PROBLEM SET #7

Issued: Thursday, Nov. 29, 2007

Due (at 5 p.m.): Tuesday, Dec. 11, 2007 (Slip it under Prof. Nguyen's door.)

1. For this problem, the gyroscope mechanical element summarized in Problem Set #6 is packaged in vacuum, where gas damping mechanisms disappear, resulting in much higher drive and sense mode Q's for the structure. It is then hooked up into the sensor circuit shown in the figure (on the next page) and is actually used as a gyroscope (where it was not in Problem Set #6). The circuit surrounding the gyroscope mechanical element is complete with an amplifier loop that sustains resonance vibration in the drive mode direction; and a sense amplifier that picks up the output vibration in the sense mode direction induced by a rotation around the indicated "sensitive" axis.

(Note that in an actual implementation, the sense amplifier often provides a balanced differential input that senses the differential current from both sense electrodes to maximize performance. In other words, the grounded electrodes in the figure would not be grounded, but would also be directed to an amplifier input. The present problem uses a single-ended pick-off configuration only to simplify things.)

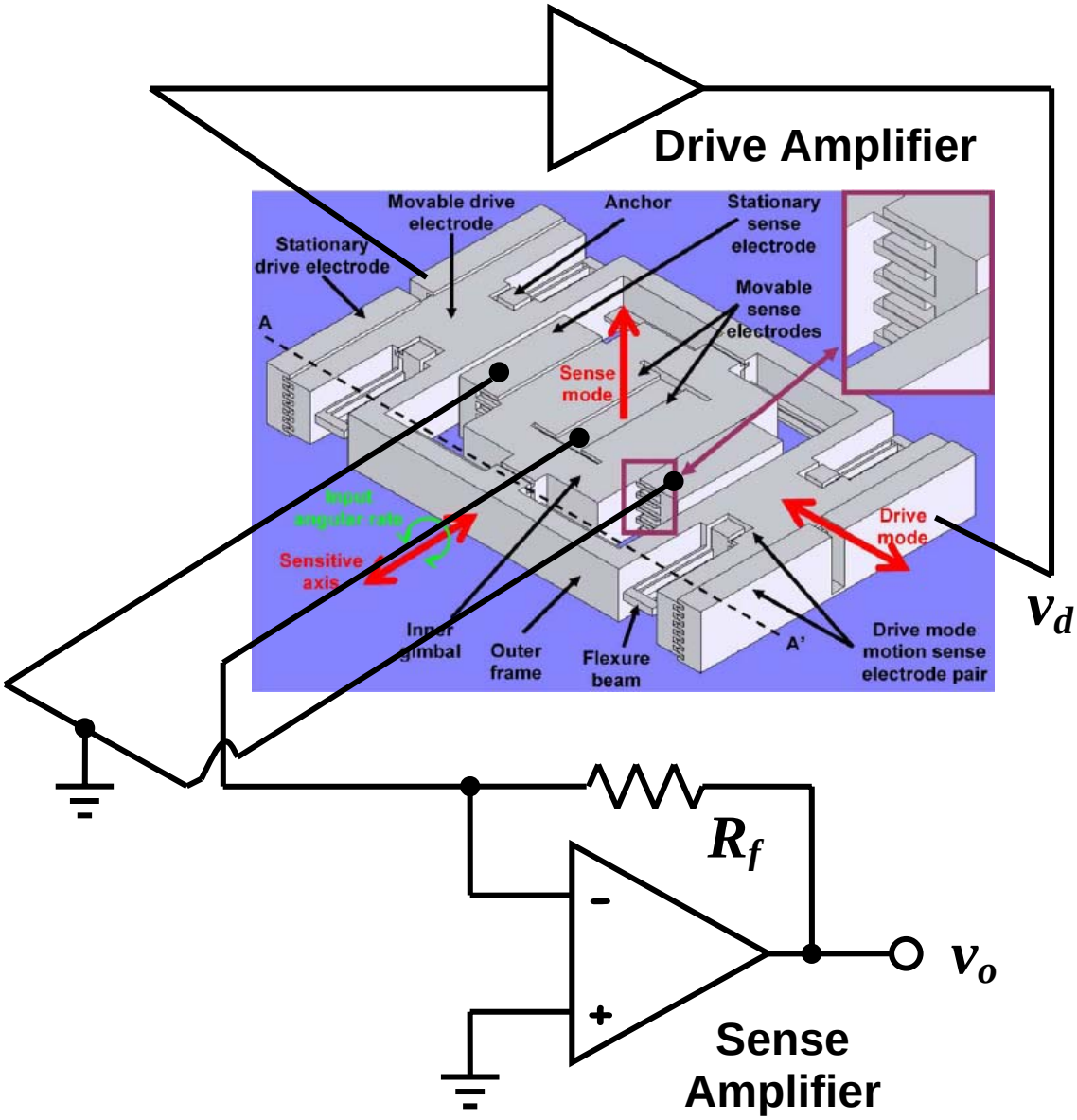
Use the data from Problem Set #6 and in the box in the figure to answer the following questions regarding this gyroscope circuit.

(a) Assume that during steady-state oscillation along the drive axis the drive amplifier delivers an ac voltage v_d with an amplitude of 2.5V and a frequency equal to the resonance frequency of the drive mode. Also, assume that the input of the drive amplifier detects velocity and that its input resistance is very small. Determine the rotation rate-to-output current scale factor for this gyroscope. Give an expression and calculate its numerical value.

(b) Determine the value of the output voltage noise spectral density of the gyro circuit (i.e., at the output node, where v_o is indicated in the figure) at the frequency of the drive mode.

(c) Draw equivalent circuits for the linked drive and sense axes of the complete gyroscope circuit and use SPICE to verify your answers to parts (a) and (b).

(d) What is the minimum detectable angular rate of this gyro circuit if the output sense circuit is limited by a low-pass filter to a 1 kHz bandwidth?



Gyro Mechanical Element Data:

Drive Mode Q in vacuum: Q_d = 10,000; Sense Mode Q in vacuum: Q_s = 10,000

All gyro mechanical element dimensions given in Problem Set #6.

Electrical Circuit Data:

R_f = 10 MΩ

Op amp SPICE model given at the end of this problem set (and online).

Op amp input-referred noise generators:

$$\sqrt{\frac{\overline{v_{ia}^2}}{\Delta f}} = 12\ \text{nV}/\sqrt{\text{Hz}} \qquad \sqrt{\frac{\overline{i_{ia}^2}}{\Delta f}} = 0.01\ \text{pA}/\sqrt{\text{Hz}}$$

Op Amp SPICE Input Deck: (electronic version online with this homework assignment)

```
*LF156 Monolithic JFET-Input OP-AMP MACRO-MODEL
*//////////////////////////////////////////////////////
*
* connections:    non-inverting input
*                 |   inverting input
*                 |   |   positive power supply
*                 |   |   |   negative power supply
*                 |   |   |   |   output
*                 |   |   |   |   |
*                 |   |   |   |   |
.SUBCKT LF156/NS  1   2  99  50  28
*
*Features:
*Low input bias current =          30pA
*Low input offset current =         3pA
*High input impedance =            1Tohm
*Low input offset voltage =         1mV
*
****************INPUT STAGE**************
*
IOS 2 1 3P
*^Input offset current
R1 1 3 1E12
R2 3 2 1E12
I1 99 4 100U
J1 5 2 4 JX
J2 6 7 4 JX
R3 5 50 20K
R4 6 50 20K
*Fp2=20 MHz
C4 5 6 1.9894E-13
*
***********COMMON MODE EFFECT***********
*
I2 99 50 4.65MA
*^Quiescent supply current
EOS 7 1 POLY(1) 16 49 3E-3 1
*Input offset voltage.^
R8 99 49 50K
R9 49 50 50K
*
*********OUTPUT VOLTAGE LIMITING********
V2 99 8 2.63
D1 9 8 DX
D2 10 9 DX
V3 10 50 2.63
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*
**************SECOND STAGE**************
*
EH 99 98 99 49 1
F1 9 98 POLY(1) VA3 0 0 0 1.5944E7
G1 98 9 5 6 2E-3
R5 98 9 100MEG
VA3 9 11 0
*Fp1=31.96 HZ
C3 98 11 49.9798P
*
*********COMMON-MODE ZERO STAGE*********
*
G4 98 16 3 49 1E-8
L2 98 17 530.52M
R13 17 16 1K
*
**************OUTPUT STAGE**************
*
F6  99 50 VA7 1
F5  99 23 VA8 1
D5  21 23 DX
VA7 99 21 0
D6  23 99 DX
E1  99 26 99 9 1
VA8 26 27 0
R16 27 28 20
V5  28 25 -.25
D4  25  9 DX
V4  24 28 -.25
D3   9 24 DX
*
***************MODELS USED**************
*
.MODEL DX D(IS=1E-15)
.MODEL JX PJF(BETA=1.25E-5 VTO=-2.00 IS=30E-12)
*
.ENDS LF156/NS
*$
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