- **Announcements**:
- Lecture schedule for the next 4 weeks:

Monday	Tuesday	Wednesday	Thursday	Friday
Oct. 5 Lecture 5-6:30 241 Cory	6 Lecture 3:30-5 Moffitt	7	8 Discuss 3:30-5 Moffitt	9
12 Lecture 5-6:30 241 Cory	13	14	15 Discuss 3:30-5 Moffitt	16
19 Lecture 5-6:30 241 Cory	20 Lecture 3:30-5 Moffitt	21	22 Lecture 3:30-5 Moffitt	23
26 Discuss 3:30-5 Moffitt	27	28	29 Lecture 3:30-5 Moffitt	30
Nov. 2 Lecture 5-6:30 241 Cory	3 Midterm 3:30-5 Moffitt	4	5 Lecture 3:30-5 Moffitt	6

- Midterm Exam:
  - Tu, Nov. 3, during regular lecture
  - 1.5 hours, but might go longer
- HW#3 due Thursday, 10/15, at 7 p.m.

- **Today**:
- Reading: Senturia, Chpt. 8
- Lecture Topics:
  - Stress, strain, etc., for isotropic materials
  - Thin films: thermal stress, residual stress, and stress gradients
  - Internal dissipation
  - MEMS material properties and performance metrics
- ---------------------------------
- Reading: Senturia, Chpt. 9
- Lecture Topics:
  - Bending of beams
  - Cantilever beam under small deflections
  - Combining cantilevers in series and parallel
  - Folded suspensions
  - Design implications of residual stress and stress gradients
- ---------------------------------

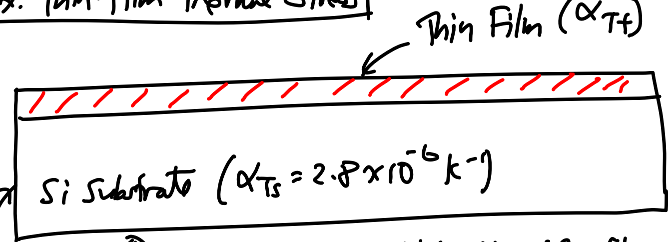
$\boxed{\text{Linear Thermal Expansion}}$

temperature ↑ ⟶ solids expand in volume

Definition: linear thermal expansion coefficient

$$\boxed{\left.\begin{array}{l}\text{Linear Thermal}\\ \text{Exp. Coeff.}\end{array}\right\} \triangleq \alpha_T = \frac{d\epsilon_x}{dT} \quad [\text{kelvin}^{-1}]}$$

$\boxed{\text{Ex. Thin-Film Thermal Stress}}$



Assume: ① substrate is much thicker than the film.
② film is deposited stress free @ T_d
③ then whole thing is cooled to room temp, T_r

substrate wins

Thermal Strain of the Substrate: (in one plane dimension)

$$\epsilon_s = -\alpha_{Ts}\Delta T, \text{ where } \Delta T = T_d - T_r$$

If the film were not attached to the substrate:

$$\epsilon_{f,free} = -\alpha_{Tf}\Delta T$$

But the film *is* attached → so the actual strain is that of the substrate

$$\epsilon_{f,attached} = -\alpha_{Ts}\Delta T$$

Thus:

$$\boxed{\text{Thermal Mismatch Strain} = \epsilon_{f,mismatch} = (\alpha_{Tf} - \alpha_{Ts})\Delta T}$$

Note this is biaxial strain (assuming isotropic film)

$$\sigma_{f,mismatch} = \underbrace{\left(\frac{E}{1-\nu}\right)}_{=E'} \epsilon_{f,mismatch}$$

Ex. Thin-film is polyimide
$\alpha_{Tf} = 70\times10^{-6}\,K^{-1}$, $E' = 4\,GPa$

$\underbrace{\text{deposited @ } 250°C\text{, then cooled to RT} = 25°C}_{\Delta T = 225K}$

$$\epsilon_{f,mismatch} = (70-2.8)\mu(225) = 1.5\times10^{-2}$$

$[\mu = 10^{-6},\ m = 10^{-3},\ k = 10^{3},\ G = 10^{9}]$

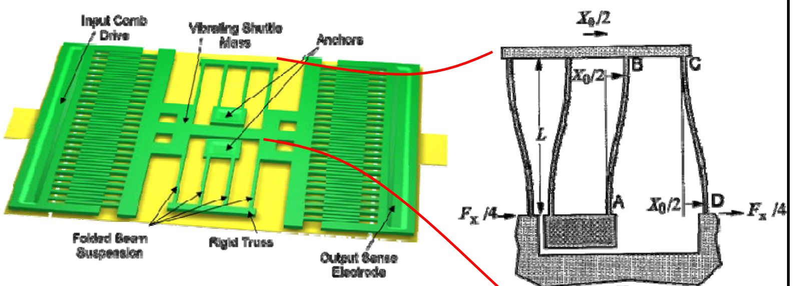
$$\sigma_{f,mismatch} = (4G)(1.5\times10^{-2}) = 60.5\,MPa$$

10^9

stress is (+) ∴ tensile
[(−) would be compressive]
SiO_2

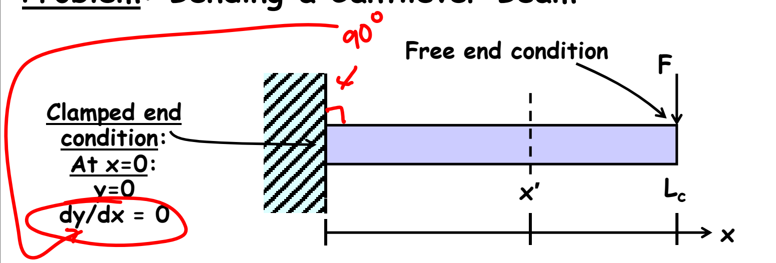
- Go through Module 7 slides 41-49
- New Topic: Bending of beams
  - Cantilever beam under small deflections
  - Combining cantilevers in series and parallel
  - Folded suspensions
  - Design implications of residual stress and stress gradients

- Springs and suspensions very common in MEMS
- Coils are popular in the macro-world; but not easy to make in the micro-world
- Beams: simpler to fabricate and analyze; become "stronger" on the micro-scale → use beams for MEMS



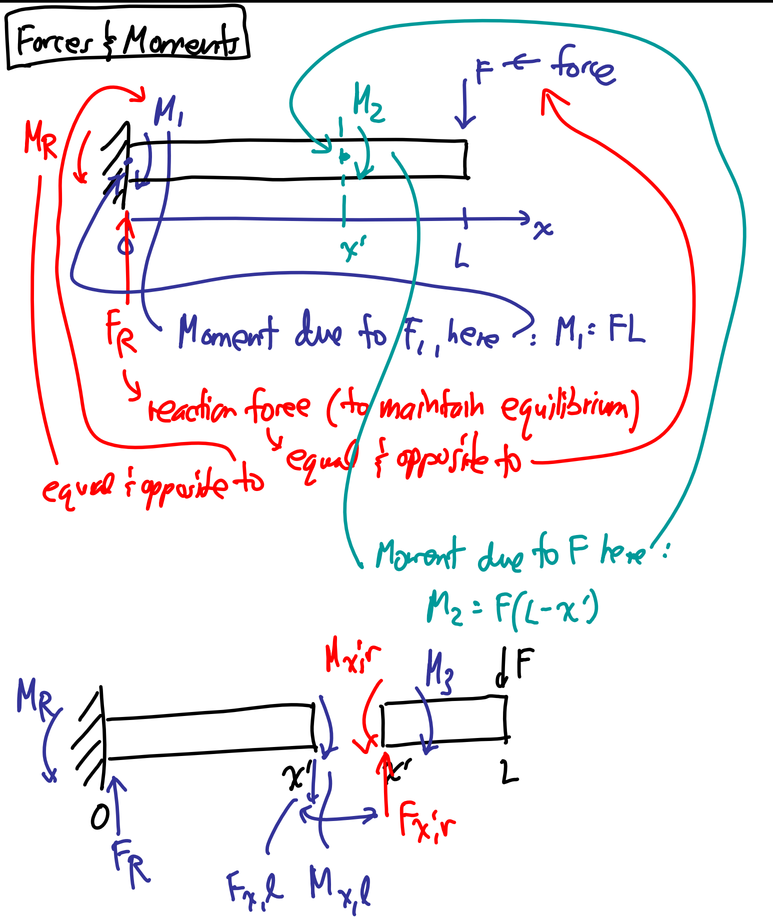
Comb-Driven Folded Beam Actuator

Problem: Bending a Cantilever Beam



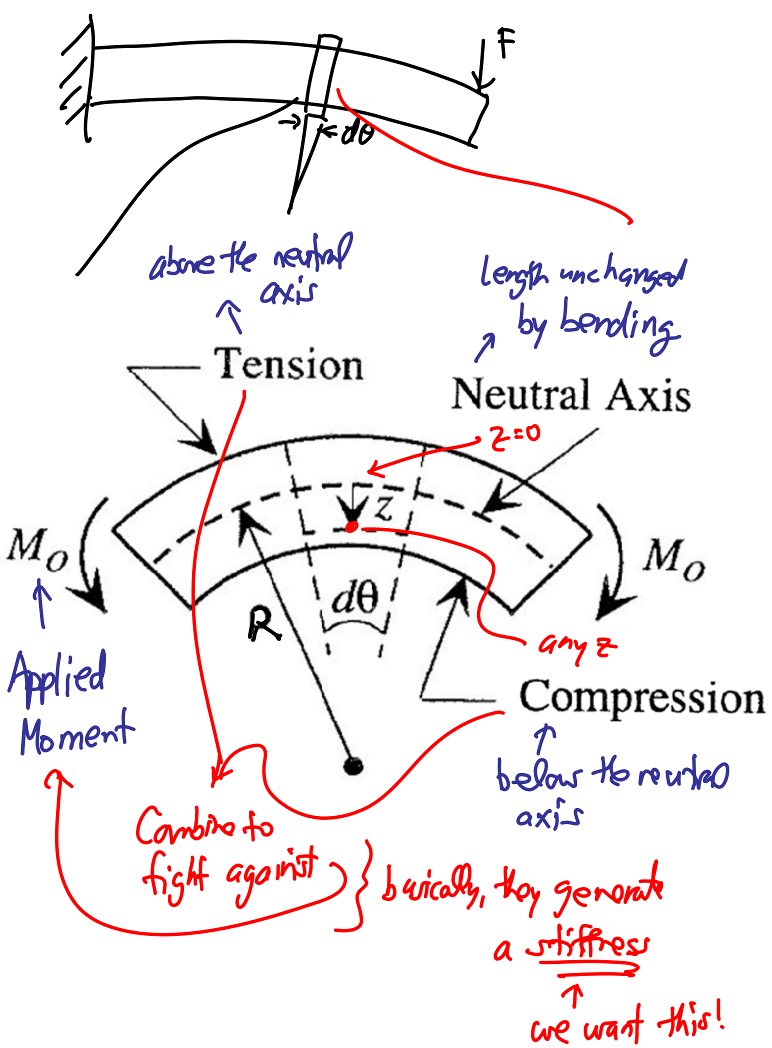
- Objective: Find relation between tip deflection $y(x=L_c)$ and applied load F
- Assumptions:
  1. Tip deflection is small compared with beam length
  2. Plane sections (normal to beam's axis) remain plane and normal during bending, i.e., "pure bending"
  3. Shear stresses are negligible

Forces & Moments



over

Bent Beam (by applied force):



Beam Segment in Pure Bending

⇒ consider the segment bounded by the dashed lines defining $d\theta$:

At $z=0$: neutral axis → segment length $= dx = R\,d\theta$ (1)

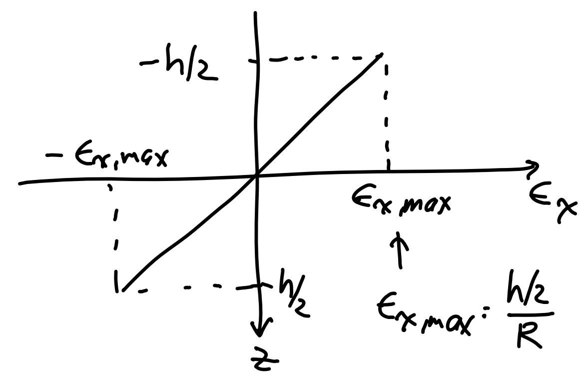
At any z: segment length $= dL = (R-z)\,d\theta$ (2)

Combine (1) & (2): $dL = dx - z\,d\theta = dx - \frac{z}{R}dx$

Thus, the axial strain @ z: $\varepsilon_x = \frac{dL-dx}{dx} = -\frac{z}{R}$

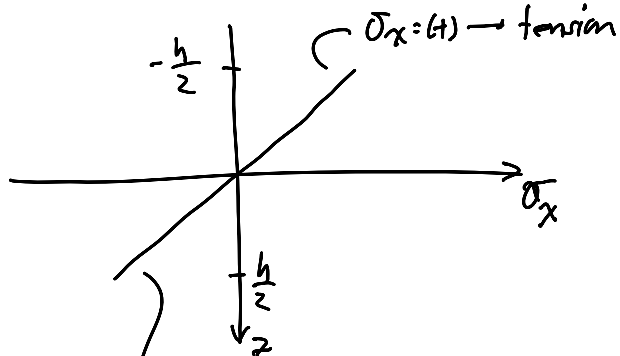
$$\boxed{\varepsilon_x = -\frac{z}{R}}$$

Thus, the strain varies linearly along the beam thickness:



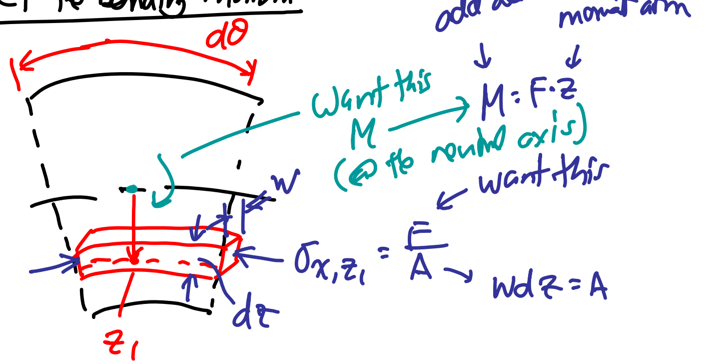
Of course, there is a corresponding axial stress:

$$\sigma_x = \epsilon_x E = \boxed{-\frac{zE}{R} = \sigma_x}$$



This gradient in stress then generates a $\underbrace{\text{bending moment}}$! in response to the original applied moment!

Get the bending moment:



→ integrate stress through the thickness of the beam:

$$M = \int_{-h/2}^{h/2} \Big[\underbrace{(W dz)\,\sigma_x}_{\text{force}}\Big] \cdot z$$

$$= -\int_{-h/2}^{h/2} \frac{EWz^2}{R}\, dz \Rightarrow \boxed{M = -\left(\frac{1}{12}Wh^3\right)\frac{E}{R}}$$

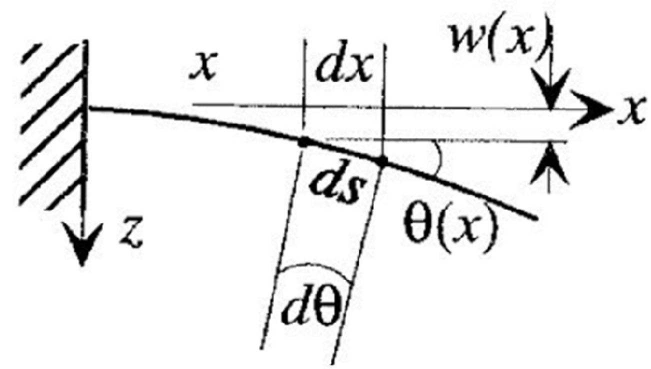
$$\left[\sigma_x = -\frac{zE}{R}\right]$$

$\frac{1}{12}Wh^3 = I \triangleq$ Moment of Inertia

$$\boxed{\frac{1}{R} = -\frac{M}{EI}}$$

Note: (+) radius of curvature
↳ (−) internal bending moment

Differential Beam Bending Equation



Write out some geometric relationships:

→ then use small angle approx.

$$\cos\theta = \frac{dx}{ds} \rightarrow ds = \frac{dx}{\cos\theta} \longrightarrow ds \simeq dx$$

$$\tan\theta = \frac{dw}{dx} = \text{slope of beam @ this point} \longrightarrow \theta \simeq \frac{dw}{dx} \quad (1)$$

$$ds = R\,d\theta \rightarrow \frac{1}{R} = \frac{d\theta}{ds} \longrightarrow \frac{1}{R} = \frac{d\theta}{dx} \quad (2)$$

Inserting: (1) into (2)

$$\frac{1}{R} = \boxed{\frac{d^2w}{dx^2} = -\frac{M}{EI}}$$

← Differential Eq. for Small Angle Beam Bending