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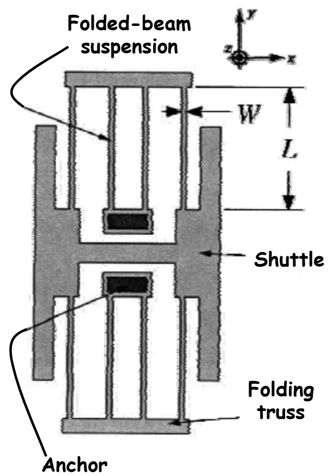
Folded-Beam Stiffness Ratios



- In the x-direction:

$$k_x = \frac{24EI_z}{L^3}$$

- In the z-direction:
  - ↳ Same flexure and boundary conditions

$$k_z = \frac{24EI_x}{L^3}$$

- In the y-direction:

[See Senturia, §9.2] $$k_y = \frac{8EWh}{L}$$

- Thus: $$\frac{k_y}{k_x} = 4\left(\frac{L}{W}\right)^2$$ Much stiffer in y-direction!

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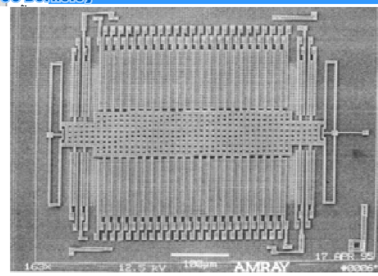
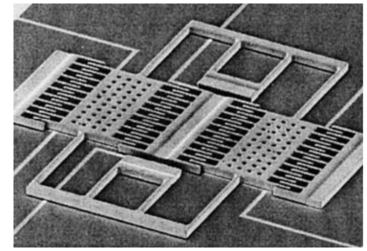
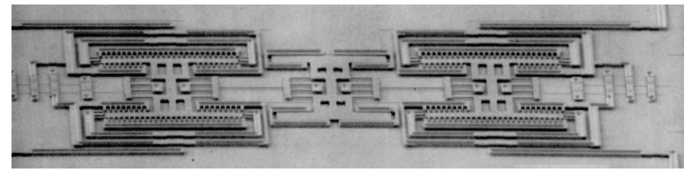
Folded-Beam Suspensions Permeate MEMS



Accelerometer [ADXL-05, Analog Devices]



Gyroscope [Draper Labs.]



Micromechanical Filter [K. Wang, Univ. of Michigan]

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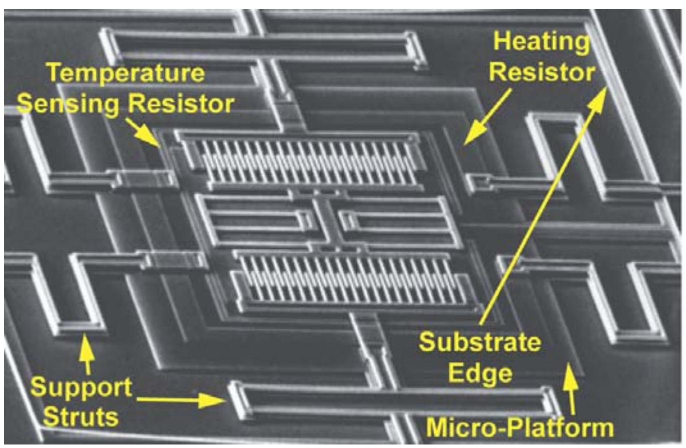
Folded-Beam Suspensions Permeate MEMS

- **Below**: Micro-Oven Controlled Folded-Beam Resonator



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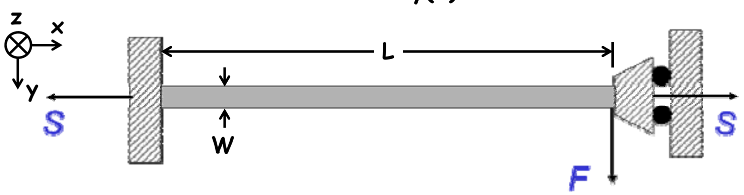
Stressed Folded-Flexures

Clamped-Guided Beam Under Axial Load

- Important case for MEMS suspensions, since the thin films comprising them are often under residual stress
- Consider small deflection case: $y(x) \ll L$



Governing differential equation: (Euler Beam Equation)

$$EI_z \frac{d^4 y}{dx^4} - S\frac{d^2 y}{dx^2} = F\delta(x-L)$$

Axial Load — Unit impulse @ x=L

Design Implications

- Straight flexures
  - Large tensile S means flexure behaves like a tensioned wire (for which k^{-1} = L/S)
  - Large compressive S can lead to buckling ($k^{-1} \rightarrow \infty$)
- Folded flexures
  - Residual stress only partially released
  - Length from truss to shuttle's centerline differs by L_s for inner and outer legs

① If polySi strain is ϵ_r, then shuttle expands by $\Delta L_s = \epsilon_r L_s$

② This then applies a load to the beams, also $\Delta L = \Delta L_s$.

③ Beam strain:

$\epsilon_b = \frac{\Delta L}{L} = \frac{\Delta L_s}{L} = \epsilon_r \frac{L_s}{L}$

over ⟹