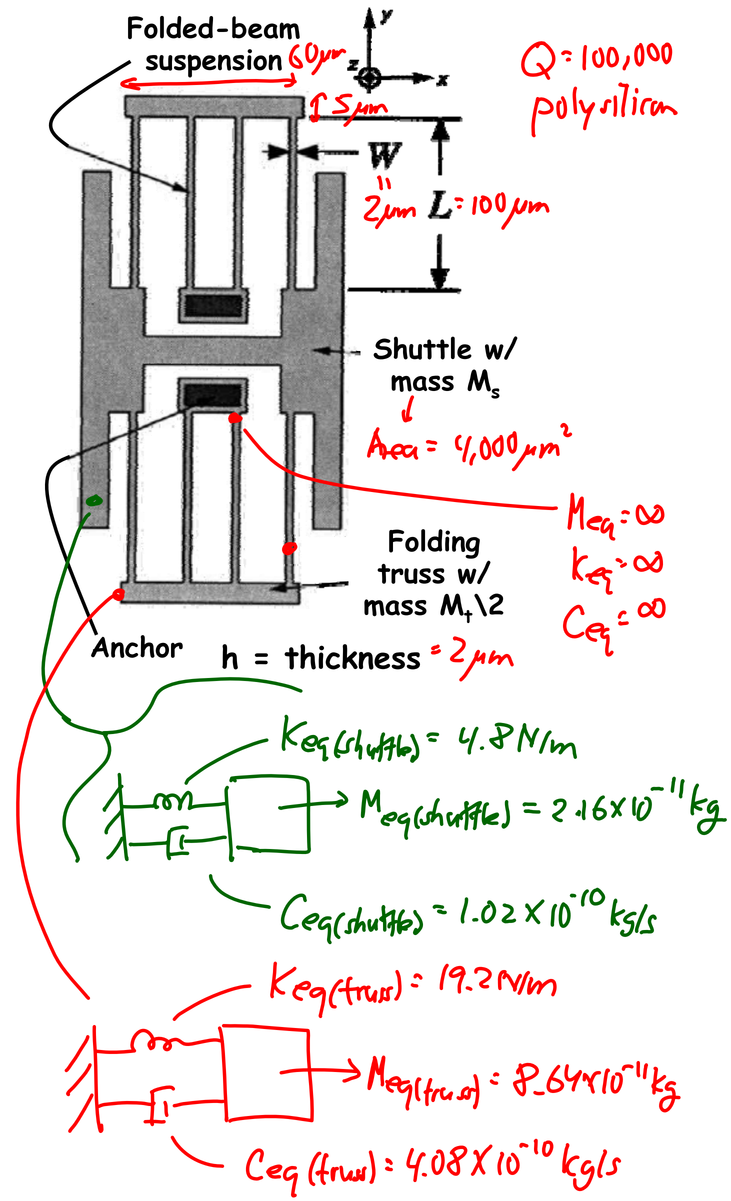
EE 245: Introduction to MEMS
Lecture 20: Equivalent Circuits I

- **Announcements**:
- Lecture schedule for the next 4 weeks:

Monday	Tuesday	Wednesday	Thursday	Friday
Nov. 2 Lecture 5-6:30 241 Cory	3 Midterm 3:30-5 Moffitt	4	5 Lecture 3:30-5 Moffitt	6
9 Lecture 5-6:30 241 Cory	10 Lecture 3:30-5 Moffitt	11	15 Lecture 3:30-5 Moffitt	13
16 Discuss 3:30-5 241 Cory	17	18	19	20
23 Lecture 5-6:30 241 Cory	24 Lecture 3:30-5 Moffitt	25	26	27

- HW#6 will be posted today
- Reading: Senturia, Chpt. 5
- Lecture Topics:
  - Lumped Mechanical Equivalent Circuits
  - Electromechanical Analogies
- Reading: Senturia, Chpt. 5, Chpt. 6
- Lecture Topics:
  - Energy Conserving Transducers
  - Parallel-Plate Capacitive Transducers

- **Last Time**:



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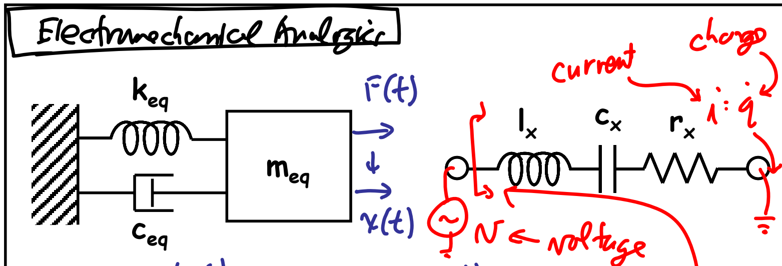
Electromechanical Analogies



$F(t) = F\cos(\omega t) \rightarrow x(t) = X\cos(\omega t)$ (off resonance)

Equation of Motion:

$$m_{eq}\ddot{x} + c_{eq}\dot{x} + k_{eq}x = F(t)$$

→ using phasor concepts:

$$F = j\omega m_{eq}\dot{x} + \frac{k_{eq}}{j\omega}\dot{x} + c_{eq}\dot{x}$$

$v(t) = V\cos\omega t \rightarrow i(t) = I\cos\omega t$

Impedance looking in:

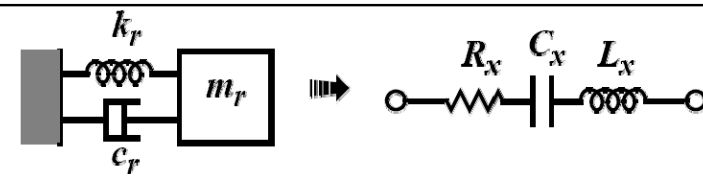
$$\frac{v}{i} = j\omega l_x + \frac{1}{j\omega c_x} + r_x$$

$$v = j\omega l_x i + \frac{(1/c_x)}{j\omega} i + r_x i$$

⇒ by analogy:

$F \rightarrow v$	$m_{eq} \rightarrow l_x$	$c_{eq} \rightarrow r_x$
$\dot{x} \rightarrow i$	$k_{eq} \rightarrow \frac{1}{c_x}$	

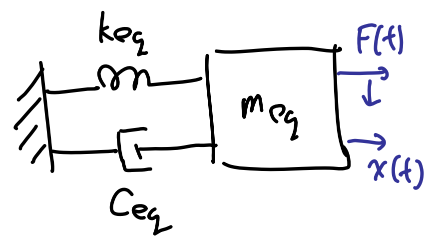
[Parameter Relationships in the Current Analogy]



- Mechanical-to-electrical correspondence in the current analogy:

Mechanical Variable	Electrical Variable
Damping, c	Resistance, R
Stiffness^{-1}, k^{-1}	Capacitance, C
Mass, m	Inductance, L
Force, f	Voltage, V
Velocity, v	Current, I

Bandpass Biquad Xfer Function



$$F = j\omega m_{eq}\dot{x} + \frac{k_{eq}}{j\omega}\dot{x} + c_{eq}\dot{x}$$

→ converting to full phasor form:

$$F = (j\omega)(j\omega X)m_{eq} + \frac{k_{eq}}{\cancel{j\omega}}(\cancel{j\omega}X) + c_{eq}(j\omega X)$$

$$\frac{X}{F}(j\omega) = \frac{1}{k_{eq}}\left[-\omega^2\frac{m_{eq}}{k_{eq}} + 1 + j\frac{c_{eq}\omega}{k_{eq}}\right]^{-1} \quad *$$

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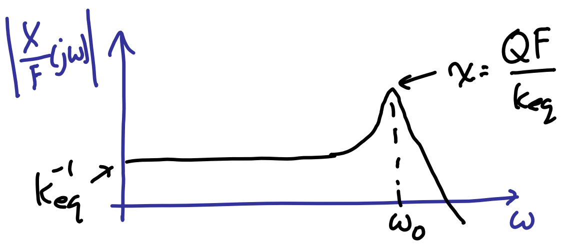
$$\left[\frac{k_{eq}}{m_{eq}} = \omega_0^2, \quad Q = \frac{m_{eq}\omega_0}{c_{eq}} = \frac{k_{eq}}{\omega_0 c_{eq}} \rightarrow \frac{k_{eq}}{c_{eq}} = Q\omega_0\right]$$

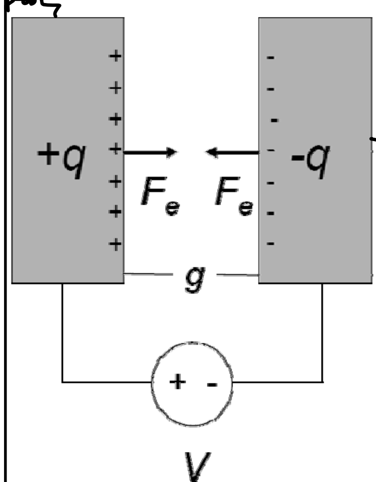
$$(*) \quad \frac{X}{F}(j\omega) = \frac{1}{k_{eq}}\left[-\left(\frac{\omega}{\omega_0}\right)^2 + 1 + j\frac{\omega}{Q\omega_0}\right]^{-1}$$

$$\Downarrow$$

$$\boxed{\frac{X}{F}(j\omega) = \frac{k_{eq}^{-1}}{1-\left(\frac{\omega}{\omega_0}\right)^2 + j\frac{\omega}{Q\omega_0}}}$$



Basic Physics of Electrostatic Actuation



Goal: Determine gap spacing g as a function of input variables

Note: Assume the plates are supported elastically

$\Downarrow$

1st: Determine the energy of the system.

What can I Δ to Δ the energy in this system?

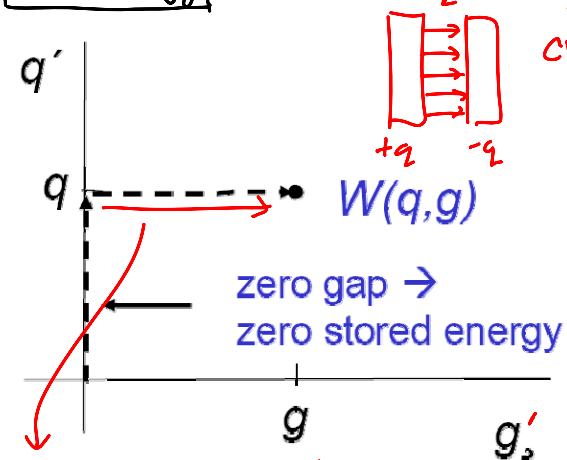
① change the charge q

② change the separation g

$$\Delta W(q,g) = V\Delta q + F_e \Delta g$$

$$dW = V dq + F_e dg$$

Stored Energy



$\mathcal{E} = \frac{q}{\epsilon A}$

↑ cross-sectional area

No change in charge: $dq = 0$

$$W = 0 + \int_0^g F_e \, dg'$$

$$F_e = \left(\frac{q}{2}\right)\mathcal{E} = \frac{1}{2}\frac{q^2}{\epsilon A} \quad \text{(independent of } g\text{)}$$

$$\therefore W = \int_0^g F_e \, dg' = F_e g' \Big|_0^g = F_e g$$

$$\boxed{W(g) = \frac{1}{2}\frac{q^2}{\epsilon A} g}$$

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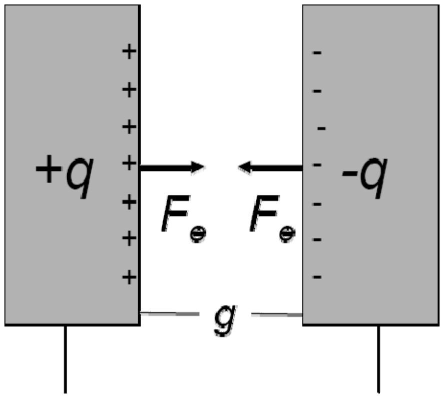
Charge-Control Case



From $dW = Vdq + F_e dg$:

⇒ Force is given by:

$$F_e = \left.\frac{\partial W(q,g)}{\partial g}\right|_{q=const.} = \frac{\partial}{\partial g}\left(\frac{1}{2}\frac{q^2}{\epsilon A}g\right)$$

$$\therefore \boxed{F_e = \frac{1}{2}\frac{q^2}{\epsilon A}}$$ ⇒ independent of gap spacing!

⇒ Voltage is given by

$$V = \left.\frac{\partial W(q,g)}{\partial q}\right|_{g=const.} = \frac{\partial}{\partial q}\left(\frac{1}{2}\frac{q^2}{\epsilon A}g\right) = \frac{qg}{\epsilon A}$$

$$\boxed{\therefore V = \frac{q}{C}}$$ ✓

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