EE C245 – ME C218
Introduction to MEMS Design
Fall 2007

Prof. Clark T.-C. Nguyen

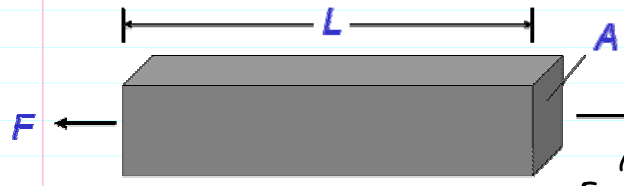
Dept. of Electrical Engineering & Computer Sciences
University of California at Berkeley
Berkeley, CA 94720

Lecture 13: Material Properties

Lecture Outline

- Reading: Senturia Chpt. 8
- Lecture Topics:
  - Elasticity: Nomenclature
    - Stress
    - Strain
    - Poisson Ratio
  - Material Properties
    - Young's modulus
    - Yield strength
    - Quality factor
  - On-chip Measurement of Material Properties
  - Anisotropic Material Properties

Normal Stress (1D)



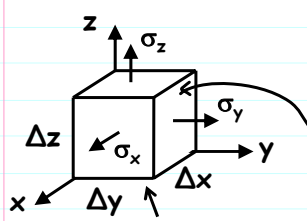
If the force acts normal to a surface, then the stress is called a **normal stress**

Force assumed uniform over the whole area A

$$\text{Stress} = \left\{\begin{matrix}\text{Force per}\\ \text{unit area}\end{matrix}\right\} = \sigma = \frac{F}{A} \quad [N/m^2 = Pa]$$

standard mks unit

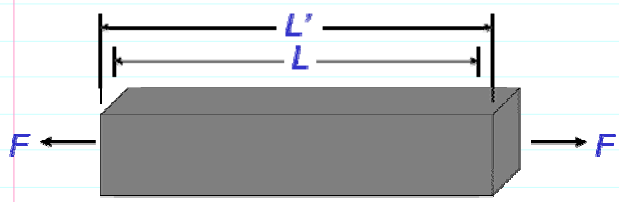
⇒ Microscopic Definition: force per unit area acting on the surface of a differential volume element of a solid body



Differential volume element

Note: force acts uniformly over each face

Strain (1D)



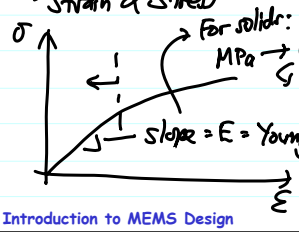
Sometimes a unit called "microstrain" is used, where $1\mu\varepsilon = \frac{\Delta L}{L}$ of 1 part in 10^6

$$\text{Strain} = \left\{\begin{matrix}\text{Fractional Change in}\\ \text{Length}\end{matrix}\right\} = \varepsilon = \frac{L'-L}{L} = \frac{\Delta L}{L} \quad \text{[unitless]}$$

$\frac{\Delta L}{L} = 10^{-6} = 1\mu\varepsilon$

In the elastic regime (i.e., for "small" stresses) & "low" temperatures

strain ∝ stress



For solids: MPa → GPa

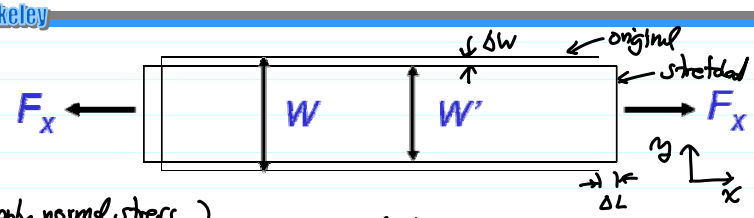
poly-Si E = 150 GPa

slope = E = Young's modulus of elasticity

$\sigma = \varepsilon E \rightarrow \boxed{\varepsilon = \frac{\sigma}{E} \quad \text{[unitless]}}$

∴ units of E same as σ → Pa

The Poisson Ratio



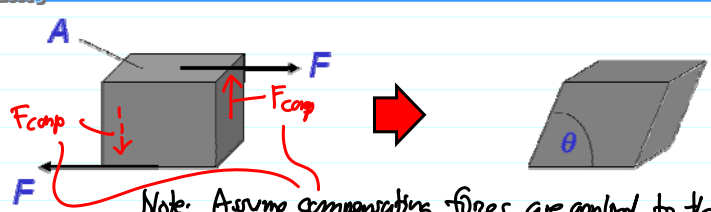
Apply normal stress to a free-standing object → uniaxial strain → ε_x
→ but also get contraction in directions transverse to the uniaxial strain

⇒ Contraction creates a (-) strain:

$$\varepsilon_y = \frac{W'-W}{W} = \frac{\delta W}{W} = -\nu \varepsilon_x$$

ν: Poisson ratio [unitless]
↳ typical values: $0 \rightarrow 0.5$
⇒ inorganic solids: $0.2 \rightarrow 0.3$
⇒ elastomers (e.g., rubber): ~ 0.5

Shear Stress & Strain (2D)



Note: Assume compensating forces are applied to the vertical faces to avoid a net torque. (This by convention)

Shear Stress: $\left\{ \begin{array}{l} \text{Force per unit area} \\ \text{Parallel to the surface} \end{array} \right\} = \tau = \frac{F}{A}$ [Pa]

↳ Generates a shear strain:

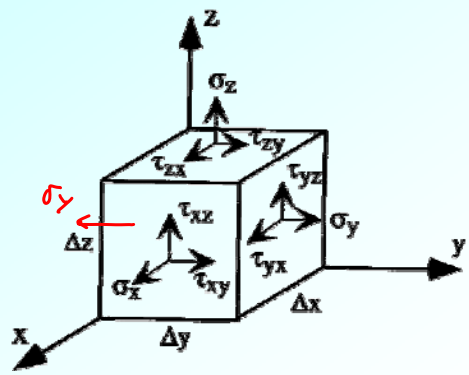
$$\text{Shear Strain} = \theta = \frac{\tau}{G}$$

← G ≡ shear modulus

$$G = \frac{E}{2(1+\nu)}$$

2D and 3D Considerations

- Important assumption: the differential volume element is in static equilibrium → no net forces or torques (i.e., rotational movements)
  - Every σ must have an equal σ in the opposite direction on the other side of the element
  - For no net torque, the shear forces on different faces must also be matched as follows:

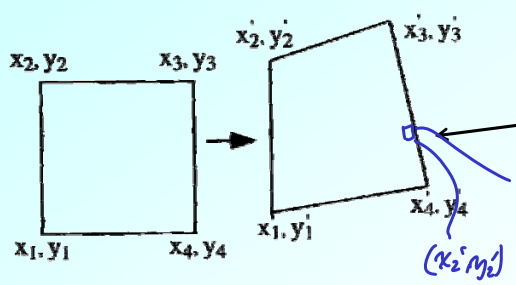


Stresses acting on a differential volume element

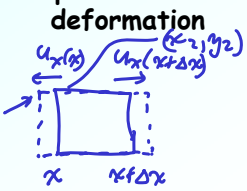
$$\tau_{xy} = \tau_{yx} \qquad \tau_{xz} = \tau_{zx} \qquad \tau_{yz} = \tau_{zy}$$

2D Strain

- In general, motion consists of
  - rigid-body displacement (motion of the center of mass)
  - rigid-body rotation (rotation about the center of mass)
  - Deformation relative to displacement and rotation

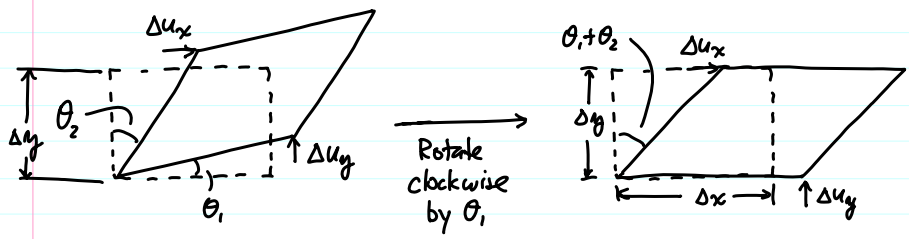


Area element experiences both displacement and deformation



- Must work with displacement vectors
- Differential definition of axial strain: ⟶ $\varepsilon_x = \dfrac{u_x(x+\Delta x) - u_x(x)}{\Delta x} = \dfrac{\partial u_x}{\partial x}$

2D Shear Strain



⇒ For shear strains, must remove any rigid body rotation that accompanies the deformation

↳ use a symmetric definition of shear strain:

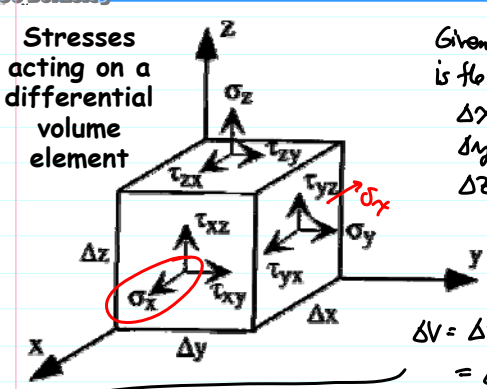
$$\gamma_{xy} = \theta_2 + \theta_1 \simeq \left(\frac{\Delta u_x}{\Delta y} + \frac{\Delta u_y}{\Delta x}\right) = \left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right)$$

For small amplitude deformations

↳ $\tan\theta \approx \theta$

Volume Change for a Uniaxial Stress

Stresses acting on a differential volume element



Given an x-directed uniaxial stress, σ_x, what is the ΔV?

$\Delta x \rightarrow \Delta x(1+\epsilon_x)$

$\Delta y \rightarrow \Delta y(1-\nu\epsilon_x)$

$\Delta z \rightarrow \Delta z(1-\nu\epsilon_x)$

↓ The resulting change in volume ΔV

$$\Delta V = \Delta x\Delta y\Delta z(1+\epsilon_x)(1-\nu\epsilon_x)^2 - \Delta x\Delta y\Delta z$$

$$= \Delta x\Delta y\Delta z\left[(1+\epsilon_x)(1-\nu\epsilon_x)^2 - 1\right]$$

[Assume small strains] ⇒ $\Delta V = \Delta x\Delta y\Delta z\left[(1+\epsilon_x)(1-2\nu\epsilon_x) - 1\right]$

$\left[(1+mx)^n \approx 1+nmx\right]$

↳ $\boxed{\Delta V \approx \Delta x\Delta y\Delta z(1-2\nu)\epsilon_x}$

For $\nu = 0.5$ (rubber) → no ΔV!

$\nu < 0.5$ → finite ΔV

Isotropic Elasticity in 3D

- Isotropic = same in all directions
- The complete stress-strain relations for an isotropic elastic solid in 3D: (i.e., a generalized Hooke's Law)

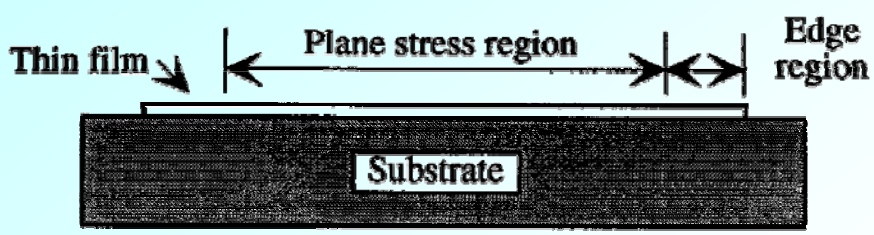
$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right] \qquad \gamma_{xy} = \frac{1}{G}\tau_{xy}$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu\left(\sigma_z + \sigma_x\right)\right] \qquad \gamma_{yz} = \frac{1}{G}\tau_{yz}$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \underbrace{\nu\left(\sigma_x + \sigma_y\right)}_{\text{Basically, add in off-axis strains from normal stresses in other directions}}\right] \qquad \gamma_{zx} = \frac{1}{G}\tau_{zx}$$

Important Case: Plane Stress

- Common case: very thin film coating a thin, relatively rigid substrate (e.g., a silicon wafer)



- At regions more than 3 thicknesses from edges, the top surface is stress-free → $\sigma_z = 0$
- Get two components of in-plane stress:

$$\varepsilon_x = (1/E)[\sigma_x - \nu(\sigma_y + 0)]$$

$$\varepsilon_y = (1/E)[\sigma_y - \nu(\sigma_x + 0)]$$

Important Case: Plane Stress (cont.)

- Symmetry in the xy-plane → $\sigma_x = \sigma_y = \sigma$
- Thus, the in-plane strain components are: $\varepsilon_x = \varepsilon_y = \varepsilon$ where

$$\varepsilon_x = (1/E)[\sigma - \nu\sigma] = \frac{\sigma}{[E/(1-\nu)]} = \frac{\sigma}{E'}$$

and where

$$\text{Biaxial Modulus} \triangleq E' = \frac{E}{1-\nu}$$

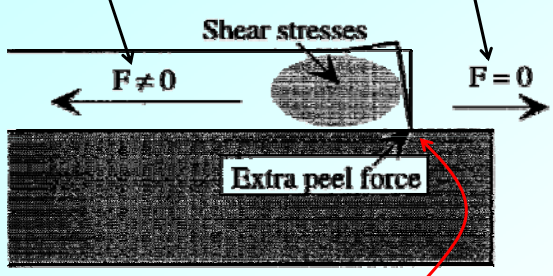
Edge Region of a Tensile (σ>0) Film

Net non-zero in-plane force (that we just analyzed)

At free edge, in-plane force must be zero

Film must be bent back, here

There's no Poisson contraction, so the film is slightly thicker, here



Discontinuity of stress at the attached corner → stress concentration

Peel forces that can peel the film off the surface

Linear Thermal Expansion

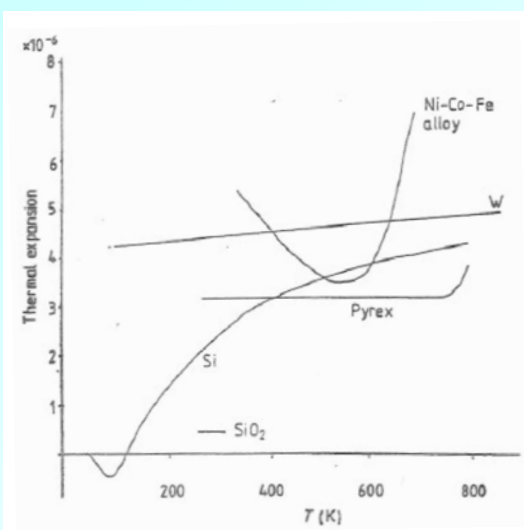
- As temperature increases, most solids expand in volume
- Definition: linear thermal expansion coefficient

$$\left.\begin{array}{r}\text{Linear thermal}\\ \text{expansion coefficient}\end{array}\right\} \triangleq \alpha_T = \frac{d\varepsilon_x}{dT} \qquad [\text{Kelvin}^{-1}]$$

Remarks:

- α_T values tend to be in the 10^{-6} to 10^{-7} range
- Can capture the 10^{-6} by using dimensions of μstrain/K, where 10^{-6} K^{-1} = 1 μstrain/K
- In 3D, get volume thermal expansion coefficient $\longrightarrow$ $\dfrac{\Delta V}{V} = 3\alpha_T \Delta T$
- For moderate temperature excursions, α_T can be treated as a constant of the material, but in actuality, it is a function of temperature

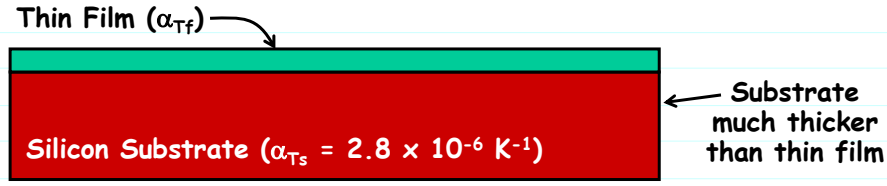
α_T As a Function of Temperature



[Madou, Fundamentals of Microfabrication, CRC Press, 1998]

- Cubic symmetry implies that α is independent of direction

Thin-Film Thermal Stress



- Assume film is deposited stress-free at a temperature T_d, then the whole thing is cooled to room temperature T_r
- Substrate much thicker than thin film → substrate dictates the amount of contraction for both it and the thin film

Thermal strain of the substrate: (in one in-plane dimension)

$$\epsilon_s = -\alpha_{Ts}\Delta T, \text{ where } \Delta T = T_d - T_r$$

How about the film?

If the film were not attached to the substrate: $\epsilon_{f,free} = -\alpha_{Tf}\Delta T$ ← eq

Linear Thermal Expansion

But the film is attached to the substrate, so the actual strain in the film:

$$\epsilon_{f,attached} = \epsilon_s = -\alpha_{Ts}\Delta T$$

Thus:

$$\boxed{\text{Thermal Mismatch Strain} = \epsilon_{f,mismatch} = (\alpha_{Tf} - \alpha_{Ts})\Delta T}$$

↳ Note that this is a biaxial strain

↳ it can only be developed by an in-plane biaxial stress:

$$\sigma_{f,mismatch} = \left(\frac{E}{1-\nu}\right)\epsilon_{f,mismatch}$$

Ex. Thin-film is polyimide → $\alpha_{Tf} = 70\times10^{-6}\,K^{-1}$, $E' = 4$ GPa

deposited @ 250°C, then cooled to RT = 25°C → $\Delta T = 225$ K

$$\epsilon_{f,mismatch} = (70 - 2.8)\times10^{-6}(225) = 1.5\times10^{-2}$$

$$\sigma_{f,mismatch} = (4G)(1.5\times10^{-2}) = 60.5\,MPa$$

(10^9)

stress is (+) ∴ tensile [if (−), would be compressive]

MEMS Material Properties

Material Properties for MEMS

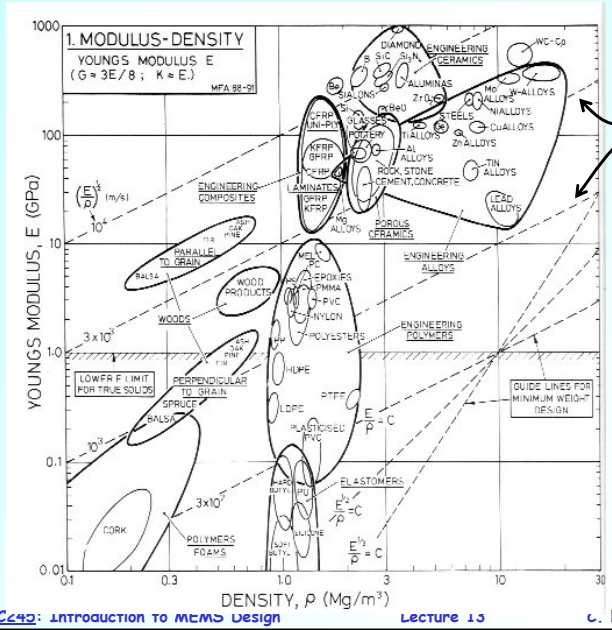
Material	Density, ρ, Kg/m³	Modulus, E, GPa	E/ ρ GN/kg-m
Silicon	2330	165	72
Silicon Oxide	2200	73	36
Silicon Nitride	3300	304	92
Nickel	8900	207	23
Aluminum	2710	69	25
Aluminum Oxide	3970	393	99
Silicon Carbide	3300	430	130
Diamond	3510	1035	295

Units: $(m/s)^2$

$\sqrt{(E/\rho)}$ is acoustic velocity

[Mark Spearing, MIT]

Young's Modulus Versus Density



Lines of constant acoustic velocity

[Ashby, Mechanics of Materials, Pergamon, 1992]

Yield Strength

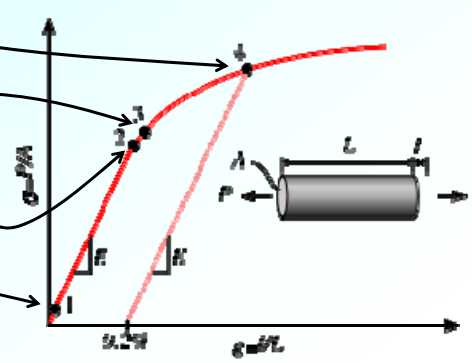
- Definition: the stress at which a material experiences significant plastic deformation (defined at 0.2% offset pt.)
- Below the yield point: material deforms elastically → returns to its original shape when the applied stress is removed
- Beyond the yield point: some fraction of the deformation is permanent and non-reversible

Yield Strength: defined at 0.2% offset pt.

Elastic Limit: stress at which permanent deformation begins

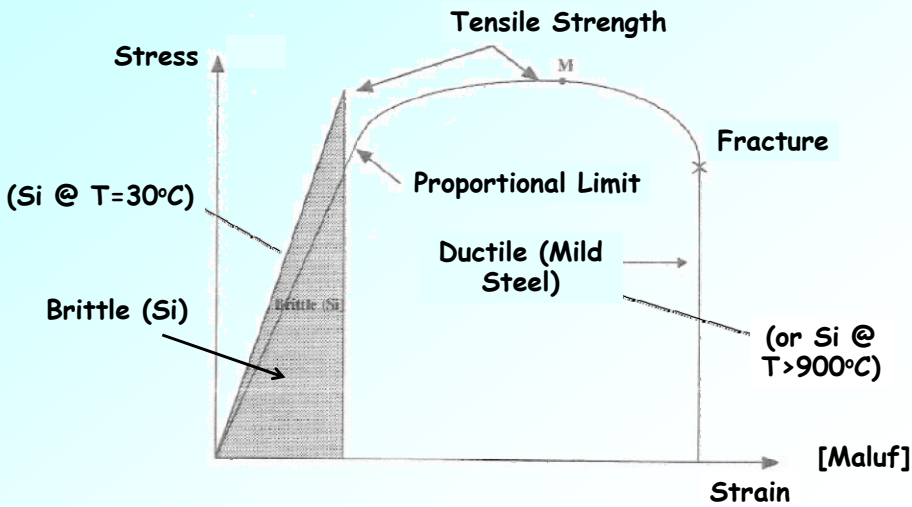
Proportionality Limit: point at which curve goes nonlinear

True Elastic Limit: lowest stress at which dislocations move



Yield Strength (cont.)

- **Below**: typical stress vs. strain curves for brittle (e.g., Si) and ductile (e.g. steel) materials



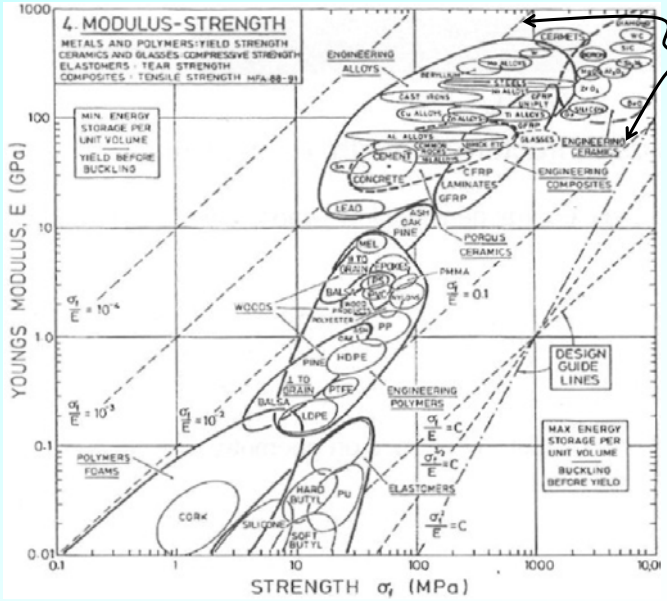
Young's Modulus and Useful Strength

Stored mechanical energy

Material	Modulus, E, GPa	Useful Strength*, σ_f, MPa	$\frac{\sigma_f}{E}$ (-) x 10^{-3}	$\frac{\sigma_f^2}{E}$ MJ/m^3
Silicon	165	4000	24	97
Silicon Oxide	73	1000	13	14
Silicon Nitride	304	1000	3	4
Nickel	207	500	2	1.2
Aluminum	69	300	4	1.3
Aluminum Oxide	393	2000	5	10
Silicon Carbide	430	2000	4	9.3
Diamond	1035	1000	1	0.9

From Mark Spearing, MIT, *Future of MEMS Workshop*, Cambridge, England, May 2003

Young's Modulus Versus Strength



Lines of constant maximum strain

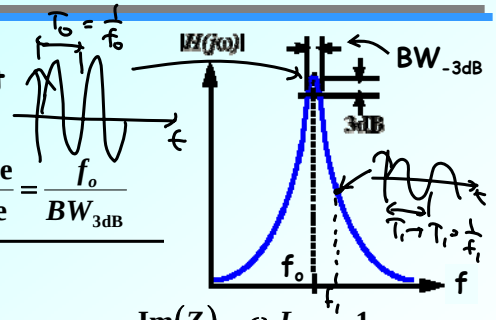
[Ashby, Mechanics of Materials, Pergamon, 1992]

Quality Factor (or Q)

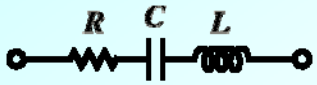
Quality Factor (or Q)

- Measure of the frequency selectivity of a tuned circuit
- Definition:

$$Q = \frac{\text{Total Energy Per Cycle}}{\text{Energy Lost Per Cycle}} = \frac{f_o}{BW_{3dB}}$$

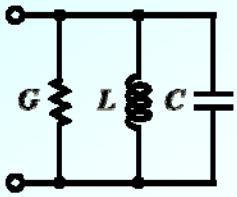


- Example: series LCR circuit



$$\Rightarrow \quad Q = \frac{\mathrm{Im}(Z)}{\mathrm{Re}(Z)} = \frac{\omega_o L}{R} = \frac{1}{\omega_o CR}$$

- Example: parallel LCR circuit

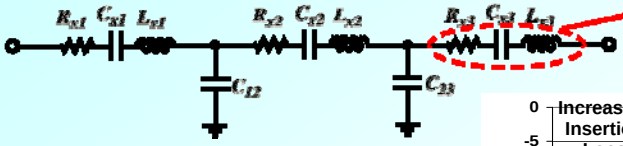


$$\Rightarrow \quad Q = \frac{\mathrm{Im}(Y)}{\mathrm{Re}(Y)} = \frac{\omega_o C}{G} = \frac{1}{\omega_o LG}$$

Selective Low-Loss Filters: Need Q

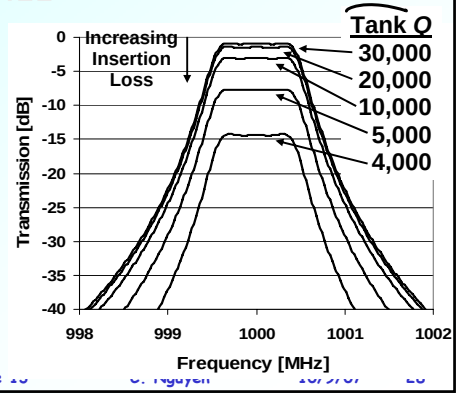
General BPF Implementation

Typical LC implementation:



Resonator "Tank Q"

- In resonator-based filters: high tank Q ⇔ low insertion loss
- At right: a 0.1% bandwidth, 3-res filter @ 1 GHz (simulated)
  - ↳ heavy insertion loss for resonator Q < 10,000



Oscillator: Need for High Q

- **Main Function**: provide a stable output frequency
- **Difficulty**: superposed noise degrades frequency stability