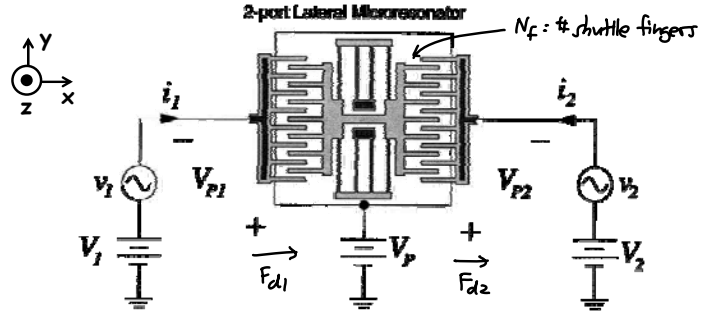
Typical Drive & Sense Configuration



Simple Analysis:

$$F_{d1} = \frac{1}{2}\frac{\partial C_1}{\partial x}(v_1 - V_{P1})^2 = \frac{1}{2}\left(\frac{\epsilon_0 h}{d_1}\right)(v_1^2 - 2V_{P1}v_1 + V_{P1}^2)(2N_f)$$

$$F_{d2} = \frac{1}{2}\frac{\partial C_2}{\partial x}(v_2 - V_{P2})^2 = \frac{1}{2}\left(\frac{\epsilon_0 h}{d_2}\right)(v_2^2 - 2V_{P2}v_2 + V_{P2}^2)(2N_f)$$

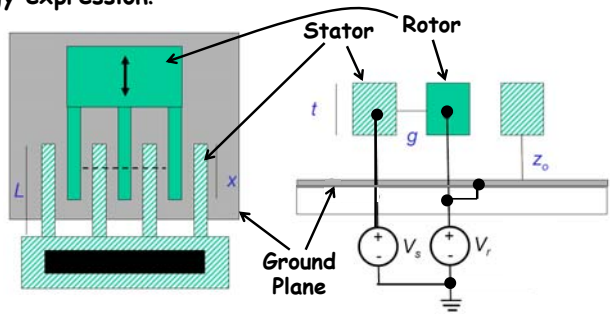
$$\therefore F_{net} = F_{d1} + F_{d2} = \frac{1}{2}\left(\frac{\epsilon_0 h}{d}\right)(v_2^2 - v_1^2 - 2(V_{P2}v_2 - V_{P1}v_1) + V_{P2}^2 - V_{P1}^2)(2N_f)$$

For $V_1 = V_2$, $v_1 = -v_2$

$$F_{net} = 2(2N_f)\left(\frac{\epsilon_0 h}{d}\right)V_{P1}v_1$$

Comb-Drive Force Equation (2nd Pass)

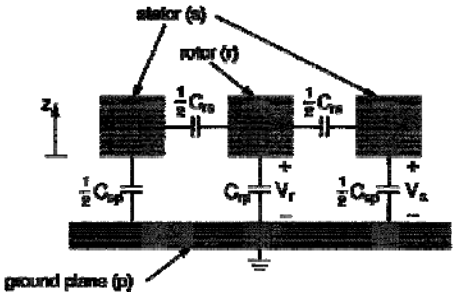
- In our 1st pass, we neglected
  - Fringing fields
  - Parallel-plate capacitance between stator and rotor
  - Capacitance to the substrate
- All of these capacitors must be included when evaluating the energy expression!



Comb-Drive Force With Ground Plane Correction

- Finger displacement changes not only the capacitance between stator and rotor, but also between these structures and the ground plane → modifies the capacitive energy

$$F_{e,x} = \frac{\partial W'}{\partial x} = \frac{1}{2}\frac{dC_{sp}}{dx}V_s^2 + \frac{1}{2}\frac{dC_{rp}}{dx}V_r^2 + \frac{1}{2}\frac{dC_{rs}}{dx}(V_s - V_r)^2$$



[Gary Fedder, Ph.D., UC Berkeley, 1994]

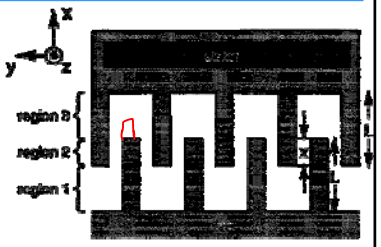
Capacitance Expressions

- Case: $V_r = V_P = 0V$
- C_{sp} depends on whether or not fingers are engaged

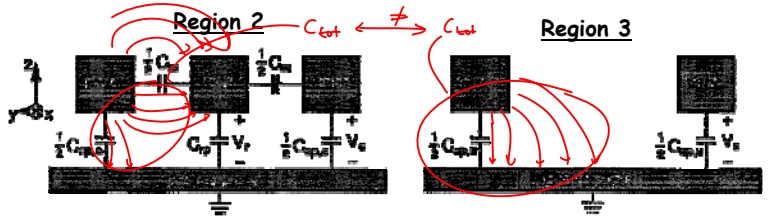
$$C_{sp} = N[C'_{sp,e}x + C'_{sp,u}(L - x)]$$

$$C_{rs} = NC'_{rs}x$$

Capacitance per unit length



Region 2 $C_{tot} \neq C_{tot}$ Region 3



[Gary Fedder, Ph.D., UC Berkeley, 1994]

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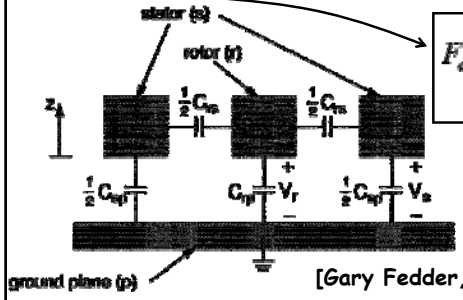


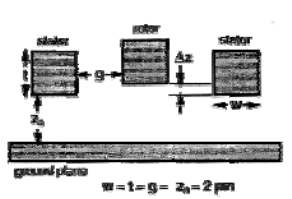
$$F_{e,x} = \frac{N}{2}(C'_{rs} + C'_{sp,e} - C'_{sp,u})V_s^2$$

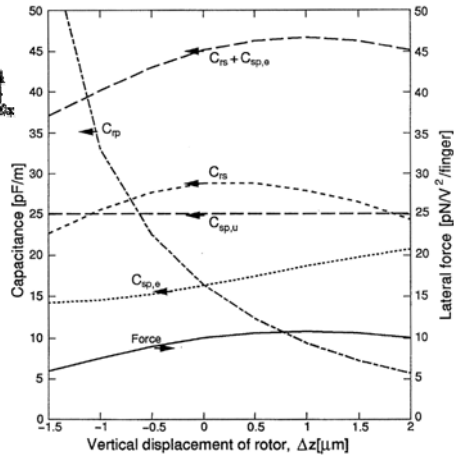
(for $V_r = V_p = 0$)

[Gary Fedder, Ph.D., UC Berkeley, 1994]

Simulate to Get Capacitors → Force

- Below: 2D finite element simulation

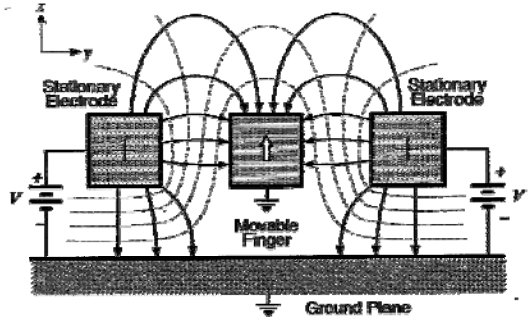




$$F_{e,x} = \frac{N}{2}(C'_{rs} + C'_{sp,e} - C'_{sp,u})V_s^2$$

20-40% reduction of $F_{e,x}$

Vertical Force (Levitation)

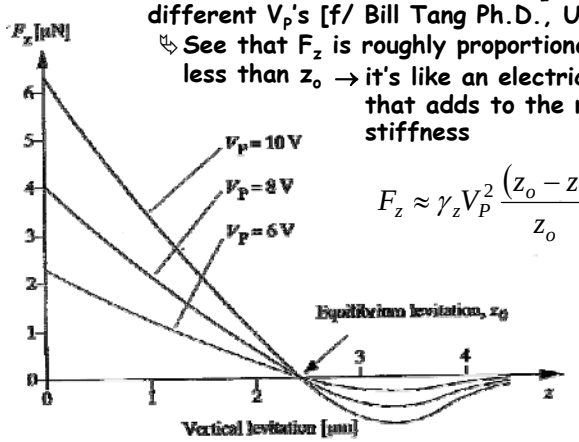


$$F_{e,z} = \frac{\partial W'}{\partial z} = \frac{1}{2}\frac{dC_{sp}}{dz}V_s^2 + \frac{1}{2}\frac{dC_{rp}}{dz}V_r^2 + \frac{1}{2}\frac{dC_{rs}}{dz}(V_s - V_r)^2$$

- For V_r = 0V (as shown): $F_{e,z} = \frac{1}{2}Nx\left[\frac{d(C'_{sp,e} + C'_{rs})}{dz}\right]V_s^2$

Simulated Levitation Force

- Below: simulated vertical force F_z vs. z at different V_p's [f/ Bill Tang Ph.D., UCB, 1990]
  - See that F_z is roughly proportional to –z for z less than z_o → it's like an electrical stiffness that adds to the mechanical stiffness



$$F_z \approx \gamma_z V_P^2 \frac{(z_o - z)}{z_o} = k_e(z_o - z)$$

Electrical Stiffness

Vertical Resonance Frequency

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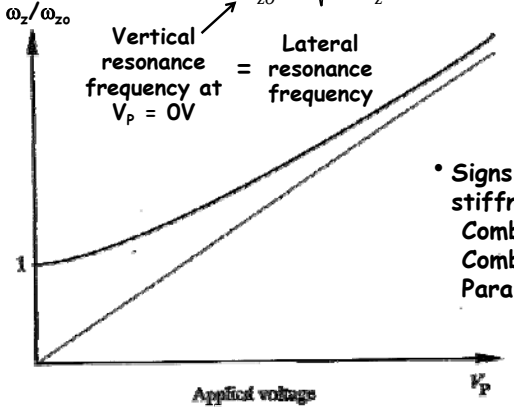
Vertical resonance frequency → $\frac{\omega_z}{\omega_{zo}} = \sqrt{\frac{k_z + k_e}{k_z}}$ where $k_e = \left(\frac{\gamma_z}{z_o}\right) V^2$

Vertical resonance frequency at V_P = 0V = Lateral resonance frequency



- Signs of electrical stiffnesses in MEMS:
  - Comb (x-axis) → $k_e = 0$
  - Comb (z-axis) → $k_e > 0$
  - Parallel Plate → $k_e < 0$

Suppressing Levitation

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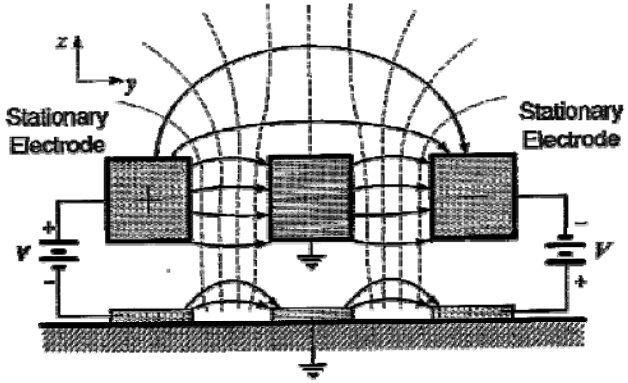

- Pattern ground plane polysilicon into differentially excited electrodes to minimize field lines terminating on top of comb
- Penalty: x-axis force is reduced

Force of Comb-Drive vs. Parallel-Plate

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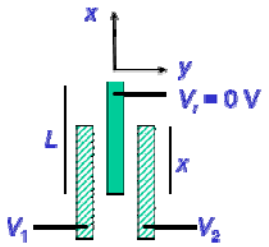

gap = g = 1 μm,
thickness = t = 2 μm
finger length = L = 100 μm
overlap length x = 75 μm

- Comb drive (x-direction)
  - $V_1 = V_2 = V_S = 1V$

$$F_{e,x} = \frac{\varepsilon_o t}{g} V_s^2$$

- Differential Parallel-Plate (y-direction)
  - $V_1 = 0V$, $V_2 = 1V$

$$F_{e,y} = \frac{1}{2}\frac{\varepsilon_o t x}{g^2} V_2^2$$

$$\frac{F_{e,y}}{F_{e,x}} = \frac{\frac{\varepsilon_o t x}{g^2} V_2^2}{\frac{2\varepsilon_o t}{g} V_s^2} = \frac{1}{2}\frac{x}{g}$$

Parallel-plate generates a much larger force; but at the cost of linearity