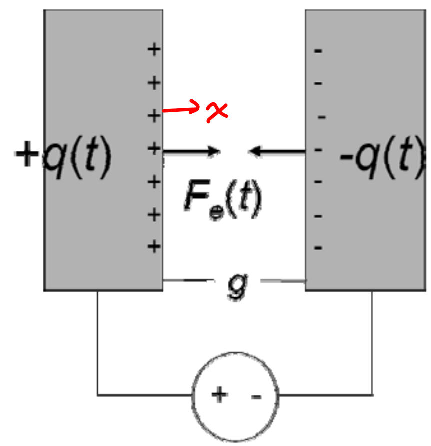
- **Announcements**:
- Lecture schedule for the next 3 weeks:

Monday	Tuesday	Wednesday	Thursday	Friday
9 Lecture 5-6:30 241 Cory	10 Lecture 3:30-5 Moffitt	11	15 Lecture 3:30-5 Moffitt	13
16 Discuss 3:30-5 241 Cory	17	18	19	20
23 Lecture 5-6:30 241 Cory	24 Lecture 3:30-5 Moffitt	25	26	27

- Reading: Senturia, Chpt. 5, Chpt. 6
- Lecture Topics:
  - Energy Conserving Transducers
    - Charge Control
    - Voltage Control
  - Parallel-Plate Capacitive Transducers
    - Linearizing Capacitive Actuators
    - Electrical Stiffness
  - Electrostatic Comb-Drive
    - 1st Order Analysis
    - 2nd Order Analysis

Advantages of Electrostatic Actuators:
- Nonlinear voltage-to-force transfer function
- Relatively weak compared with other transducers (e.g., piezoelectric), but things get better as dimensions scale

$\boxed{\text{Linearizing the Voltage-to-Force Xfer Function}}$



$$v(t) = V_P + v_i(t)$$

Bias (DC)   Signal (ac)

$$F_e(t) = \frac{\partial W'}{\partial x} = \frac{\partial}{\partial x}\left[\frac{1}{2} C\,(v(t))^2\right]$$

$$= \frac{1}{2}\frac{\partial C}{\partial x}[v(t)]^2 = \frac{1}{2}\frac{\partial C}{\partial x}[V_p + v_i(t)]^2$$

$$= \frac{1}{2}\left[V_p^2 + 2V_p v_i(t) + \cancel{[v_i(t)]^2}\right]\frac{\partial C}{\partial x}$$

$$[V_p >> v_i(t)] \Rightarrow F_e(t) = \underbrace{\frac{1}{2}V_P^2\frac{\partial C}{\partial x}}_{\text{DC offset}} + \underbrace{V_P\frac{\partial C}{\partial x}v_i(t)}_{\text{AC drive signal}}$$

$$\left[C_o = \frac{\epsilon A}{g_o}\right]$$

$$C(x) = \frac{\epsilon A}{g_0 - x} = C_0\left(1 - \frac{x}{g_0}\right)^{-1} \approx C_0\left(1 + \frac{x}{g_0}\right)$$

$[x \ll g_0]$

static plate-to-plate overlap capacitance

$$\therefore \frac{\partial C}{\partial x} = \frac{C_0}{g_0}$$

$$\Rightarrow F_e(t) = \frac{1}{2}\frac{C_0}{g_0}V_P^2 + V_P \frac{C_0}{g_0} v_i(t)$$

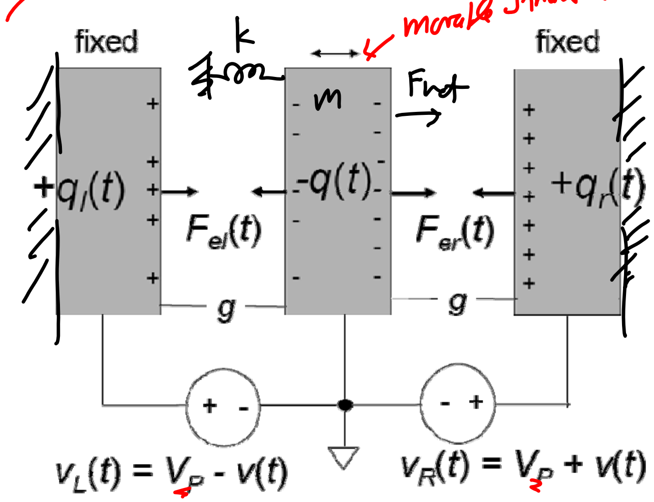
linear!

Can cancel by symmetry (later)

~const. for small amplitudes

This is small-signal analysis in the mechanical domain!

movable structure



$$F_{net}(t) = F_{er}(t) - F_{el}(t)$$

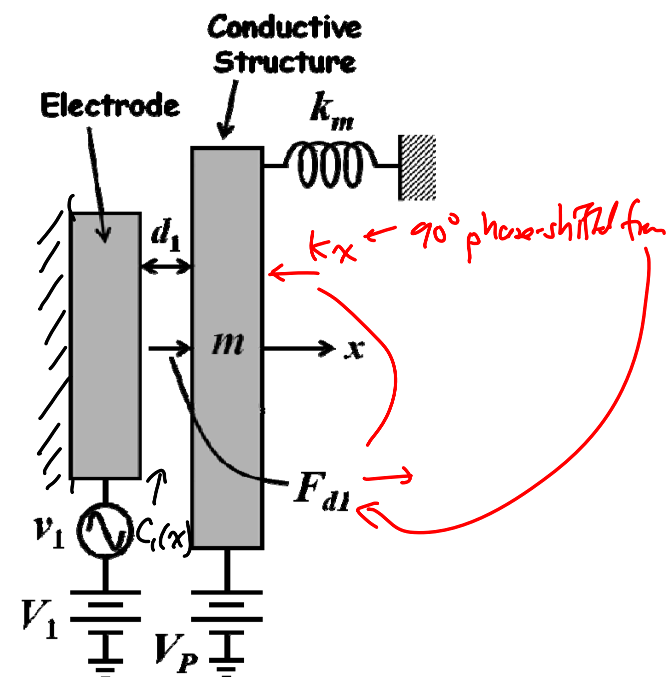
$$= \frac{1}{2}\frac{\partial C}{\partial x}\left\{[v_R(t)]^2 - [v_L(t)]^2\right\}$$

$$= \frac{1}{2}\frac{\partial C}{\partial x}\left\{\left(\cancel{V_P^2} + 2V_P v(t) + \cancel{[v(t)]^2}\right) - \left(\cancel{V_P^2} - 2V_P v(t) + \cancel{[v(t)]^2}\right)\right\}$$

$$\therefore \boxed{F_{net}(t) = 2V_P\frac{\partial C}{\partial x}v(t) = 2V_P\frac{C_0}{g_0}v(t)}$$

$\Rightarrow$ linear w/ $v(t)$! (for small signals, i.e., small displacements)

Nonlinearity still affects us!



More Complete Expression)

$$C_1(x) = \frac{\epsilon A}{d_1+x} = C_o\left(1+\frac{x}{d_1}\right)^{-1} \rightarrow \frac{\partial C_1}{\partial x} = -\frac{C_{o1}}{d_1}\left(1+\frac{x}{d_1}\right)^{-2}$$

[Expand the Taylor series further]

Before, took only this term ...

$$\frac{\partial C_1}{\partial x} = -\frac{C_{o1}}{d_1}\left(1+A_1x+A_2x^2+A_3x^3+\cdots\right)$$

where $A_1 = -\frac{2}{d_1},\ A_2 = \frac{3}{d_1^2},\ A_3 = -\frac{4}{d_1^3},\ \ldots$

$$F_{d1} = \frac{1}{2}\frac{\partial C_1}{\partial x}(V_P - V_1 - v_1)^2 = \frac{1}{2}\frac{\partial C}{\partial x}(V_{P1} - v_1)^2$$

[small displacements: $x \ll d_1$]

$v_1 = V_1\cos\omega t$

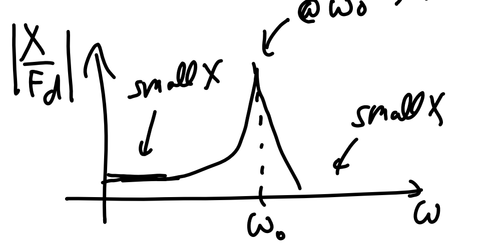
$$F_{d1} = \frac{1}{2}\left(-\frac{C_{o1}}{d_1}\right)(1+A_1x)\left(V_{P1}^2 - 2V_{P1}v_1 + v_1^2\right)$$

@ ω_o

$$= \frac{1}{2}\left(-\frac{C_{o1}}{d_1}\right)\left\{V_{P1}^2 - 2V_{P1}v_1 + v_1^2 + A_1V_{P1}^2x - 2A_1V_{P1}xv_1 + A_1xv_1^2\right\}$$

$x = X\sin\omega_o t$

Resonance:



@ resonance:

$$x = \frac{QF_{d1}}{jk} = \frac{Q}{jk}\frac{\partial C}{\partial x}V_{P1}v_1$$

j ← 90° phase shift

Thus:

$v_1 = |V_1|\cos\omega t \rightarrow x = |X|\sin\omega_o t$

90° phase shifted from v_1

Force terms only @ ω_o:

$$F_{d1}\big|_{\omega_o} = \underbrace{V_{P1}\frac{C_{o1}}{d_1}|V_1|\cos\omega_o t}_{\text{forcing term}} + V_{P1}^2\frac{C_{o1}}{d_1^2}|X|\sin\omega_o t$$

$k_e \rightarrow$ electrical stiffness

proportional to x

90°

Electrical Stiffness:

① A negative spring constant!

② Dependent on V_P

$$\boxed{k_e = V_{P1}^2\frac{C_{o1}}{d_1^2} = V_{P1}^2\frac{\epsilon A}{d_1^3}}$$

What does this do for us?

↳ it affects resonance freq.!

$$\omega_o = \sqrt{\frac{k_m}{m}}$$

k_m ← mechanical spring constant

m ← mass

↳ get this w/o V_P

Apply V_P:

$$\omega_o' = \sqrt{\frac{k}{m}} = \sqrt{\frac{k_m - k_e}{m}} = \sqrt{\frac{k_m}{m}}\left(1 - \frac{k_e}{k_m}\right)^{1/2}$$

$$\boxed{\omega_o' = \omega_o\left(1 - \frac{V_{P1}^2}{k_m}\frac{\epsilon A}{d_1^3}\right)^{1/2}}$$