EE 245: Introduction to MEMS
Lecture 18: Midterm Info & Resonance Frequency

- **Announcements**:
- Lecture schedule for the next 4 weeks:

Monday	Tuesday	Wednesday	Thursday	Friday
Oct. 5 Lecture 5-6:30 241 Cory	6 Lecture 3:30-5 Moffitt	7	8 Discuss 3:30-5 Moffitt	9
12 Lecture 5-6:30 241 Cory	13	14	15 Discuss 3:30-5 Moffitt	16
19 Lecture 5-6:30 241 Cory	20 Lecture 3:30-5 Moffitt	21	22 Lecture 3:30-5 Moffitt	23
26 Discuss 3:30-5 Moffitt	27	28	29 Lecture 3:30-5 Moffitt	30
Nov. 2 Lecture 5-6:30 241 Cory	3 Midterm 3:30-5 Moffitt	4	5 Lecture 3:30-5 Moffitt	6

- Midterm Exam:
  - Tu, Nov. 3, during regular lecture
  - 1.5 hours, but might go longer
- HW#5 due Thursday morning at 10 a.m. in the EE245 box

- Reading: Senturia, Chpt. 10
- Lecture Topics:
  - Energy Methods
  - Virtual Work
  - Energy Formulations
  - Tapered Beam Example
  - Estimating Resonance Frequency

Estimating Resonance Freq.

$x(t) = X_o \cos \omega t$

Potential Energy

$$W(t) = \frac{1}{2} k x^2(t) = \frac{1}{2} k X_o^2 \cos^2 \omega t$$

Kinetic Energy

$$K(t) = \frac{1}{2} M \dot{x}^2(t) = \frac{1}{2} M X_o^2 \omega^2 \sin^2 \omega t$$

$$\dot{x} = \frac{dx}{dt} = \text{velocity}$$

Remarks.

(1) Energy must be conserved.

(2) Total Energy = Potential Energy + Kinetic Energy at all times & locations on the structure

$$W_{max} = \frac{1}{2}k\cancel{X_0^2} = K_{max} = \frac{1}{2}M\omega^2\cancel{X_0^2}$$

maximum potential energy

peak displacement

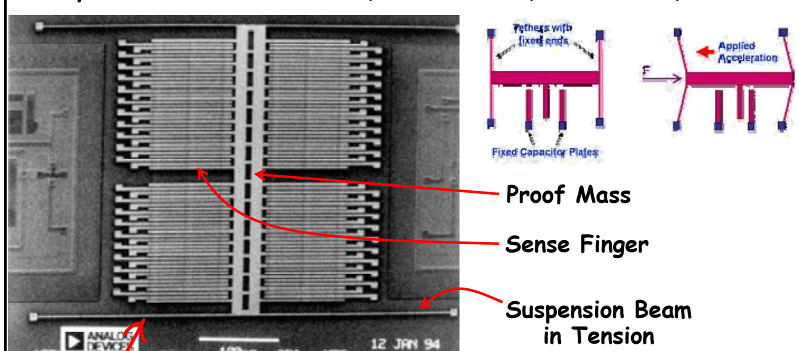
Maximum kinetic energy

radian frequency

$$\therefore \boxed{\omega = \sqrt{\frac{k}{M}}}$$

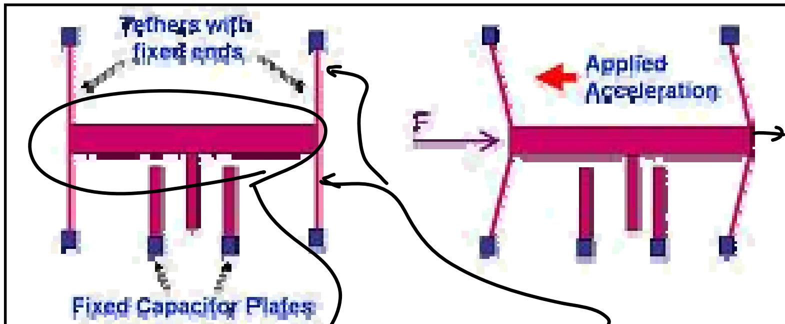
⇒ good for problems whose mass & stiffness can be separated...

- The proof mass of the ADXL-50 is many times larger than the effective mass of its suspension beams
  - ↳ Can ignore the mass of the suspension beams (which greatly simplifies the analysis)
- Suspension Beam: L = 260 μm, h = 2.3 μm, W = 2 μm



In fabrication: purposely introduce a tensile stress in the beams

a large one!



mass of structure >> mass of springs

$\therefore$ ignore the mass of the springs

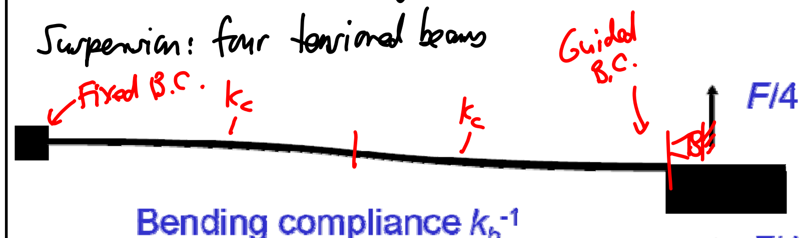
stiffness of springs << stiffness of structure

$\therefore$ ignore stiffness of the structure

For the ADXL-50: 60% of mass from the sense fingers

↳ $M = 162$ ng

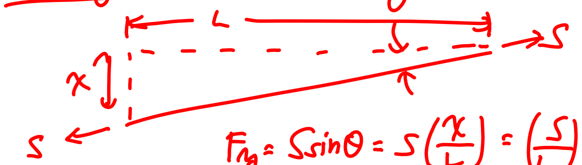
Suspension: four torsional beams



Bending Contribution

$$k_b^{-1} = \left(\frac{1}{k_c}+\frac{1}{k_c}\right) = 2\left(\frac{(L/2)^3}{3E(Wh^3/12)}\right) = \frac{L^3}{EWh^3} = 4.2\,\mu m/\mu N$$

Stretching Contribution



$F_y = S\sin\theta = S\left(\frac{x}{L}\right) = \underbrace{\left(\frac{S}{L}\right)}_{= k_{st}} x$

$k_{st}^{-1} = \frac{L}{S} = \frac{L}{\sigma_r W h} = 1.14\,\mu m/\mu N$

To get total spring constant → add bending stiffness to stretching:

$k = 4(k_b + k_{st}) = 4(0.24 + 0.88) = 4.5\,\mu N/\mu m$

Now, get resonance freq.:

$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{4.48\ N/m}{162\times10^{-12}\,kg}} = 26.5\,kHz$

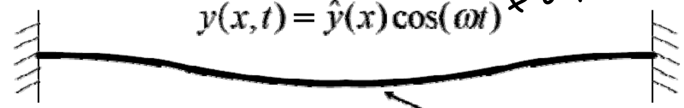
ADXL-50 Data Sheet: $f_o = 24\,kHz$ — difference?

capacitive transducer shifts the freq.

introduces a neg. stiffness, k_e ← electrical

- Vibrating structure displacement function:

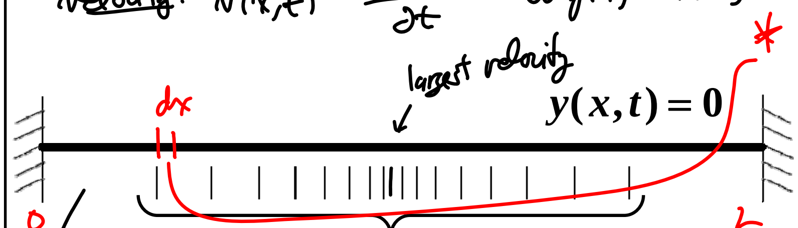
$y(x,t) = \hat{y}(x)\cos(\omega t)$ ← displacement



- Procedure for determining resonance frequency:
  - Use the static displacement of the structure as a trial function and find the strain energy W_{max} at the point of maximum displacement (e.g., when t=0, π/ω, …)
  - Determine the maximum kinetic energy when the beam is at zero displacement (e.g., when it experiences its maximum velocity)
  - Equate energies and solve for frequency

Get Maximum Kinetic Energy

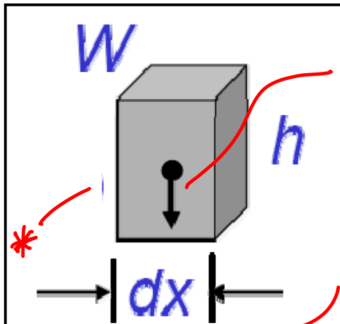
velocity: $v(x,t) = \frac{\partial y(x,t)}{\partial t} = -\omega\hat{y}(x)\sin(\omega t)$



when $y(x,t)=0$, all energy in the structure is kinetic: (since $W=0$)

$v\left(x, \frac{(2n+1)\pi}{2\omega}\right) = -\omega\hat{y}(x)$

↳ $t = \frac{\pi}{2\omega}, \frac{3\pi}{2\omega}, \ldots$



velocity:

$v = -\omega \hat{y}(x)$

$dK = \frac{1}{2} \cdot dm \cdot [v(x,t)]^2$

$dm = \rho(Wh \cdot dx)$

$\therefore$ Maximum Kinetic:

$$K_{max} = \int_0^L \frac{1}{2}\rho Whdx\, v^2(x,t') = \int_0^L \frac{1}{2}\rho Wh \omega^2 \hat{y}^2(x)\, dx$$

To get frequency:

$K_{max} = W_{max}$

$$\therefore \boxed{\omega = \sqrt{\frac{W_{max}}{\int_0^L \frac{1}{2}\rho Wh \hat{y}^2(x)\, dx}}}$$ [radian/s]

ω: res. freq.

W_{max}: maximum potential energy

ρ: density of the structural material

W: beam width

h: " thickness

$\hat{y}(x)$: resonance mode shape