-j\psi} I_0}{I_0} = e^{-j\psi} \quad \text{where} \quad \psi = \frac{\pi}{\lambda z}\left[(x_2^2 + y_2^2) - (x_1^2 + y_1^2)\right]}$$